Article

Tilted Plane Symmetric Magnetized Cosmological Models

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Abstract

In this paper we have investigated tilted plane symmetric cosmological models in presence and absence of magnetic field. To get the deterministic model, we have assumed the supplementary conditions p = 0 and $B = A^n$ where A and B are metric potentials and n is constant. Some geometric aspects of the model are also discussed.

Keywords: tilted models, plane symmetric, dust fluid.

1. Introduction

An anisotropic cosmological model plays an important role in the large scale behavior of the universe. The many researchers working on cosmology by using relativistic cosmological models have not given proper reasons to believe in a regular expansion for the description of the early stages of the universe. There are some experimental data of CMR and theoretical arguments which supports the existence of anisotropic universe (Verma et.al [1], Chimento [2], Misner [3], Land et .al [4], M. Demianski [5], Chawala et al [6]).

The considerable interest has been focused in investigating spatially homogeneous and anisotropic universes in which the matter does not move orthogonally to the hyper surface of homogeneity in recent years. These are called tilted universes. King and Ellis [7]; Ellis and King [8]; Collins and Ellis [9] have studied the general dynamic of tilted cosmological models.

Dunn and Tupper [10,11] have been studied tilted Bianchi type I cosmological model for perfect fluid and shown that a Bianchi tilting universe is possible when electromagnetic field is present. Many other researchers like Matravens et al. [12], Bali and Sharma [13], Horwood et al. [14], Hewit et al. [15], Aposotolopoulos [16] have studied different aspects of tilted cosmological models. Bianchi type V tilted cosmological models in the Scale-covariant theory derived by

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Beesham [17]. Bali and Meena [18] have investigated tilted cosmological models filled with disordered radiation of perfect fluid, heat flow. Pradhan and Rai [19] have obtained tilted Bianchi type V cosmological models filled with disordered radiation in the presence of bulk viscous fluid and heat flow. Bhaware et al. [20] studied tilted cosmological models with varying Λ. Pawar et al. [21] have investigated tilted plane symmetric cosmological models with heat conduction and disordered radiation. Pawar and Dagwal [22,23] have studied confirmally flat tilted cosmological models and recently two fluids tilted cosmological models in general relativity.

Banerjee et al. [24] have studied an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field. LRS Bianchi type string dust-magnetized cosmological models have been investigated by Bali and Upadhyay [25]. Stationary distribution of dust and electromagnetic field in general relativity has been investigated by Banerjee and Banerjee [26]. Pawar et al. [27, 28] have studied bulk viscous fluid with plane symmetric string dust magnetized cosmological model in general relativity and Lyra manifold. Patil et al. [29, 30] obtained on thick domain walls with viscous field coupled with electromagnetic field in general relativity and Lyra geometry. Bayaskar et al. [31] derived cosmological models of perfect fluid and massless scalar field with electromagnetic field. Bali et al. [32] have studied magnetized tilted universe for perfect fluid distribution in general relativity. Bagora et al. [33] have investigated tilted Bianchi type I dust fluid magnetized cosmological model in general relativity.

2. Field Equation

We consider metric in the form –

$$ds^2 = -dt^2 + A^2\left(dx^2 + dy^2\right) + B^2 dz^2 \ , \tag{1}$$

where A and B are the functions of t alone.

The Einstein's field equation is –

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j e \tag{2}$$

The energy-momentum tensor for perfect fluid distribution with heat conduction is given by –

$$T_i^j = (\varepsilon + p)\, v_i v^j + p\, g_i^j + q_i v^j + v_i q^j + E_i^j \,. \tag{3}$$

With

$$g_{ij} v^i v^j = -1, \tag{4}$$

$$q_i q^j \rangle\ 0\,, \tag{5}$$

$$q_i v^i = 0\,. \tag{6}$$

Here E_i^j is the energy momentum tensor of electromagnetic field given by

$$E_i^j \equiv \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right], \tag{7}$$

where $\bar{\mu}$ is magnetic permeability and the magnetic flux vector h_i is given by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}}\varepsilon_{ijk\lambda}F^{k\lambda}v^j\,, \tag{8}$$

$F_{k\lambda}$ is the electromagnetic field tensor, $\varepsilon_{ijk\lambda}$ is the Levi-Civita tensor density, P is pressure, ε is density and q^i is heat conduction vector orthogonal to the fluid flow vector v^i. The fluid flow vector v^i has the components $\left(0,0,\frac{\sin h\alpha}{B},\cos h\alpha\right)$, satisfying condition (4); and α is the tilt angle.

