Article

A Derivation of GravitoElectroMagnetic Proca-type Equations in Fractional Space

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Abstract

In a recent paper, M. Zubair *et al*. described a novel approach for fractional space generalization of the differential electromagnetic equations. A new form of vector differential operator Del and its related differential operators are formulated in fractional space. Using these modified vector differential operators, the classical Maxwell equations have been worked out for fractal media. In the meantime, there are other papers discussing fractional Maxwell equations. However, so far there is no derivation of Proca equations and GravitoElectroMagnetic Proca-type equations in fractional space. In this paper, I present for the first time a derivation of GravitoElectroMagnetic (GEM) Proca-type equations in fractional space. Considering that Proca equations may be used to explain some electromagnetic effect in superconductor, I suggest that fractional GEM Proca-type equations may be used to explain some gravitomagnetic effects of superconductor for fractal media. It is hoped that this paper may stimulate further investigations and experiments on gravitomagnetic effects.

Key Words: fractional space, Proca equation, gravitoelectromagnetic, GEM, Maxwell equations.

1. Introduction

There have been much interests to study different physical phenomenon in fractional dimensional space during the last few decades. It is also important to mention that the experimental measurement of the dimension of real world is 3 ± 10^{-6}, not exactly 3 [1].

In a recent paper, M. Zubair et al. described a novel approach for fractional space generalization of the differential electromagnetic equations [1].A new form of vector differential operator Del, and its related differential operators, is formulated in fractional space. Using these modified vector differential operators, the classical Maxwell equations have been worked out for fractal media. In the meantime, there are other papers discussing fractional Maxwell equations [2-3]. However, so far there is no derivation of Proca equations and GravitoElectroMagnetic Proca-type equations in fractional space. Therefore in this paper I present for the first time a derivation of GravitoElectroMagnetic (GEM) Proca-type equations in fractional space. Considering that Proca equations may be used to explain some electromagnetic effect in superconductor[4], then fractional GEM Proca-type equations may be expected to explain some gravitomagnetic effects of superconductor for fractal media.[5] It is our hope, that this paper may stimulate further investigation and experiments in particular with respect to gravitomagnetic effects.

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The present paper is intended to be a follow-up paper of my preceding paper, reviewing Shpenkov's interpretation of classical wave equation and its role to explain periodic table of elements and other phenomena [16].

2. A review of previous result - Maxwell equations in fractional space

I will not re-derive Maxwell equations here. For a good reference on *Classical Electrodynamics*, see for example Julian Schwinger et al.'s book [9]. Penrose also discusses Maxwell equations shortly in his book:*The Road to Reality*[10].

Zubair et al. were able to write a differential form of Maxwell equations in far-field region in the fractional space as follows [1]:

$$div_D D = \rho_v , \tag{1}$$

$$div_D B = 0 , \tag{2}$$

$$curl_D E = -\frac{\partial B}{\partial t} , \tag{3}$$

$$curl_D H = J + \frac{\partial D}{\partial t} , \tag{4}$$

and the continuity equation in fractional space as:

$$div_D J = -\frac{\partial \rho_v}{\partial t} , \tag{5}$$

where div_D and $curl_D$ are defined as follows [1]:

$$div_D F = \nabla_D \cdot F = \frac{\partial F_x}{\partial x} + \frac{1}{2}\frac{(\alpha_1 - 1)F_x}{x} + \frac{\partial F_y}{\partial y} + \frac{1}{2}\frac{(\alpha_2 - 1)F_y}{y} + \frac{\partial F_z}{\partial z} + \frac{1}{2}\frac{(\alpha_3 - 1)F_z}{z} , \tag{6}$$

$$curl_D F = \nabla_D \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} + \frac{1}{2}\frac{\alpha_1 - 1}{x} & \frac{\partial}{\partial y} + \frac{1}{2}\frac{\alpha_2 - 1}{y} & \frac{\partial}{\partial z} + \frac{1}{2}\frac{\alpha_3 - 1}{z} \\ F_x & F_y & F_z \end{vmatrix} , \tag{7}$$

where parameters ($0 < \alpha_1 \le 1$, $0 < \alpha_2 \le 1$ and $0 < \alpha_3 \le 1$) are used to describe the measure distribution of space where each one is acting independently on a single coordinate and the total dimension of the system is $D = \alpha_1 + \alpha_2 + \alpha_3$. [1]

