

Origin of Gravitation Mass in Lorentz-invariant Gravitation Theory (Part I)

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Abstract

To construct a Lorentz-invariant gravitation theory (LIGT), the origin of the gravitational mass (charge) is a fundamental problem. This article is devoted to the solution of this task. The result of our analysis allows us to limit the number of possible theories of gravitation and ground the axioms of LIGT that will be done in the following articles.

Part I of this article includes: 1. Statement of the problem; 2. The theories of mass generation; 3. Quantum wave theory of matter; 4. Axiomatics of the non-linear theory of elementary particles; 5. The photon theory; and 6. Massive intermediate boson theory of QED and the mechanism of mass generation in NTEP.

Keywords: Lorentz-invariant gravitation theory, non-linear quantum theory, mass origin.

1. Statement of the problem

According to the modern theory of gravity - general relativity, mass is a source of gravity, as well as the energy and momentum of the field, which, after being divided by the square of the speed of light, are identical to the distributed mass and mass flow. In this way, in the general case, mass can be considered as a cause of gravity. Mass as a source of gravitation is called gravitational mass or gravitational charge. Currently, the origin of the gravitational mass is unknown. But we know that it is equal with great precision to inertial mass, which appears in the laws of motion in mechanics. Thus, if we find out the origin of inertial mass, we can conclude that gravitational mass has the same origin. The question now is, what do we know about the origin of inertial mass?

2. The theories of mass generation

Historically there are two theories of generation of inertial mass, which have a serious experimental basis. The first of them, created by J.J. Thomson and H. Lorentz (1881 - 1926), lies entirely in the field of classical electromagnetic theory. According to this theory, the inertial mass has electromagnetic origin. Unfortunately, attempts to apply this theory to quantum theory has not been undertaken. Nevertheless, there is still no evidence that the inertial mass is not fully electromagnetic (Feynman, 1964a):

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There is definite experimental evidence of the existence of electromagnetic inertia – there is evidence that the mass of charged particles is electromagnetic in origin...

So the electromagnetic mass seems to be in general agreement with electromagnetic theory, if we stop our integrals of the field energy at the same radius obtained by these other methods. That's why we believe that the differences do represent electromagnetic mass... So we come back again to the original idea of Lorentz - may be all the mass of an electron is purely electromagnetic, maybe the whole 0.511 MeV is due to electrodynamics. Is it or isn't it? We haven't got a theory, so we cannot say.

The second mass theory is the, so-called, Higgs mechanism of the Standard Model theory (SM) (Quigg, 2007; Dawson, 1999). Unfortunately, the results of the CM can not be used in the gravitation theory of any whatsoever type. In fact, the Higgs mechanism, under certain assumptions, can describe only the generation of masses of fundamental elementary particles: intermediate bosons, leptons and quarks. But as we know (Quigg, 2007), more than 98.5% of the visible mass in the Universe is composed by the non-fundamental (composite) particles: protons, neutrons and other hadrons. Therefore we can not rely on this theory to describe the generation of gravitational field. Between the Higgs mechanism and the theory of mass of Thomson -Lorentz there are some parallels in the interpretation of results, but mathematically they have nothing in common.

Another theory of mass generation also exists. It is mathematically similar to the CM, but ideologically it is the development of the theory of Thomson-Lorentz on a quantum-field basis. This is the theory of mass generation in the framework of nonlinear quantum theory of elementary particles (in short, NTEP), which is the development of the D. Ivanenko and W. Heisenberg non-linear approach to the description of elementary particles. The mechanism of mass generation in NTEP is uniform for all elementary particles and fields. That is why we will rely on it.

NTEP has the following differences from the SM:

1) As we know (Quigg, 2007), the CM is constructed as a heuristic theory: by the trial and error method. At the basis of finding suitable mathematical description forms the various conservation laws (symmetry) and their violation are widely used here.

NTEP is constructed as a strictly axiomatic theory. All its results follow logically from the selected axioms without additional hypotheses. Symmetries themselves and their violation are consequence of the theory, but not its foundation.

2) Another major difference between the SM and NTEP is closely related to the interpretation of the wave functions. The existence of two possible choices of wave functions is known for a long time. This fact is noted, for example, by Enrico Fermi (Fermi, 1952) and R. Feynman (Feynman, 1964b):

There are many changes in what concepts are important when we go from classical to quantum mechanics... In particular, the force concept gradually fades away, while the concepts of energy and momentum become of paramount importance...The momenta and energies, which determine the phases of wave functions, are therefore the important

quantities in quantum mechanics... It is, in fact, just because momentum and energy play a central role in QM that [vector and scalar potentials] \vec{A} and φ provide the most direct way of introducing electromagnetic effects into quantum description.

In the SM, the wave functions of the gauge bosons (e.g., photons or intermediate bosons) are 4-potentials; in the same time the wave functions of the spinor particles do not a physical meaning. In NTEP the wave functions of all particles are the vectors of field strengths of the corresponding particles.

Note, that the choice of the wave function of the gauge bosons (in particular, photons) was known in the early time of development of the theory of elementary particles (Fermi, 1952; Akhiezer and Berestetskii, 1965; Levich, etc, 1973). But later, preference was given to potentials. In NTEP it is shown that the use of field strengths as wave functions greatly simplifies the calculations, and also makes clear the physical meaning of equations and mathematical calculations.

3) The most important assumption in the SM is local gauge (or phase) symmetry: symmetry with respect to local rotation of the wave functions in the interior space of the particles. At the same time, to achieve this symmetry it is necessary to enter (pick) manually the additional (compensating) terms, related to the characteristics of the particles. These terms define the interaction between the particles.

Instead, in NTEP postulate of rotation of the particle field (wave function) is introduced. This postulate is the source of particle self-action. The additional terms appear automatically by means of transport of the field vectors along a curved path. These terms disclose a simple physical meaning.

4) The CM is based on the hypothesis that all the particles at the time of generation of the Universe were massless. According to the Higgs mechanism, the appearance of a mass of fundamental particles requires their interaction with some physical vacuum - the Higgs vacuum. Violation of the initial gauge symmetry of this vacuum allows all of them, except for the photon, to acquire masses.

