Article

LRS Bianchi Type-V Dust Filled Universe with Varying Λ(t) in Creation Field Theory of Gravitation

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Abstract

We have studied the Hoyle-Narlikar C-field cosmology for LRS Bianchi type-V space times with varying cosmological constant Λ(t) when the universe is filled with dust distribution. To get deterministic solution, we assumed that $\Lambda = 1/A^2$ as considered by Chen & Wu (Phys. Rev. D 41:695, 1990), where A is a scale factor. The physical aspects of the model are also studied.

Keywords: creation field, cosmology, varying cosmological constant.

1. Introduction

In the study of early universe, all the investigations dealing with physical process are successfully explained by the big- bang model based on Einstein's field equations. This big-bang model is described by Robertson-Walker line element and a matter density source which obeys the equation of state $\rho = 3p$. The big-bang model explains three important observational results *viz*. the phenomenon of expanding universe, Primordial nucleosynthesis and the observed isotropy of the Cosmic Microwave Background Radiation (CMBR). The astronomical predictions of the Friedman-Robertson-Walker model do not always meet the requirements as was believed earlier [1]. The theoretical explanations given from the big-bang type of the model were in contrasts with some puzzling results regarding the red-shifts from the extra galactic objects.

Alternative theories of gravitation were proposed from time to time to overcome the drawbacks of big-bang model. Also CMBR discovery did not prove it to be an outcome of big-bang theory. The possibility of non-relic interpretation of CMBR had been proved by Narlikar *et al*. [2]. Bondi and Gold [3] proposed a steady state theory in which the universe does not have any singular beginning or an end on the cosmic time scale where the matter density is throughout constant. They considered a very slow but continuous creation of matter in contrasts to explosive creation of standard model to maintain constancy of matter density. The steady state theory was discarded for not giving any physical justification to continuous creation of matter. Hoyle and Narlikar [4, 5, 6] adopted a field theoretic approach by introducing a massless and chargeless scalar field C in Einstein-Hilbert action to account for the creation of matter. Narlikar & Padmanabhan [7] obtained a solution of Einstein's field equations admitting radiations with

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negative energy massless scalar creation field C. Chatterjee & Banerjee [8] carried out the study of Hoyle-Narlikar theory to higher dimensional space times. Singh & Chaubey [9] investigated Bianchi type cosmological models in the C-field theory of gravitation. Bali *et al*. [10, 11] have studied FRW cosmological models with variable gravitational constant G in different context. Adhav *et al.* [12, 13] have investigated LRS Bianchi type-I and LRS Bianchi type-V space-times in Creation field cosmology. Recently, Katore [14] has studied plane symmetric universe in C-field cosmology.

A number of studies have concluded that the universe is undergoing an accelerated expansion. Cosmological constant (Λ) is one of the most important candidates which is responsible for expansion of universe. Einstein introduces Λ term into field equations as a blunder mistake of his life. But later on attempts are made by many researchers to discuss cosmological models using Λ term. Astronomical work such as gravitational lens statistics [15], CMB anisotropies [16, 17] & CMB intensity [18], structure formation [19] and cluster abundance [20] continue on using Λ as constant term. A wide range of observations by cosmologist and field theory researchers like Schmidt *et al.* [21], Garnavich *et al.* [22] suggest that the universe possesses a non-zero cosmological constant. Bertolami [23] was the first who consider cosmological models with a variable cosmological constant of the form $\Lambda \sim t^{-2}$. Berman and Som [24] have shown that $\Lambda \sim t^{-2}$ plays a very important role in cosmology. Chen and Wu [25] have also solved the problem by considering $\Lambda \sim R^{-2}$, where R is the scale factor in the Robertson-Walker space times. Carvalho, Lima and Waga [26] generalized the proposed model of Chen and Wu by including a term proportional to H^2 on the time dependence of Λ.

At the same time cosmologist *viz.* Barr [27], Pebbles & Ratra [28], Moffat [29], Frieman *et al.* [30], Abramo *et al*. [31] and many other authors had shown that cosmological term decays with time. The recent observations from astronomy indicate that $\Lambda \sim 10^{-56} cm^{-2}$ but the theory of physics of elementary particles predicts that the value of Λ must have been 10^{120} times larger in the past. It is worth noting that cosmological models based on Einstein field equations with a time-dependent cosmological constant Λ had been the subject of numerous models in recent years. Abdel Rahman [32] and Lima *et al.* [33] have constructed cosmological models with more general form of variable cosmological constant Λ term. Overduin [34] have investigated FRW cosmological model by assuming $\Lambda \sim t^{-2}$ and $\Lambda \sim H^{-2}$ and shows their compability with various observations. Several attempts have made by many researchers *viz.* Vishwakarma [35], Lui & Wesson [36], Pradhan *et al.* [37] and Singh *et al*. [38] in the favor of time dependent $\Lambda \sim t^{-2}$ in different contexts. Katore *et al.* [39] solved anisotropic plane symmetric magnetized model with cosmological constant. Baghel & Singh [40] obtained Bianchi type V universe with bulk viscous matter and time varying gravitational and cosmological constant. Recently, Bali and Saraf [41, 42, 43] have investigated different cosmological models with varying Λ in Creation field theory of gravitation.