3. Proca Equations in Fractional Space

Proca equations can be considered as an extension of Maxwell equations, and they have been derived in various ways, see for instance [4, 6, 7]. It can be shown that Proca equations can be derived from first principles [6], and also that Proca equations may have link with Klein-Gordon equation [7]. However, in this paper I will not attempt to re-derive Proca equations. Instead, I will use Proca equations as described in [6]. Then I will derive the Proca equations in fractional space, in accordance with Zubair et al.'s approach as outlined above[1].

According to Blackledge, Proca equations can be written as follows [7]

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2 \phi, \tag{8}$$

$$\nabla \cdot \vec{B} = 0, \tag{9}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{10}$$

$$\nabla \times \vec{B} = \mu_0 j + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \kappa^2 \vec{A}, \tag{11}$$

where:

$$\nabla \phi = -\frac{\partial \vec{A}}{\partial t} - \vec{E}, \tag{12}$$

$$\vec{B} = \nabla \times \vec{A}, \tag{13}$$

$$\kappa = \frac{mc_0}{\hbar}. \tag{14}$$

Therefore, by using the definitions in equation (6) and (7), we can arrive at Proca equations in fractional space from (8) through (13), as follows:

$$div_D \vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2 \phi, \tag{15}$$

$$div_D \vec{B} = 0, \tag{16}$$

$$curl_D \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{17}$$

$$curl_D \vec{B} = \mu_0 j + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \kappa^2 \vec{A}, \tag{18}$$

where:

$$\nabla_D \phi = -\frac{\partial \vec{A}}{\partial t} - \vec{E}\,, \tag{19}$$

$$\vec{B} = curl_D \vec{A}\,, \tag{20}$$

and Del operator ∇_D can be defined as follows [1]:

$$\nabla_D = \left(\frac{\partial}{\partial x} + \frac{1}{2}\frac{\alpha_1 - 1}{x}\right)\hat{x} + \left(\frac{\partial}{\partial y} + \frac{1}{2}\frac{\alpha_2 - 1}{y}\right)\hat{y} + \left(\frac{\partial}{\partial z} + \frac{1}{2}\frac{\alpha_3 - 1}{z}\right)\hat{z}\,. \tag{21}$$

To my best knowledge so far, the above expression of Proca equations in fractional space has not been proposed elsewhere before.

Since according to Blackledge, the Proca equations can be viewed as a unified wavefield model of electromagnetic phenomena [7], then we can also regard the Proca equations in fractional space as further generalization of this unified wavefield picture.

4. GravitoElectroMagnetic (GEM) Proca-type Equations in Fractional Space

The term GravitoElectroMagnetism (GEM) refers to the formal analogies between Newton's law of gravitation and Coulomb's law of electricity. The theoretical analogy between the electromagnetic and the gravitational field equations has been first suggested by Heaviside in 1893, see for example [8]. The fields of GEM can be defined in close analogy with the classical electrodynamics. Therefore, if we can consider Proca equations as generalization and extension of Maxwell equations, then we can also find **GravitoElectroMagnetic** Proca-type equations.

In accordance with Demir [8], the GravitoElectroMagnetic Proca-type equations can be expressed straightforward from their electromagnetic counterpart as follows (Here I use Demir's notations instead of Blackledge's notations):

$$\nabla \cdot \vec{E}_g = -\rho_e - \kappa_g^2 \phi\,, \tag{22}$$

$$\nabla \cdot \vec{H}_g = 0\,, \tag{23}$$

$$\nabla \times \vec{E}_g = -\frac{\partial \vec{H}_g}{\partial t}\,, \tag{24}$$

$$\nabla \times \vec{H}_g = -J_g^e + \frac{\partial \vec{E}_g}{\partial t} + \kappa_g{}^2 \vec{A}_g^e\,, \tag{25}$$

where the fields E_g and H_g can be defined in terms of the potentials just as given in equation (12) and (13), and the term κ_g represents the inverse Compton wavelength of the graviton, [8]