The NTEP comes from the existing experimental fact of the massless photon and massiveness of all other particles in the free state (however, in NTEP it is shown that fundamental particles can be considered as massless in the bound state, and only at the time of transition to a free state, they acquire mass). And, by doing so, a symmetry breaking of the initial field occurs, but the relationship of this field with the vacuum is veiled and is not explicitly used.

Next we will very briefly consider the mechanism of mass generation in NTEP, which we conventionally call "quantum-wave theory of the masses" (for details, see (Kyriakos, 2009) and search the later publications in "Prespacetime Journal" <http://www.prespacetime.com/index.php/index/search/search> under the same name).

3. Quantum wave theory of matter

3.1 Energy and momentum of electromagnetic wave (photon)

The fact that EM wave has an energy and a momentum, it was discovered already into the 19th century. The EM wave presses the metallic wall, and also it can revolve a light rotator. By this we can assume that EM wave (photon) has a mass.

For the time average of the pressure of the train of EM waves with area s and length l , the following expression (Becker, 1982) is obtained: $P = \frac{1}{8\pi} (\vec{E}^2 + \vec{H}^2) = u$, where u is the energy density of EM wave. The important dependence between energy and momentum of wave is already included in this equation. The total momentum, transmitted from EM train to wall will be equal to: $p = u \cdot s \cdot t$, where $t = l/c$ is the time of action of train. Thus, the transmitted momentum is equal to: $p = u \cdot s \cdot l/c$. Since the numerator $u \cdot s \cdot l = \varepsilon$ is the energy of train, we obtain $p = \varepsilon/c$. If we assign to EM wave a mass m' , then it is possible to consider that $p = m'c$. In that case we obtain $\varepsilon = m'c^2$ - the known relationship of Einstein.

Nevertheless, later it was proven that photon is a mass-free particle in the sense that its rest mass is equal to zero. But if we interpret the collision of EM wave with the wall as the stoppage of EM wave, then it is possible to say that the “stopped” photon acquires mass m' .

This result led, evidently, to a study of other methods of the “stoppage” of EM waves for the purpose of understanding the origin of mechanical mass of the material bodies.

3.2 “The Mass of a Box Full of Light”

“The experimental confirmation of the pressure of light in 1901 led to new theoretical work. In 1906, Max Abraham computed the pressure produced by radiation upon a moving surface, when the beam of light reaches the surface in a mirror in any angle. Starting from Abraham's results, Friedrich Hasenoehrl (1876-1916) studied the dynamics of a box full of radiation.

Imagine a cubic box with perfectly reflecting internal surfaces, full of light. When the box is at rest, the radiation produces equal forces upon all those surfaces, Now suppose that the box is accelerated, in such a way that one of its surfaces moves in the x direction. It is possible to prove that, when the radiation inside the box strikes this surface, the pressure will be smaller, and when it strikes the opposite surface, the pressure will be greater, than in the case when the box is at rest (or in uniform motion). Therefore, the radiation inside the box will produce a resultant force against the motion of the box. So, to accelerate a box full of light requires a greater force than to accelerate the same box without light. In other words, the radiation increases the inertia of the box. In the case when the radiation inside the box is isotropic, there is a very simple relation between its total energy E and its contribution m to the inertia of the box (Hasenoehrl, 1906): $m = 4\varepsilon/3c^2$. Note that here, as in the theory of the classical electron, there appears a numerical factor $4/3$.

Hasenoehrl also computed the change of the radiation energy as the box was accelerated. He proved that the total radiation energy would be a function of the speed of the box. Therefore, when the box is

accelerated, part of the work done by the external forces is transformed into the extra radiation energy. Since the inertia of the radiation is proportional to its energy, and since this energy increases with the speed of the box, the inertia of the box will increase with its speed. Of course, if the internal temperature of the box were increased, the radiation energy would augment, and the inertia of the box would also increase. Therefore, Hasenohrl stated that the mass of a body depends on its kinetic energy and temperature” (The authors, 2006).

By Pauli (Pauli, 1958, p.138) “The case of black-body radiation in a moving cavity is of historical interest, since it can be treated entirely on the basis of electrodynamics, without relativity. When this is done, one comes to the inevitable conclusion that a momentum, and thus also an inertial mass, must be ascribed to the moving radiation, energy. It is of interest that this result should have been found by Hasenohrl already before the theory of relativity had been formulated”.

3.3 A “Box Full of Light” as massive body

Since the box full of light has energy and mass, corresponding to it, let us conditionally name the totality of EM waves in a box as ‘EM-particle’. From the foresaid above it is obvious that the mass of ‘EM-particle’, calculated according to Lorentz’s theory, will also have a coefficient of 4/3 like the mass of classical electron. Obviously, upon consideration of the stresses of Poincare we will obtain the coefficient one. The stresses of Poincare were introduced for the stabilization of the electrostatic field of classical electron. In the case in question the stability exists due to interaction of EM wave with the walls of the box. These interactions play in this case the role of the stresses of Poincare, which ensure the stability of ‘EM particle’. Naturally, if we take into account the presence of these stresses, we will also obtain the coefficient one (of course this result will also appear, if we use Einstein’s approach).

Really, with the perpendicular fall of EM waves on the walls of the box the stress is pressure. With inclined fall the components of stress will formally consist both of pressures and tangent stresses (as a result of the resolution of momentum on perpendicular and tangential components). The stress tensor of Maxwell (and generally, continuous medium tensor) consists precisely of such components (Kyriakos, 2013). In this example the origin of the stresses are not mechanical: EM waves interact with the electrons of the wall atoms by means of EM Lorentz’s forces. Nevertheless, these stresses are external with respect to EM waves in the box, i.e., they are not organized by the EM waves themselves. This is one of many examples when electromagnetism can be interpret mechanically.

The question arises: are such conditions possible, when EM wave can ensure themselves the stability of ‘EM-particle’ without the presence of external actions? In this case we will actually have a massive “particle”, generated by EM waves. Obviously, this case can be realized only as a result of the self-interaction of parts of EM waves. This means that the equation of EM wave-particle must be nonlinear.