In this paper, we have investigated LRS Bianchi type-V space-time with varying cosmological constant $\Lambda(t)$ in the creation field theory of gravitation. The solution of the field equations are obtained by assuming a relation $\Lambda = \frac{1}{A^2}$ (Chen and Wu (1990)), where A is a scale factor. This work is organized as follows. In Section 2, the model and field equations have been presented. The solution of field equations with special cases $m = 0, \pm 1$, has been discussed in Section 3. Then in Section 4, the physical aspects of the model have been discussed. In the last Section 5 concluding remarks have been expressed.

2. The Metric and Field Equations

We consider LRS Bianchi type-V metric considered in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2mx}(dy^2 + dz^2), \tag{1}$$

where A and B are functions of time t only and m is a constant.

Einstein field equations by introduction of C-Field are modified by Hoyle and Narlikar (1964a,b,c) as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G\left(\underset{(m)}{T_i^j} + \underset{(c)}{T_i^j}\right) - \Lambda g_i^j \ . \tag{2}$$

The energy momentum tensor $\underset{(m)}{T_i^j}$ for perfect fluid and $\underset{(c)}{T_i^j}$ for creation field are given by

$$\underset{(m)}{T_i^j} = (\rho + p) v_i v^j - p g_i^j \tag{3}$$

and
$$\underset{(c)}{T_i^j} = -f\left(c_i c^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right), \tag{4}$$

Here ρ is the energy density of massive particle and p is the pressure. v_i are co-moving four velocities which obeys the relation $v_i v^j = 1$. The coupling constant between matter and creation field is greater than zero. It is assumed that creation field C is a function of time only i.e. $C(x,t) = C(t)$.

With the help of equations (3) and (4), the Hoyle-Narlikar field equations (2) for the metric (1) lead to

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} - 3\frac{m^2}{A^2} = 8\pi G\left(\rho - \frac{1}{2} f \dot{C}^2\right) + \Lambda \tag{5}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{m^2}{A^2} = 8\pi G\left(-p + \frac{1}{2} f \dot{C}^2\right) + \Lambda \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = 8\pi G\left(-p + \frac{1}{2} f \dot{C}^2\right) + \Lambda \tag{7}$$

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} \ . \tag{8}$$

where overhead dot $\left(\dot{}\right)$ denotes differentiation with respect to time t.

From equation (8), we get

$$B = KA, \tag{9}$$

where K is a constant of integration.

Using equation (9), Field equations (5)-(7) reduce to

$$3\frac{\dot{A}^2}{A^2} - 3\frac{m^2}{A^2} = 8\pi G\left(\rho - \frac{1}{2} f \dot{C}^2\right) + \Lambda \tag{10}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{m^2}{A^2} = 8\pi G\left(-p + \frac{1}{2} f \dot{C}^2\right) + \Lambda . \tag{11}$$

The conservation equation

$$(8\pi G T_i^j + \Lambda g_i^j)_{;j} = 0 \tag{12}$$

leads to

$$8\pi\dot{G}\left[\rho - \frac{1}{2} f \dot{C}^2\right] + 8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + 3\rho\frac{\dot{A}}{A} - 3f\dot{C}^2\frac{\dot{A}}{A} + 3p\frac{\dot{A}}{A}\right] + \dot{\Lambda} = 0 , \tag{13}$$

p being an isotropic pressure.

3. Solution of field equation

Following Hoyle and Narlikar (1964a,b,c) for dust distribution, we have taken $p = 0$. The source equation of C-field $C^j_{;i} = 0$ leads to $C = t$, for larger r. Thus $\dot{C} = 1$.

Using $p = 0$, in equation (11), we get

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{m^2}{A^2} = 4\pi G f \dot{C}^2 + \Lambda . \tag{14}$$

Also, using $\dot{C} = 1$, in equations (10) and (14) therein, we have

$$3\frac{\dot{A}^2}{A^2} - 3\frac{m^2}{A^2} = 8\pi G\left(\rho - \frac{1}{2} f\right) + \Lambda \tag{15}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{m^2}{A^2} = 4\pi G f + \Lambda . \tag{16}$$

Since, Gravitational constant G = constant and $p = 0$, equation (13) transfers to

$$8\pi G\dot{\rho} - 8\pi G f\dot{C}\ddot{C} + 24\pi G\rho\frac{\dot{A}}{A} - 24\pi G f\dot{C}^2\frac{\dot{A}}{A} + \dot{\Lambda} = 0 . \tag{17}$$

Solving equations (15) and (16), we get

$$\frac{\ddot{A}}{A} + 2\frac{\dot{A}^2}{A^2} - \frac{(2m^2 + 1)}{A^2} = 4\pi G\rho . \tag{18}$$

To get deterministic solution in terms of cosmic time t, we assume that $\Lambda = \frac{1}{A^2}$. {Chen & Wu (*Phys. Rev. D* 41:695, 1990)}.