$$\kappa_g = \frac{m_g c}{\hbar}. \tag{26}$$

Now I will present the Proca-type equations for GEM in fractional space using the same method as described in the previous section and equation (6) and (7), which can be written as follows:

$$div_D \vec{E}_g = -\rho_e - \kappa_g^2 \phi, \tag{27}$$

$$div_D \vec{H}_g = 0, \tag{28}$$

$$curl_D \vec{E}_g = -\frac{\partial \vec{H}_g}{\partial t}, \tag{29}$$

$$curl_D \vec{H}_g = -J_g^e + \frac{\partial \vec{E}_g}{\partial t} + \kappa_g{}^2 \vec{A}_g^e, \tag{30}$$

To the best of my knowledge, the above expression of Proca-type equations for GEM in fractional space has not been proposed elsewhere before.

5. Fractional Helmholtz equation and solution of classical wave equation in fractional space

It is worth noting here that Zubair et al. also wrote Helmholtz equations in fractional space for E and H field as a consequence of Maxwell equations in fractional space, as follows [1]:

$$\nabla_D^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0, \tag{31}$$

$$\nabla_D^2 H - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0. \tag{32}$$

In another paper, Zubair, Mughal & Naqvi give a solution of this kind of Helmholtz equation in fractional space [11]. The Laplacian operator in D-dimensional fractional space is defined as follows:

$$\nabla_D^2 = \frac{\partial^2}{\partial x^2} + \frac{\alpha_1 - 1}{x}\frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} + \frac{\alpha_2 - 1}{y}\frac{\partial}{\partial y} + \frac{\partial^2}{\partial z^2} + \frac{\alpha_3 - 1}{z}\frac{\partial}{\partial z}. \tag{33}$$

Then they derive a solution of equation (31) with the help of Bessel equation [11].

It is also interesting to note here that Shpenkov has suggested that a classical wave equation - which is essentially the same with Helmholtz equation- can be used to derive a periodic table of elements which is close to Mendeleyev's periodic table [12-13]. This result is in contradiction to spherical solution of Schrodinger equation which does not explain any atom, except perhaps hydrogen [14-15]. Therefore, it seems worth to study what the effect of an extension of classical wave equation in fractional space to the structure of atoms and molecules is. A review of Shpenkov's interpretation and use of classical wave equation can be found here [16].

Since the classical wave equation as described by Shpenkov is the same with the equation of vibrating string in 3-dimension [16], it seems possible to compare the solution of equation (31) with solution of *fractal vibrating string*. A few papers have been written discussing this fractal vibrating string in detail, which can be found elsewhere [17-18]. It seems worthwhile to study spherical solution of this fractal vibrating string equation in order to verify Shpenkov's results, including his periodic table of elements. One possible way to find such a solution of fractal vibrating string is by obtaining numerical solution of such an equation by a method of converting fractional differential equation to partial differential equation as proposed by He & Li [19]. After a partial differential equation (PDE) is obtained, it seems not so difficult to find its numerical solution with computer algebra packages like Mathematica, Maple. Maxima, or MatLab.

Concluding remarks

In a recent paper, M. Zubair et al. described a novel approach for fractional space generalization of the differential electromagnetic equations. A new form of vector differential operator Del, and its related differential operators, is formulated in fractional space. Using these modified vector differential operators, the classical Maxwell equations have been worked out for fractal media. In the meantime, there are other papers discussing fractional Maxwell equations. However, so far there is no derivation of Proca equations and GravitoElectroMagnetic Proca-type equations in fractional space. Therefore in this paper I present for the first time a derivation of GravitoElectroMagnetic (GEM) Proca-type equations in fractional space. Considering that Proca equations may be used to explain some electromagnetic effect in superconductor, then fractional GEM Proca-type equations may be expected to explain some gravitomagnetic effects of superconductor for fractal media. It is our hope, that this paper may stimulate further investigation and experiments in particular with respect to gravitomagnetic effects.

I also propose to investigate further the spherical solution of Helmholtz equation corresponding to Proca-type equations for GEM in fractional space. This kind of investigation may be useful for the study of gravitomagnetic effect and gravitational wave.

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