We can improve our model for the purpose to do approach the quantum field theory. Let us select a box with mirror walls of the size of the order of a wavelength λ . If we consider resonance conditions, the box itself will select the appropriate wavelength. This corresponds to the case when we placed into this box one photon. In the case of quantum theory we can speak about the photon in a cell of

phase size. If we ignore the presence of walls, it is possible to consider photon in the box as particle. This particle possesses spin one and mass, determined by its energy $m' = \varepsilon/c^2 = \hbar/\lambda c$. In other words, we have a model of the neutral massive boson, similar to intermediate boson. The mathematical description of this model in the classical case can be given on the basis of the theory of waveguides and resonators (Crawford, 1968; Broglie, 1941). As is known, the motion of waves is determined by the dispersion equation (or by another dispersion relationship).

Dispersion equation is the relationship, which connects angular frequencies ω and wave vectors k of natural harmonic waves (normal waves) in linear uniform systems: continuous media, waveguides, transmission lines and others. Dispersion equation is written in the form

$$\omega = \omega(k), \quad (3.1)$$

Dispersion equations are the consequence of the dynamic (in the general case integrodifferential) equations of motion and of boundary conditions. And also, vice versa, on the base of the form of dispersion equation the dynamic equations of processes can be restored with the replacement:

$$i\omega \rightarrow \frac{\partial}{\partial t}, \quad ik_x \rightarrow -\frac{\partial}{\partial x}, \quad \frac{1}{i\omega} \rightarrow \int (...)dt, \quad \frac{1}{ik_x} \rightarrow \int (...)dx, \quad (3.2)$$

It is easy to obtain the dispersion equation for the infinite wave without any limiting conditions, $\vec{\Phi} = \vec{\Phi}_0 e^{-i(\omega t - ky)}$, using the homogeneous wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \vec{\Phi} = 0, \quad (3.3)$$

where $\vec{\Phi}$ are in our case any vector components of electrical and magnetic field. Putting this solution, we obtain $\omega^2 - v^2 k^2 = 0$ or $\omega = v \cdot k$.

In the case of the presence of limitations, superimposed on the wave by medium or by it self, the equation becomes heterogeneous:

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \vec{\Phi} = \vec{\Phi}_0, \quad (3.4)$$

where $\vec{\Phi}_0$ is certain function of the electromagnetic fields. In this case dispersion relationship becomes more complex: new terms are introduced and its linearity is disrupted.

The same relationship dispersion equation:

$$\omega^2 = \omega_0^2 + v^2 k^2, \quad (3.5)$$

can correspond to: 1) EM waves in the isotropic plasma; 2) plasma waves; 3) waves in the waveguides; 4) waves in the acoustic waveguides; 5) elementary particle in relativistic wave mechanics ($v = c$, $\omega_0 = m_0 c^2 / \hbar$, m_0 is rest mass).

In the latter case the discussion deals with de Broglie wave dispersion relation.

Energy, momentum, and mass of particles are connected through the relativistic relation

$$\varepsilon^2 = (m_0 c^2)^2 + (pc)^2, \quad (3.6)$$

Elementary particles, atomic nuclei, atoms, and even molecules behave in some context as matter waves. According to the de Broglie relations, their kinetic energy ε can be expressed as a frequency ω : $\varepsilon = \hbar\omega$, and their momentum p as a wave number k : $p = \hbar k$.

The relationships, obtained for EM wave in a waveguides or in a box (Broglie, 1941): , are completely analogous to those, which exist in wave mechanics, in which the rectilinear and uniform particle motion with the rest mass m_0 depicts in the form of propagation of plane simple harmonic wave $\psi = \psi_0 e^{i(\omega t - kr)}$.

As we noted, $\omega = ck$ corresponds to the propagation of EM wave in the vacuum. But if EM wave is in the waveguide, then between ω and k we have the relationship (3.6), where ω_0 is different from zero and it is equal to one of its eigenvalues, which correspond to the form of the waveguide in question. From the point of view of wave mechanics everything happens as if the photon had its own mass, determined by the form of waveguide and by the eigenvalue $\omega_{0i} = m_{0i}/\hbar$. Thus, it is possible to say that in this waveguide the photon can possess a series of possible own masses.

From a contemporary point of view we can interpret the appearance of photon mass as follows. A photon, until its entry into a waveguide or resonator, obeys to the linear equation

$$\left(\Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \bar{\Phi} \equiv \sum_{\nu} \frac{\partial^2}{\partial x_{\nu}^2} \bar{\Phi} \equiv \partial_{\nu} \partial^{\nu} \bar{\Phi} = 0, \quad (3.7)$$

Lagrangian of which

$$L = \frac{1}{2} \left\{ \left(\frac{\partial \bar{\Phi}}{\partial t} \right)^2 - c^2 (\bar{\nabla} \bar{\Phi})^2 \right\} \equiv \frac{1}{2} c^2 \sum_{\nu} \left(\frac{\partial \bar{\Phi}}{\partial x_{\nu}} \right)^2 \equiv \partial_{\nu} \bar{\Phi} \partial^{\nu} \bar{\Phi}, \quad (3.8)$$

describes the mass-free field. After entry to a box the photon experiences a certain spontaneous transformation and becomes massive particle. Each component of the field of this massive particle obeys to Klein-Gordon wave equation (Wentzel, 2003):

$$\left(\Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - m^2 \right) \bar{\Phi} \equiv \left(\sum_{\nu} \frac{\partial^2}{\partial x_{\nu}^2} - m^2 \right) \bar{\Phi} \equiv (\partial_{\nu} \partial^{\nu} - m^2) \bar{\Phi} = 0, \quad (3.9)$$

This is achieved by choosing the following, evidently Lorentz-invariant Lagrangian:

$$L = \frac{1}{2} \left\{ \left(\frac{\partial \bar{\Phi}}{\partial t} \right)^2 - c^2 (\bar{\nabla} \bar{\Phi})^2 - c^2 m^2 \bar{\Phi}^2 \right\} \equiv -\frac{1}{2} c^2 \left\{ \sum_{\nu} \left(\frac{\partial \bar{\Phi}}{\partial x_{\nu}} \right)^2 + m^2 \bar{\Phi} \bar{\Phi}^+ \right\} \equiv \partial_{\nu} \bar{\Phi} \partial^{\nu} \bar{\Phi} - c^2 m^2 \bar{\Phi}^2 \quad (3.10)$$

Could we likewise the generation of a massive elementary particle? Let us turn to the implementation of these ideas in the mathematical theory.