The incident magnetic field is taken on z-axis so that

$$h_1 = 0\,,\ h_2 = 0\,,\ h_3 \neq 0\,,\ h_4 \neq 0\,. \tag{9}$$

The first set of Maxwell's equation is

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0\,. \tag{10}$$

Here

$$F_{12} = \text{constant} = \text{M(say)}\,. \tag{11}$$

Here $F_{14} = F_{24} = F_{34} = 0$, due to assumption of infinite electrical conductivity.

The only non-vanishing component of F_{ij} is F_{12}.

Hence

$$h_3 = \frac{BM}{\bar{\mu}A^2}\cos h\alpha\,, \tag{12}$$

$$h_4 = \frac{M}{\bar{\mu}}\sin h\alpha \tag{13}$$

and

$$|h|^2 = h_i h^i = \frac{M^2}{\bar{\mu}^2 A^4}\,. \tag{14}$$

From (7) we have,

$$E_1^1 = E_2^2 = \frac{M^2}{2\bar{\mu}A^4} = -E_3^3 = -E_4^4\,. \tag{15}$$

The field equation (2) for metric (1) reduces to

$$\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{B}}{B}=-8\pi\left[p+\frac{M^2}{2\bar{\mu}A^4}\right], \tag{16}$$

$$\frac{\dot{A}^2}{A^2}+\frac{2\ddot{A}}{A}=-8\pi\left[(\varepsilon+p)\sin^2 h\alpha+p+2q_3\frac{\sin h\alpha}{B}-\frac{M^2}{2\bar{\mu}A^4}\right], \tag{17}$$

$$\frac{\dot{A}^2}{A^2}+\frac{2\dot{A}\dot{B}}{AB}=-8\pi\left[-(\varepsilon+p)\cos^2 h\alpha+p-2q_3\frac{\sin h\alpha}{B}-\frac{M^2}{2\bar{\mu}A^4}\right], \tag{18}$$

$$(\varepsilon+p)B\sin h\alpha\cos h\alpha+q_3\cos h\alpha+q_3\frac{\sin^2 h\alpha}{\cos h\alpha}=0\,. \tag{19}$$

Here the dot (.) over a field variable denotes the differentiation with respect to time t.

3. Solution of the Field Equations

The set (16) – (19) being highly non-linear containing six unknowns (A, B, α, ε, p and q_3). So to obtain a determinate solution we have to use two additional constraints. Let us first assume that the model is filled with dust which leads to

$$P=0 \tag{20}$$

and secondly we consider that the scalar expansion θ is proportional to the shear scalar σ leads to

$$B=A^n\,, \tag{21}$$

where *n* is constant.

Equation (17), (18) lead to

$$\frac{2\dot{A}^2}{A^2}+\frac{2\ddot{A}}{A}+\frac{2\dot{A}\dot{B}}{AB}=8\pi\varepsilon+\frac{8\pi M^2}{\bar{\mu}A^4}\;. \tag{22}$$

Equating (16) and (20) we have

$$\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{B}}{B}=\frac{-8\pi M^2}{2\bar{\mu}A^4}\;. \tag{23}$$