Using $\Lambda = \frac{1}{A^2}$ in equation (16), we get

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{(m^2+1)}{A^2} = 4\pi Gf \quad , \tag{19}$$

which again leads to

$$2\ddot{A} + \frac{\dot{A}^2}{A} - \frac{(m^2+1)}{A} = 4\pi GfA. \tag{20}$$

To find the solution of equation (20), Let $\dot{A} = F(A)$.

which implies $\ddot{A} = FF'$, where $F' = \frac{dF}{dA}$.

Substituting in equation (20), it leads to

$$\frac{dF^2}{dA} + \frac{1}{A}F^2 = 4\pi GfA + \frac{(m^2+1)}{A}. \tag{21}$$

On integration simplifies to

$$F^2 = \frac{4\pi Gf}{3}A^2 + (m^2+1). \tag{22}$$

For simplicity, the integration constant taken to be zero.

Using $\dot{A} = F(A)$, equation (22) simplifies to

$$\frac{dA}{\sqrt{A^2 + \frac{3(1+m^2)}{4\pi Gf}}} = \sqrt{\frac{4\pi Gf}{3}}dt, \tag{23}$$

which leads to

$$\frac{dA}{\sqrt{A^2+\alpha^2}} = \beta\, dt, \tag{24}$$

$$\text{where} \quad \alpha^2 = \frac{3(1+m^2)}{4\pi Gf}, \quad \beta = \sqrt{\frac{4\pi Gf}{3}}. \tag{25}$$

Equation (24) on integration gives

$$A = \alpha \sinh \beta t. \tag{26}$$

In particular for $\beta = 1$, we have

$$A^2 = (1+m^2)\sinh^2 t \tag{27}$$

$$\text{and} \quad \Lambda = \frac{1}{A^2} = \frac{\operatorname{cosech}^2 t}{1+m^2} \tag{28}$$

Using equations (27), (28) in equation (15), we have

$$\rho = \frac{1}{8\pi G}\left[3\frac{\dot{A}^2}{A^2} - \frac{(3m^2+1)}{A^2} + 4\pi G f\right]. \tag{29}$$

Using equation (27) in metric (1), we get

$$ds^2 = dt^2 - (1+m^2)\sinh^2 t\left[dx^2 - e^{-2mx}\left(dy^2 + dz^2\right)\right]. \tag{30}$$

Case (i): $m = \pm 1$

Equation (22) leads to

$$\frac{dA}{\sqrt{A^2+2}} = dt \ , \tag{31}$$

where $\frac{4\pi G f}{3} = 1$.

Equation (31) on integration gives

$$A = \sqrt{2}\sinh t. \tag{32}$$

Using equations (29) and (32) in equation (17), we have

$$\frac{d\dot{C}^2}{dt} + (6\coth t)\dot{C}^2 = 6\coth t. \tag{33}$$

Equation (33) leads to

$$\dot{C}^2 \sinh^6 t = 6\int \coth t.\sinh^6 t\, dt \ . \tag{34}$$

On simplification equation (34) reduce to

$$\dot{C} = 1, \tag{35}$$

which leads to

$$C = t \ , \tag{36}$$

which agrees with the value used in the source equation. Thus creation field C increases with time t.

Case (ii): $m = 0$

Equation (21) leads to

$$\frac{dA}{\sqrt{A^2+1}} = dt, \tag{37}$$

where $\frac{4\pi G f}{3} = 1$.

Equation (37) on integration gives

$$A = \sinh t. \tag{38}$$

Using equations (29) and (38) in equation (17), we have

$$\frac{d\dot{C}^2}{dt}+(6\coth t)\dot{C}^2=6\coth t\,. \tag{39}$$

Equation (39) leads to

$$\dot{C}^2\sinh^6 t=6\int \coth t.\sinh^6 t\,dt \tag{40}$$

On simplification equation (40) reduce to

$$\dot{C}=1, \tag{41}$$

which leads to

$$C=t\,, \tag{42}$$

which agrees with the value used in the source equation. Thus creation field C increases with time t.

4. Physical Aspects

The Energy mass density (ρ), the cosmological constant (Λ) and the deceleration parameter (q) for the model (30) are given by

$$8\pi G\rho=\frac{2\coth^2 t+2}{1+m^2}, \tag{43}$$

$$\Lambda=\frac{1}{(1+m^2)\sinh^2 t}, \tag{44}$$

$$A=\sqrt{(1+m^2)}\,\sinh t\quad . \tag{45}$$

$$q=-\tanh^2 t, \tag{46}$$

where $4\pi Gf=3$ and $\beta=1$ assumed.

5. Conclusion

The Spatial volume V increases for large values of t. The creation field C increases with time t. The homogeneous mass density $\rho>0$, for all values of m. The deceleration parameter $q<0$ indicating that the model (30) representing an accelerating universe, which is singularity free model. The cosmological constant $\Lambda\sim t^{-2}$, which matches with latest astronomical observations. Our model resembles exactly same with the model investigated by Bali & Saraf (2013) for FRW space times.

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