4. Axiomatics of the non-linear theory of elementary particles

The axiomatic basis of the proposed theory is composed by 5 postulates, from which the first 4 are the postulates of contemporary field theory. Postulate 5 expresses the specific nonlinearity of theory, but it does not contradict to the results of contemporary physics.

1) Postulate of fundamentality of the electromagnetic field: *Maxwell's equation for the field without sources:*

$$\frac{1}{c} \frac{\partial \vec{E}}{\partial t} - \text{rot} \vec{H} = 0, \quad (4.1)$$

$$\frac{1}{c} \frac{\partial \vec{H}}{\partial t} + \text{rot} \vec{E} = 0, \quad (4.2)$$

are fundamental independent equations of motion of fields.

Definition 1: A self-propagated in space, alternated electric and magnetic fields is called electromagnetic (EM) wave.

2) The postulate of the quantization of EM wave fields: *electromagnetic wave fields consist of the elementary electromagnetic wave formations (particles) – photons.*

3) Quantization postulates of Planck and de Broglie: *the relationship between the energy, frequency and wavelength of photon is determined by the following formulas:*

$$\varepsilon_{ph} = h\nu = \hbar\omega, \quad (4.3)$$

$$\lambda = \frac{h}{p_{ph}} = \frac{hc}{\varepsilon}, \quad (4.4)$$

4) The postulate of superposition of wave fields: *in the general case electromagnetic waves are the superposition of elementary wave fields, the simplest of which are photons.*

5) Postulate of the massive particles' generation: *for generation of the massive particles the field of photon must undergo the rotation transformation.*

Here: \vec{E} and \vec{H} are the vectors of strength of electrical and magnetic fields; ε is energy, \vec{p} is momentum, λ is wavelength, c is speed of light, h or $\hbar = h/2\pi$ are Planck constants.

Let us note that the Maxwell equations of the first postulate are linear equations. As we show further, postulate 5 introduces nonlinearity into the theory (recall that this postulate, according to its results, is equivalent to gauge transformation).

We do not consider the question, if this system of axioms is complete. Moreover, judging by the formulation of other axiomatic theories, it is possible to use another system of axioms.

5. The photon theory

The quantum theory of the photon is part of the theory of quantum electrodynamics (QED) (Akhiezer and Berestetskiy, 1969).

5.1 Linear EM wave equation in matrix form

Within the framework of the proposed nonlinear theory of elementary particles (NTEP), the photon is considered as an object of QED described by a known linear wave equation. The only differences of our theory from the QED relate to the form of equations and the interpretation of their characteristics. As is known the field (electromagnetic form of the photon equation, in view of the Planck quantization condition, is identical to its quantum form (Feynman, 1964c)

5.2 Wave equation of a photon in matrix form

Let us consider the general case of a circularly polarized electromagnetic (EM) wave that is moving, for instance, along the y - axis (the use of any other direction does not change anything in the theory). This wave is the superposition of two plane-polarized waves with mutually perpendicular vectors of the EM fields: \vec{E}_x, \vec{H}_z and \vec{E}_z, \vec{H}_x .

The EM wave equation has the following known form (Jackson, 1999):

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \vec{\Phi}(y) = 0, \quad (5.1)$$

where $\vec{\Phi}(y)$ is any of the above electromagnetic wave field vectors (5.2). In other words, this equation represents four equations: one for each vector of the electromagnetic field.

An electromagnetic wave propagating in any direction can have two plane polarizations; it contains only four field vectors. For example, in the case of y -direction, we have:

$$\vec{\Phi}(y) = \{E_x, E_z, H_x, H_z\}, \quad (5.2)$$

and $E_y = H_y = 0$ for all transformations. Here, note that the Dirac bispinor also has only four components.

The solution of equation (5.1) can be written in a complex form as follows:

$$\begin{cases} \vec{E} = \vec{E}_o e^{-i(\omega t - ky)} + \vec{E}_o^* e^{i(\omega t - ky)} \\ \vec{H} = \vec{H}_o e^{-i(\omega t - ky)} + \vec{H}_o^* e^{i(\omega t - ky)} \end{cases}, \quad (5.3)$$

where k is wave number (here, for simplicity, the constant phase is adopted as zero).

We can also write this equation in the following operator form:

$$\left(\hat{\varepsilon}^2 - c^2 \hat{p}^2\right) \Phi(y) = 0, \quad (5.4)$$

where $\hat{\varepsilon} = i\hbar \frac{\partial}{\partial t}$, $\hat{p} = -i\hbar \vec{\nabla}$ are correspondingly the operators of energy and momentum; Φ is a matrix which consists of the four components $\vec{\Phi}(y)$.

Taking into account that $(\hat{\alpha}_0 \hat{\varepsilon})^2 = \hat{\varepsilon}^2$, $(\hat{\alpha} \hat{p})^2 = \hat{p}^2$, where $\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix}$; $\hat{\alpha} = \begin{pmatrix} 0 & \hat{\sigma} \\ \hat{\sigma} & 0 \end{pmatrix}$;

$\hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}$ are Dirac's matrices and $\hat{\sigma}_0, \hat{\sigma}$ are Pauli's matrices, equation (5.4) can also be represented in a matrix form:

$$\left[(\hat{\alpha}_0 \hat{\varepsilon})^2 - c^2 (\hat{\alpha} \hat{p})^2 \right] \Phi = 0, \quad (5.5)$$

Recall that in case of a photon $\omega = \varepsilon/\hbar$ and $k = p/\hbar$. From equation (5.5), using (5.1), we obtain $\varepsilon = cp$, which is the same as for a photon. Therefore, we can consider the wave function Φ of the equation (5.5) both as that of an EM wave and (taking into account its quantization) of a photon.