Equating (21) and (23) we have

$$2\ddot{A}+\frac{2n^2}{(n+1)}\frac{\dot{A}^2}{A}=\frac{N}{(n+1)A^3}\,, \tag{24}$$

where

$$N=\frac{-8\pi M^2}{\bar{\mu}}\;. \tag{25}$$

The equation can be rewritten as

$$\frac{df^2}{dA}+\frac{a}{A}f^2=\frac{N}{(n+1)A^3}, \tag{26}$$

where $a=\frac{2n^2}{(n+1)}$, (27)

and $\dot{A}=f(A)$, (28)

$$\ddot{A}=ff' \quad \text{when} \quad f'=\frac{df}{dA} \quad . \tag{29}$$

Equation (26) leads to

$$\left(\frac{dA}{dt}\right)^2=\frac{N}{2a_1A^2}+\frac{C}{A^a}, \tag{30}$$

where $a_1=(n^2-n-1)$, $a=\frac{2n^2}{n+1}$ & C is constant of integration .

The metric (1) reduces to form

$$ds^2=-\left(\frac{dt}{dA}\right)^2dA^2+A^2\left(dx^2+dy^2\right)+A^{2n}dz^2 \tag{31}$$

Equating (31) we get

$$ds^2=-\left[\frac{N}{2a_1T^2}+\frac{C}{T^a}\right]^{-1}dT^2+T^2\left(dX^2+dY^2\right)+T^{2n}dZ^2 \tag{32}$$

Where $A=T$, $dx=dX$, $dy=dY$, $dz=dZ$.

4. Some Physical and Geometrical Properties

The density for the model (31) is given by

$$8\pi\varepsilon=\frac{a_2N}{T^4}+\frac{a_3C}{T^{a_4}}, \quad \underline{n\neq -1} \tag{33}$$

where $a_2=\frac{-(n^2-2n-3)}{n^2-n-1}$, $a_3=\frac{2(2n^2+2n+1)}{(n+1)}$ & $a_4=\frac{2(n^2+n+1)}{(n+1)}$.

The tilt angle α is given by

$$\cos h\alpha=\left[\frac{a_5NT^{a_6}+a_7}{a_8NT^{a_6}+a_9}\right]^{\frac{1}{2}}, \tag{34}$$

where $a_5=(-n^3+2n^2+5n+2)$, $a_6=\frac{2(n^2-n-1)}{(n+1)}$,

$a_7 = (n^4 + 2n^3 - 3n^2 - 4n - 1)C$, $a_8 = 2(n^2 - 1)$,

and $a_9 = 2(-2n^4 + 4n^3 - 2n)C$.

$$\sin h\alpha = \left[\frac{a_{10}NT^{a_6} + a_{11}}{a_8NT^{a_6} + a_9}\right]^{\frac{1}{2}}, \tag{35}$$

where $a_{10} = -n^3 + 5n + 4$, $a_{11} = -(3n^4 + 6n^3 - 3n^2 + 8n + 1)C$.

The expansion (θ) calculated for the flow vector v^i is given by

$$\theta = \frac{(n+2)}{T}\left[\frac{N}{2a_1T^2} + \frac{C}{T^a}\right]\left(\frac{a_5NT^{a_6} + a_7}{a_8NT^{a_6} + a_9}\right)^{\frac{1}{2}}, \quad n \neq -2 \tag{36}$$

The flow vector v^i and heat conduction vector q^i for the metric (31) are given by

$$v^3 = \frac{1}{T^n}\left[\frac{a_{10}NT^{a_6} + a_{11}}{a_8NT^{a_6} + a_9}\right]^{\frac{1}{2}}, \tag{37}$$

$$v^4 = \left[\frac{a_5NT^{a_6} + a_7}{a_8NT^{a_6} + a_9}\right]^{\frac{1}{2}} \quad , \tag{38}$$

$$q^3 = \frac{\left[\frac{a_{10}NT^{a_6} + a_{11}}{a_8NT^{a_6} + a_9}\right]^{\frac{1}{2}}\left[\frac{a_5NT^{a_6} + a_7}{a_8NT^{a_6} + a_9}\right]}{T^n 8\pi\left[\frac{(1-n)N}{a_1T^4} + \frac{2n(n-1)C}{T^{a_4}}\right]}, \tag{39}$$

$$q^4 = \frac{\left[\frac{a_{10}NT^{a_6} + a_{11}}{a_8NT^{a_6} + a_9}\right]\left[\frac{a_5NT^{a_6} + a_7}{a_8NT^{a_6} + a_9}\right]^{\frac{1}{2}}}{8\pi\left[\frac{(1-n)N}{a_1T^4} + \frac{2n(n-1)C}{T^{a_4}}\right]}. \tag{40}$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (w_{ij}) are given by

$$\sigma^2 = \frac{2(1-n)}{3T}\left[\frac{N}{2a_1T^2} + \frac{C}{T^a}\right]^{\frac{1}{2}} \quad , \qquad \underline{n \neq 1} \tag{41}$$

$$\omega_{34} = nT^{n-1}\left[\frac{N}{2a_1T^2} + \frac{C}{T^a}\right]^{\frac{1}{2}}\left[\frac{a_{10}NT^{a_6} + a_{11}}{a_8NT^{a_6} + a_9}\right]^{\frac{1}{2}}\left[\frac{\left[\frac{a_2N}{T^4} + \frac{a_3C}{T^{a_4}}\right]}{\left[\frac{(1-n)N}{a_1T^4} + \frac{a(n-1)}{nT^{a_4}}\right]}\right]. \tag{42}$$

The rate of expansion H_i in the direction of *x*, *y* and *z*-axis respectively are given by

$$H_1 = H_2 = \frac{2}{T}\left[\frac{N}{2a_1T^2} + \frac{C}{T^a}\right]^{\frac{1}{2}}, \tag{43}$$

$$H_3 = \frac{2n}{T}\left[\frac{N}{2a_1T^2} + \frac{C}{T^a}\right]^{\frac{1}{2}}. \tag{44}$$

5. Discussion

At the initial moment energy density is infinite where as it vanishes when T is infinite. i.e The model (32) starts with big bang at T = 0 and stops at $T = \infty$.Thus model has point -type singularity at T = 0.The tilt angles and the flow vectors are

$\cos h\alpha = \sqrt{\frac{a_7}{a_9}}$, $\sin h\alpha = \sqrt{\frac{a_{11}}{a_9}}$, $v^3 = \infty, v^4 = \sqrt{\frac{a_7}{a_9}}$, when T = 0 provided $n \neq 1$,

Whereas $\cos h\alpha = \sqrt{\frac{a_5}{a_8}}$, $\sin h\alpha = \sqrt{\frac{a_{10}}{a_8}}$, $v^3 = 0$, $v^4 = \sqrt{\frac{a_5}{a_8}}$, when $T = \infty$ provided $n \neq \pm 1$. Thus tilt angles are constant for both T = 0 and $T = \infty$.

From (36), initially the rate of expansion θ is infinite, it decreases when time increases and the expansion stop at $T = \infty$. At T = 0 shear scalar σ is infinite provided $n \neq 1$ where as it vanishes when T is infinite. Initially directional Hubble parameters are infinite and it vanishes for large value of T. Heat conduction vector q^3 is infinite at T = 0 and vanishes at infinite time provided n > 4 and n > a_4 and q^4 vanishes when T = 0 but it is infinite for large value of T. Since $\lim_{T \to \infty}\left(\frac{\sigma}{\theta}\right) \neq 0$ the model not approach isotropy for large value of *T*.

In this case, the models are expanding, shearing, rotating for tilted universe.