Factoring (5.5) and multiplying it on the left by the Hermitian-conjugate function Φ^+ , we get:

$$\Phi^+ (\hat{\alpha}_0 \hat{\varepsilon} - c \hat{\alpha} \hat{p}) (\hat{\alpha}_0 \hat{\varepsilon} + c \hat{\alpha} \hat{p}) \Phi = 0, \quad (5.6)$$

Equation (5.6) may be broken down into system from two equations identical to the equation (5.1) or (5.5):

$$\Phi^+ (\hat{\alpha}_0 \hat{\varepsilon} - c \hat{\alpha} \hat{p}) = 0, \quad (5.7')$$

$$(\hat{\alpha}_0 \hat{\varepsilon} + c \hat{\alpha} \hat{p}) \Phi = 0, \quad (5.7'')$$

Let us note, first, that the equations (5.7) are identical to the Dirac electron and positron equations without mass terms. Secondly, a comparison of equation (5.1) with the system (5.7) suggests that the photon is a superposition of massless electrons and positrons.

Note that the system of equations (5.7) can be represented (Akhiezer and Berestetskiy, 1965; Levich, etc, 1973) as a system of quantum equations for a photon in Hamilton's form. At the same time in the electromagnetic interpretation they are the Maxwell equations of EM waves.

Actually, it is not difficult to show that only in the case when the Φ -matrix has the form:

$$\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \quad \Phi^+ = (E_x \quad E_z \quad -iH_x \quad -iH_z), \quad (5.8)$$

the equations (5.7) are the right Maxwell-like equations of the retarded and advanced electromagnetic waves. Using (5.8), and substituting it into (5.7), we obtain:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0 \end{array} \right. , \quad (5.9')$$

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = 0 \end{array} \right. , \quad (5.9'')$$

For waves of any other direction the same results can be obtained by cyclic transposition of indices, or by a canonical transformation of matrices and wave functions.

We will further conditionally name each of (2.7) equations the linear semi-photon equations, remembering that it was obtained by division of one wave equation of a photon into two equations of the electromagnetic waves: retarded and advanced.

Let us make two important remarks:

1) To describe the circularly polarized EM wave, as is known, two pairs of mutual-perpendicular vectors are required: electric vectors (in our case, E_x, E_z) and magnetic vector (H_x, H_z). To this corresponds the fact that the system of four equations (5.9) describes one photon with circular polarization.

In the case of plane polarization there are two separate photons, that move along the y -axis (in our case with the vectors E_x, H_z and E_z, H_x), which are described by two independent systems of equations:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} \mp \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_z}{\partial t} \mp \frac{\partial E_x}{\partial y} = 0 \end{array} \right. , \quad (5.10)$$

and

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_z}{\partial t} \pm \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} \pm \frac{\partial E_z}{\partial y} = 0 \end{array} \right. , \quad (5.11)$$

It is possible to say that system (5.9) includes this pair of photons; i.e., it is the general case of describing photons of different polarization. As we will see in the following papers, this has an important physical meaning.

2) At present as the wave vector of photon is more frequently used the 4- vector potential $A_\mu = \{i\varphi, A_i\}$ of EM of field, and the system of four equations is used as the initial wave equations:

$$\begin{aligned} \bar{\nabla}^2 \bar{A} - \frac{1}{c^2} \frac{\partial^2 \bar{A}}{\partial t^2} &= 0 \\ \bar{\nabla}^2 \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} &= 0 \end{aligned} \quad (5.12)$$

which undergoes quantization in this or in another way.

There is an essential difference between systems (5.9) and (5.12). Wave functions in the first case are the characteristics of the field of EM of wave, and a quantity of equations is determined by the polarization of EM of wave. In the second case the potentials are auxiliary mathematical characteristics, and a number of equations is determined by the formal association of the physical values.

In connection with this let us recall (Bethe, 1964) that the spinor equation of Dirac has 2 wave functions and 2 equations for them, and the bispinor Dirac equation has 4 wave functions and 4 equations for them; moreover this is in no way connected with the 4-dimensional space-time. As we will be convinced, this has a straight succession with the theory of photon and a deep physical sense.

5.3 Normalized and non-normalized representation of the wave function of the photon

Above within the framework of NTEP we used the wave function of photon, which is the strength of electromagnetic field.

The particular interpretation of wave function is one of the special features of quantum mechanics in comparison with the classical electrodynamics. The physical sense of quantum wave function consists in the fact that its square (but more precisely, the product of wave function to its conjugate function) is the probability of finding the particle in some point of space in this instant. In this case the normalization of wave function is presented by one of the basic requirements of wave mechanics.

It is understandable that this meaning of wave function is more mathematical, than physical. But this probabilistic interpretation is confirmed by all experimental data.

Thus, we must show that in the framework of NTEP the representation of the wave function in the form of the strengths of electromagnetic field does not contradict the probabilistic representation.

The wave function of photon must satisfy the requirements of the energy conservation law:

$$\int_0^\infty \frac{1}{8\pi} (\bar{\mathbf{E}}^2 + \bar{\mathbf{H}}^2) d\tau = \varepsilon, \quad (5.13)$$

where in this case ε is photon energy. Taking into account that $\Phi^+ \Phi = (\vec{E}^2 + \vec{H}^2)$, we obtain:

$$\int_0^{\infty} \Phi^+ \Phi d\tau = 8\pi\varepsilon, \quad (5.14)$$

For the passage to the probabilistic representation of wave function we will use the indication, given in (Physical encyclopedia, 1962) “The probability interpretation of EM of wave”: “From the EM energy, radiated per unit time, it is possible to pass to the probabilities of radiation of the photon by division of EM energy by the photon energy”. It is not difficult to see that if we write down the wave function of photon in the form:

$$\Psi(\vec{r}, t) = \frac{\Phi(\vec{r}, t)}{\sqrt{8\pi\varepsilon}}, \quad (5.15)$$

we will obtain wave function in the probabilistic representation. Actually, the value

$$P(\vec{r}, t) = \Psi^+ \Psi = \frac{\Phi^+ \Phi}{8\pi\varepsilon}, \quad (5.16)$$

is the dimensionless quantity, having the sense of **probability density**.

As far as one photon is considered, the value P will determine the portion of energy from the photon energy at the particular point of space-time. When we have many photons in a given volume of space, ε should be understood as the total electromagnetic energy of these photons. In this case the formula (5.16) will determine the probability of finding the photon at the particular point of the volume of space at the given point of time. Obviously, in both cases the following normalization conditions are satisfied.

$$\int_0^{\infty} \Psi^+ \Psi d\tau = 1, \quad (5.17)$$

Depending on the convenience, the one or the other normalization can be selected, taking into account that the energy of photon ε is a normalized constant, similar to unit. In this case

We will dedicate the following parts of theory to the description of the generation of massive elementary particles. We will ascertain that the physical sense of the wave function of massive particles remains the same (with the only difference that, besides energy, in this case it is possible to use the mass of particles according to Einstein's formula $\varepsilon = mc^2$).

Let us note that since energy, momentum, wave vector, frequency and wavelength of photon are one-to-one connected among themselves, it is possible to speak about normalization in relation to any of these characteristics. Of course in this case it is necessary to use a wave function in the appropriate representation.

One additional question, connected with the normalization of wave function, is, what volume of the space of integration the formula (5.17) contains. Usually the infinite space is understood here. On this base it is said that the photon is spread on the infinite space.

As we have seen above, accordingly to QED a photon is a nonlocal object with size, characterized by its wavelength. Since photon is neutral and does not possess external infinite field, physically, integration on infinity is thoughtless. Then a question arises: why the integration for formula (5.17) does not cause difficulty in the physics calculations.

Let us assume that photon occupies the limited volume τ_{ph} . It is not difficult to see that:

$$\int_0^{\infty} \Phi^+ \Phi d\tau = \int_0^{\infty - \tau_{ph}} \Phi^+ \Phi d\tau + \int_0^{\tau_{ph}} \Phi^+ \Phi d\tau = \int_0^{\tau_{ph}} \Phi^+ \Phi d\tau = 8\pi\epsilon, \quad (5.18)$$

since the integral on the space $\tau = \infty - \tau_{ph}$, where the photon is absent, is equal to zero. Therefore, the presence or absent of photon size plays no role in the calculations.

6. Massive intermediate boson theory of QED and the mechanism of mass generation in NTEP

6.1. Photon as a gauge field

As one of the simplest examples of the generation of massive fields (particles) we can consider the photoproduction of the electron-positron pair:



Actually, the photon γ is a mass-free gauge vector boson. The EM field of proton p (or some atom nucleus) initiates its transformation into two massive particles: electron and positron, *but its EM field does not disappear, similarly to Higgs's field*. The fields of the electron and positron e^+, e^- are spinors, which are not transformed like vector fields. *Thus, we can say that the reaction (A) describes the process of symmetry breaking of the initial mass-free vector field in order to generate the massive spinor particles.*

Let us examine the Feynman diagram of the above reaction of a pair production (Fig. 6.1):

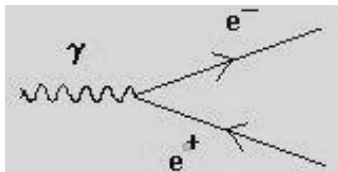


Fig. 6.1

It is known that using Feynman's diagrams within the framework of SM, we can precisely calculate all characteristics of particles with the exception of charge and mass. Nevertheless, the reaction (A) remains mysterious: we do not know, for example, how the process of transformation of the mass-free boson field into the massive fields happens, and how the electrical charge appears.

It is obvious that the interaction of the photon with proton (nucleus) takes place in the vertex of the diagram 4.1 and here the transformation of a massless photon into massive electron-positron pair begins.

We ask, what kind of transformation could this be? There are several considerations that allow us to answer this question. On the one hand Higgs's theory states that the intermediate (massive) bosons must participate.

On the other hand it is known that the propagation of EM wave in the strong electromagnetic fields is accompanied by nonlinear effects. On this base it is possible to assume that the photon must undergo a certain transformation (De Lorenci et al, 2000; Novello, et al, 2000); Denisov and Svertilov, 2003; Kim and Lee, 2011). And as we postulated above, this is a transformation of rotation.

Based on this evidence, let us assume that during the first stage in the vertex of the Feynman's diagram the rotation transformation of the “linear” photon (in the sense that it obeys a linear wave equation) into the “nonlinear” massive photon (obeyed a nonlinear wave equation) is achieved. At the second stage, the transformation of massive boson in two massive fermions must take place. Obviously, in this case the initial symmetry of photon will be broken. These ideas can be translated into mathematical language.

6.2 The rotation transformation of electromagnetic wave quantum

The rotation transformation of the “linear” photon wave to a “curvilinear” one can be conditionally written in the following form:

$$\hat{R}\Phi \rightarrow \Phi', \tag{6.1}$$

where \hat{R} is the rotation operator for the transformation of a photon wave from linear state to curvilinear state, and Φ' is some final wave function:

$$\Phi' = \begin{pmatrix} \Phi'_1 \\ \Phi'_2 \\ \Phi'_3 \\ \Phi'_4 \end{pmatrix} = \begin{pmatrix} E'_x \\ E'_z \\ iH'_x \\ iH'_z \end{pmatrix}, \tag{6.2}$$

which appears after the nonlinear transformation (6.1); here, $(E'_x \ E'_z \ -iH'_x \ -iH'_z)$ are electromagnetic field vectors after the rotation transformation, which correspond to the wave functions Φ' .

It is known that the transition of vector motion from linear to curvilinear state is described by differential geometry (Eisenhart, 1960). Note also that this transition is mathematically equivalent to a vector transition from flat space to curvilinear space, which is described by Riemann geometry. In relation to this, let us remind ourselves that the Pauli matrices, as well as the photon matrices, are the space rotation operators – 2-D and 3-D accordingly (Ryder, 1985).

6.3 The rotation transformation description in differential geometry

We do not know the real structure of a photon as a quantum of an EM wave. However, we have an idea about the description of the structure of its fields and their motion. Therefore, in the following, the word “photon” should be understood only in the sense of the known mathematical description of photon fields' characteristics.