When the magnetic field is absent ($N = 0$):

$$ds^2 = -dt^2 + (kT)^{2/k}\left(dx^2 + dy^2\right) + (kT)^{2n/k}\,dz^2 \tag{45}$$

where $T = (ct + d)$ & $k = \frac{n^2 + n + 1}{n + 1}$

The density for the model (45) is given by

$$\varepsilon = \frac{2c^2\left(n-k+2\right)}{8\pi\left(kT\right)^2} \tag{46}$$

Tilt angles are given by

$$\cos h\alpha = \sqrt{\frac{2k-3}{2\left(1-n-k\right)}} \tag{47}$$

$$\sin h\alpha = \sqrt{\frac{4k+2n-5}{2\left(1-n-k\right)}} \tag{48}$$

The flow vector v^i and heat conduction vector q^i for the metric (45) are given by

$$v^3 = \frac{1}{\left(kT\right)^{n/k}}\left[\frac{4k+2n-5}{2\left(1-n-k\right)}\right]^{\frac{1}{2}} \tag{49}$$

$$v^4 = \left[\frac{2k-3}{2\left(1-n-k\right)}\right]^{\frac{1}{2}} \tag{50}$$

$$q^3 = \frac{c^2\left(2k-3\right)}{8\pi\left(kT\right)^{\frac{(2k+n)}{k}}}\sqrt{\frac{4k+2n-5}{2\left(1-n-k\right)}} \tag{51}$$

$$q^4 = \frac{c^2\left(4k-2n-5\right)}{8\pi\left(kT\right)^2}\sqrt{\frac{2k-3}{2\left(1-n-k\right)}} \tag{52}$$

Scalar expansion (θ), the non-vanishing component of shear tensor $\left(\sigma_{ij}\right)$ and rotation tensor $\left(\omega_{ij}\right)$ respectively are given by

$$\theta = \frac{\left(n+2\right)}{kT}\sqrt{\frac{2k-3}{2\left(1-n-k\right)}} \tag{53}$$

$$\sigma^2 = \frac{2}{3}\left[\frac{c\left(1-n\right)}{kT}\right]^2 \tag{54}$$

$$\omega_{34} = nc\left(kT\right)^{\frac{n-k}{k}}\left[\frac{k-n-2}{n+k-1}\right]\sqrt{\frac{4k+2n-5}{2\left(1-n-k\right)}} \tag{55}$$

The rate of expansion H_i in the direction of *x*, *y* and *z*-axis respectively are given by

$$H_1 = H_2 = \frac{2c}{kT} \tag{56}$$

$$H_3 = \frac{2nc}{kT} \tag{57}$$

At the initial moment energy density and scalar expansion are infinite where as energy density and scalar expansion vanish when T is infinite. The tilt angles are (47) and (48) which are the same at any instant. The flow vectors v^3 is infinite at T= 0 and vanishes when time is infinite. The flow vectors v^4 is constant for both T = 0 and $T = \infty$. Initially heat conduction vector q^3 & q^4 are infinite and it vanishes for large value of T. At T = 0 shear scalar σ is infinite where as it vanishes when T is infinite. Initially directional Hubble parameters are infinite and it vanishes for large value of T.

Since $T \overset{\lim}{\to} \infty \left(\frac{\sigma}{\theta}\right) \neq 0$ the model not approach isotropy for large value of *T*.

7. Conclusion

In the present paper we have constructed tilted plane symmetric dust fluid cosmological model in presence and absence of the magnetic field. We have obtained a determinate solution by assuming the conditions that model is filled with dust (which leads to the zero pressure) and the expansion scalar ϴ is proportional to the shear scalar σ. We have discussed the physical behavior of the models in presence and absence of the magnetic field. Energy density, expansion scalar and shear scalar have the similar behavior in presence and absence of magnetic field. At the initial epoch all these parameters are infinite and decreases with increase in cosmic time. In the presence and absence of the magnetic field the models are expanding, shearing, rotating and tilted. Only variations in tilt angles are found. In the presence of the magnetic field tilt angles are the functions of time and it tends to finite number when time is infinite and they are infinite when time zero. Whereas tilted angles are constant (very small) throughout the evolution of the universe in absence of magnetic field. The model has real singularity at T=0 and it start with big bang and stops when cosmic time is infinite, in other word it becomes asymptotically empty. The present model does not approach isotropy in presence and absence of magnetic field since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$.

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