Let us consider a plane-polarized EM wave, which has the field vectors (E_x, H_z) (see fig. 6.2):

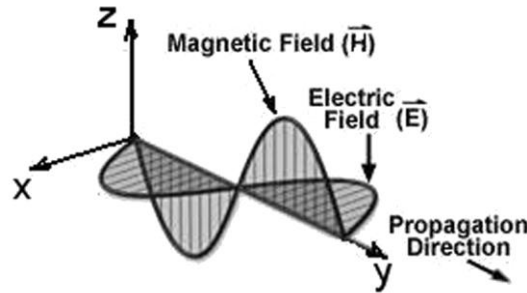


Fig. 6.2

Movement of an electromagnetic wave is completely described by means of three mutually perpendicular vectors: two vectors of electric and magnetic fields \vec{E}, \vec{H} , and the Poynting vector \vec{S} , which determines the movement of energy and momentum of the wave field. These three vectors constitute the Frenet-Serret concomitant trihedral $\vec{n}, \vec{b}, \vec{\tau}$ and reflect the relationship between electricity, magnetism and mechanical motion of electromagnetic wave.

Assume that under certain conditions this trihedral moves on a circular path of radius $s r_p$, in the plane (X', O', Y') of a fixed co-ordinate system (X', Y', Z', O') around the axis Z' , so that E_x is parallel to the plane (X', O', Y') , and H_z is perpendicular to this plane (fig. 6.3).

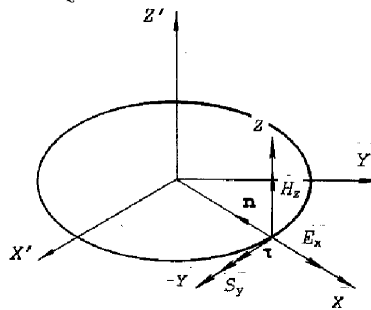


Fig. 6.3

According to Maxwell (Jackson, 1999), the following term of equations (5.6)

$$\hat{\alpha}_0 \hat{\epsilon} \Phi = i\hbar \frac{\partial \Phi}{\partial t}$$

contains the Maxwell's displacement current, which is defined by the expression:

$$j_{dis} = \frac{1}{4\pi} \frac{\partial \vec{E}}{\partial t}, \quad (6.3)$$

The electrical field vector \vec{E} above, which moves along the curvilinear trajectory (assume its direction is from the center), can be written in the form:

$$\vec{E} = -E \cdot \vec{n}, \quad (6.4)$$

where $E = |\vec{E}|$, and \vec{n} is the normal unit-vector of the curve, directed to the center. Then, the derivative of \vec{E} can be represented as follows:

$$\frac{\partial \vec{E}}{\partial t} = -\frac{\partial \vec{E}}{\partial t} \vec{n} - E \frac{\partial \vec{n}}{\partial t}, \quad (6.5)$$

Here, the first term has the same direction as \vec{E} . The existence of the second term shows that at the rotation transformation of the wave an additional displacement current appears. It is not difficult to show that it has a direction tangential to the ring:

$$\frac{\partial \vec{n}}{\partial t} = -v_p K \vec{\tau}, \quad (6.6)$$

where $\vec{\tau}$ is the tangential unit-vector, $v_p \equiv c$ is the electromagnetic wave velocity, $K = \frac{1}{r_p}$ is the curvature of the trajectory, and r_p is the curvature radius. Thus, the displacement current of the plane wave moving along the ring can be written in the following form:

$$\vec{j}_{dis} = -\frac{1}{4\pi} \frac{\partial E}{\partial t} \vec{n} + \frac{1}{4\pi} \omega_p E \cdot \vec{\tau}, \quad (6.7)$$

where $\omega_p = \frac{m_p c^2}{\hbar} = \frac{v_p}{r_p} \equiv cK$ is an angular velocity. Furthermore, here, $m_p c^2 = \varepsilon_p$ is photon energy, where m_p is some mass, corresponding to the energy ε_p .

Obviously, the terms $\vec{j}_n = \frac{1}{4\pi} \frac{\partial E}{\partial t} \vec{n}$ and $\vec{j}_\tau = \frac{\omega_p}{4\pi} E \cdot \vec{\tau} \equiv \frac{m_p c^2}{\hbar} \frac{1}{4\pi} E \equiv \frac{v_p}{r_p} \frac{1}{4\pi} E \equiv K \frac{c}{4\pi} E$ are the normal and tangent components of the displacement current of the rotated electromagnetic wave accordingly. Thus:

$$\vec{j}_{dis} = \vec{j}_n + \vec{j}_\tau, \quad (6.8)$$

This is a remarkable fact that the currents \vec{j}_n and \vec{j}_τ are always mutually perpendicular, so that we can write (6.8) in complex form as follows:

$$j_{dis} = j_n + ij_\tau, \quad (6.8')$$

Thus, the appearance of the tangent current leads to origination of the imaginary unit in a complex form of particles' equation. So, we can assume that the appearance of the imaginary unit in the quantum mechanics is tied to the appearance of tangent currents.

6.4. A description of the rotation transformation in curvilinear space

We can consider the Maxwell wave equations (5.5) with the wave function (5.8) as Dirac's equation without mass.

$$\hat{\alpha}_\mu \partial_\mu \Phi = 0, \quad (6.9)$$

where $\hat{\alpha}_\mu = \{\hat{\alpha}_0, \hat{\alpha}\}$ and $\partial_\mu \equiv \partial / \partial x_\mu$ ($\mu = 0, 1, 2, 3$ are indexes of summing)

The generalization of the Dirac equation on the curvilinear geometry is connected to the parallel transport of the spinor in curvilinear space (Fock, 1929; Fock and Ivanenko, 1929; Schroedinger, 1932; Infeld und Van der Waerden, 1933; Goenner, 2004).

In order to generalize the Dirac equation in the form of curvilinear geometry, we replace the usual derivative $\partial_\mu \equiv \partial / \partial x_\mu$ (where x_μ are the co-ordinates in the 4-space) with the covariant derivative, which will be sufficient:

$$D_\mu = \partial_\mu + \Gamma_\mu, \quad (6.10)$$

where $\mu = 0, 1, 2, 3$ are the summing indices, and Γ_μ is the analogue of Christoffel's symbols in the case of spinor theory, which are called Ricci symbols (or connection coefficients).

When a spinor moves along a straight line, all the symbols $\Gamma_\mu = 0$, and we have the usual derivative. However, if the spinor moves along the curvilinear trajectory, not all Γ_μ are equal to zero, and in this case an additional term appears.

Typically, the last term is not the derivative, but is equal to a product of the spinor itself with some coefficient Γ_μ , which is an increment in the spinor. It is easy to see that the tangent current j_τ corresponds to the Ricci connection coefficients (symbols) Γ_μ .

According to the general theory (Sokolov and Ivanenko, 1952), we can obtain as an additional term of equations (b) the following term: $\hat{\alpha}_\mu \Gamma_\mu = \hat{\alpha}_i p_i + i \hat{\alpha}_0 p_0$, where p_i and p_0 are real values. Since the increment in spinor Γ_μ has the form and the dimension of the energy-momentum 4-vector, it is logical to identify Γ_μ with a 4-vector of the energy-momentum of the photon's electromagnetic field:

$$\Gamma_\mu = \{\varepsilon_p, c\vec{p}_p\}, \quad (6.11)$$

where ε_p and p_p are the photon's energy and momentum respectively (not the operators). In other words, we have:

$$\hat{\alpha}_\mu \Gamma_\mu = \hat{\alpha}_0 \varepsilon_p + \vec{\hat{\alpha}} \vec{p}_p, \quad (6.12)$$

Taking into account that the energy conservation law $\varepsilon^2 - c^2 \vec{p}^2 - m_p c^2 = 0$ in linear form is $\hat{\alpha}_0 \varepsilon_p \mp \vec{\hat{\alpha}} \vec{p}_p = \pm \hat{\beta} m_p c^2$, we can see that the additional term contains mass of the transformed wave as a tangential current (6.9).

$$\hat{\alpha}_\mu \Gamma_\mu = \hat{\beta} m_p c^2, \quad (6.13)$$

6.5 An equation of the massive intermediate photon

As it follows from the previous sections, some additional terms $K = \hat{\beta} m_p c^2$, corresponding to tangent components of the displacement current, must appear in reaction (A) due to a curvilinear motion of the electromagnetic wave:

$$\left(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \cdot \hat{p} - K\right) \left(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \cdot \hat{p} + K\right) \Phi' = 0, \quad (6.14)$$

Thus, in the case of the curvilinear transformation of the electromagnetic fields of a photon, we obtain the following Klein-Gordon-like equation with mass (Schiff, 1955), instead of equation (6.14):

$$\left(\hat{\varepsilon}^2 - c^2 \hat{p}^2 - m_p^2 c^4\right) \Phi' = 0, \quad (6.15)$$

This is remarkable that due to a rotation transformation of the initial photon the tangential current is formed. At the same time, the current characteristics are unambiguously related to the mass of transformed photon. This mass is equal to its energy divided by a square of the speed of light. This, by the way, explains why mass divergence in electron theory is always connected with the divergence of its electrical charge.

Equation (6.15) is similar to the Klein-Gordon equation. However, the latter describes the **scalar** field, i.e. the massive boson with zero spin of the type of the hypothetical Higgs boson (let us also recall that Higgs's mechanism of mass generation is based on the scalar equation of Klein-Gordon).. It is not difficult to prove, using an electromagnetic form that (6.15) is an equation of a massive **vector** particle.

As we can see, the Φ' -function that appears after the transformation of the electromagnetic wave, and that satisfies equation (6.15), is not identical to the Φ -function before the transformation. The Φ -function is a classical linear electromagnetic wave field that satisfies the wave equation. At the same time, the Φ' -function is a non-classical curvilinear electromagnetic wave field that satisfies equation (6.15).

Moreover, equation (6.15), whose wave function is a 4×1 - matrix with electromagnetic field components, *cannot be a scalar field equation*. Let us analyze the objects, which this equation describes.

It follows from the Maxwell's equations that each of the components E_x, E_z, H_x, H_z of vectors of the EM wave fields \vec{E}, \vec{H} is included into the same scalar wave equations. In the case of a linear wave, all field components are independent. So, studying each of \vec{E}, \vec{H} vector components, we can consider the vector field as scalar. However, we cannot proceed to scalar theory after the curvilinear transformation when a tangential current appears. In fact, the components of vector \vec{E}' are not independent functions, as it follows from the condition (which is the Maxwell law) $\vec{\nabla} \cdot \vec{E}' = \frac{4\pi}{c} \vec{c}^o \cdot \vec{j}$, where \vec{c}^o is a unit vector of wave velocity.

With this regard, this equation plays a role of the Procá equation. The Procá equation can be recorded in a form, similar to equation (6.15)

$$\left(\hat{\varepsilon}^2 - c^2 \hat{p}^2 - m_p^2 c^4\right) A_\mu = 0, \quad (6.16)$$

As it is known (Ryder, 1985), this equation is considered in SM as equation of intermediate bosons. The Procá equation is an equation for a four-dimensional vector potential, which can be used to describe a massive particle with spin equal to one.

The difference of (6.3) from the equation (6.15) lies in the fact that the free term of Procá equation is written through the 4-potential and is not gauge invariant in the case when m_p is a particle mass. In our case the mass term is expressed through the field strengths, i.e., through the wave function itself, and does not disrupt the invariance of the equations.

According to the results presented above, we need to assume that the more detailed Feynman's diagram of photoproduction of an electron-positron pair must include a massive intermediate photon. So, the diagram must have the following form (Fig. 6.4):

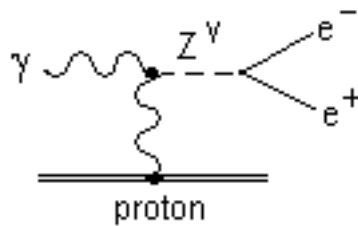


Fig. 6.4

where we designated the nonlinear photon Z^γ like as an neutral intermediate massive Z^0 -boson, described by the electro-weak theory within the framework of the Standard Model

The question arises of whether the neutral intermediate boson Z^γ is somehow connected with neutral intermediate boson Z^0 . Obviously, there are many similarities between them (Practicum, 2004). They are massive intermediate vector bosons, which at the virtual level can be presented as dipoles, consisting of particle-antiparticle pairs. The only significant difference between them is the mass values: Z^γ boson is much lighter than Z^0 . But perhaps the difference between them is the fact that the Z^γ provides electromagnetic processes, while Z^0 - weak processes. As it is shown that these processes can be unified, it is possible to think that at a certain energy, the difference between them is eliminated.

(Continued on Part II; References are listed at the end of Part II)