Article

Exact Kantowski-Sach Cosmological Models with Constant EoS Parameter in Barber's Second Self-Creation Theory

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Abstract

We have constructed spatially homogeneous and anisotropic Exact Kantowski-Sach dark energy cosmological models with constant equation of state (EoS) Parameter in Baber′s second self-creation theory of gravitation. The observed values of dark energy EoS parameter ω and deviation parameter δ are not necessarily the function of cosmic time but their determinate values for derived models are in good agreement with recent observations of SNe Ia data. In order to obtain the exact solution of the Einstein's field equation we have considered the spatial law of variation of Hubble′s parameter which yields a negative constant deceleration parameter and in view of the anisotropy space time, the scalar expansion θ in the model is proportional to shear σ which leads to $A = R^n$. Some physical and geometrical behaviours of the cosmological models are also studied.

Keywords: Kantowski-Sach, dark energy, Barber, self-creation.

1. Introduction

In recent years a considerable interest has been focused by cosmologists in formation of alternative theories of gravitations. Brans and Dicke [1] is one of them who have attracted the many workers towards scalar tensor theories of gravitations. For the more time according to the observations of the cosmologists it was the best alternative for the Einstein Theory of gravitations. In an attempt to produce continuous creation theories, Barber [2] has proposed two self creation theories modifying the Brans Dicke theory of gravitation and Einstein theory of general relativity.

According to Brans and Dicke [3] Baber's first self creation theory is unsatisfactory because of the violation in equivalence principle. Second self creation theory is the modification of general theory of relativity to a variable G which includes continuous creation of matter but in the limit of observations. This modified theory creates the universe out of self contained gravitational and

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matter field. In this theory scalar field does not gravitate directly, but it simply divides the matter tensor, acting as a reciprocal gravitational constant. This theory postulates that scalar field used couples to the trace of the energy momentum tensors. Hence field equations used in Baber's second theory are:

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\varphi} T_{ij} \quad \text{and} \quad \Box\varphi = \frac{8\pi}{3} \lambda T$$

Where λ is a coupling constant and φ is a Barber's scalar. In the limit $\lambda \to 0$ the theory approaches Einsten's theory of general relativity in every respect. According to the cosmologists Reiss,A.G.,et al. and Perlmutter,s et al.[4-5] dark energy plays an important role in order to explain and investigate the cosmic accelerated expansion of the universe. The cosmological observations of the type SNe Ia supernovae [6-8] indicate that the current universe is not only expanding but also accelerating. In addition to this to Milkinson Microwave Anisotropy Probe (WMAP) this acceleration is cosmic on the combinations of results from the large scale distribution of galaxies and the most precise data on the cosmic microwave background (CMB).

Now it is confirmed that the dark energy with negative pressure is the most important component responsible for the cosmic accelerated expansion of the universe. According to Dobado [9] introduction of cosmological constant with equation of state ω_Λ= -1 is also responsible for an accelerated expansion of the universe. Apart from the cosmological constant there are other candidates to dark energy responsible for expansion of the universe. The phantom fields, the quintessence, chaplygingas, tachyon field [10-14] etc are various alternative candidates for dark energy.

In general theory of relativity relationship between temperature, pressure, energy, vacuum energy density parameterize. The evolution of the expansion rate of universe is parameterized by the relationship between temperature, pressure and combined matter, energy and vacuum energy density in general theory of relativity. According to Carroll [15], today, dark energy model is characterized by the equation of state (EoS) parameter $\omega = p/\rho$ which is not necessarily constant. In the present model we have observed that EoS parameter is independent of cosmic time where as the experimental data obtained by Sahni and Starobinsky [16] and analysis done by Sahni, et al [17] the EoS parameter is a function of cosmic time.

The phantom energy ($\omega < -1$), quintessence are the simple candidates for the dark energy having time dependent EoS parameter but usually EoS parameter is considered as constant with phase wise value -1,0,-1/3 and+1 for vacuum field, dust fluid, radiation and stiff fluid dominated universe respectively but according to our observations in the present paper $\omega = -1$ is not admissible value for the Exact Kantowski-Sach cosmological model in Barber's second self creation theory. It is due to because of lack of observational evidence it is difficult to distinguish between constants and variables ω [18-19]. But in general EoS parameter is a function of time or red shift [20-21].

Steinhardt,P.J. ,et al [22-23] have used Kaluza Klein cosmological model with variable EoS parameter. Soleng [24] have discussed F-R-W models in Barber's second theory of self creation. Reddy and Venkateshwarlu [25] have studied the Bianchi type VI_0 cosmological model in Barber's second theory of gravitation. Pawar et al. [26] investigated string cosmological model in presence of massless scalar field in modified theory of general relativity.

2. The Metric and Field Equations

We consider the Exact Kantowski-Sach space time

$$ds^2 = dt^2 - A^2 dr^2 - B^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) , \tag{2.1}$$

where the metric potentials A, B are the functions of cosmic time t.

The field equations in Barber's second self creation theory are

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\varphi} T_{ij} , \tag{2.2}$$

and $\qquad \Box\varphi = \varphi_{;k}{}^{,k} = \frac{8\pi}{3}\lambda T ,$ (2.3)

where T is the trace of the energy momentum tensors, φ is the Barber's scalar and λ is coupling constant. In the limit $\lambda \to 0$ the theory approaches Einstein's theory of general relativity in every respect.

The energy momentum tensor for the perfect fluid distribution is given by

$$T_i^j = diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right] \tag{2.4}$$

Here we are dealing with an anisotropic dark energy . We can parameterize eq^n.(2.4) as

$$T_i^j = diag\left[-p_r, -p_\theta, -p_\phi, \rho\right] \tag{2.5}$$

where ρ is the energy density of the fluid; p_r, p_θ, p_ϕ are the pressures along the co-ordinate axes. The anisotropic fluid is characterized by EoS $p = \omega\rho$ where EoS parameter ω is not necessarily function of cosmic time t. Here we have adjusted the EoS parameter along r-axis by setting $\omega_r = \omega$ so that we are free to introduce skewness parameter δ which is the deviation from ω along θ and ϕ axes.

Thus we have energy momentum tensor as

$$T_i^j = diag\left[-\omega_r, -\omega_\theta, -\omega_\phi, 1\right]$$
$$T_i^j = diag\left[-\omega, -(\omega+\delta), -(\omega+\delta), 1\right]\rho \tag{2.6}$$

By using co-moving co-ordinate system, Barber's field equations (2.2),(2.3) and(2.6) space time eq^n.(2.1) reduces to

$$\frac{2\ddot{B}}{B} + \left(\frac{\ddot{B}}{B}\right)^2 + \frac{1}{B^2} = -\frac{8\pi\omega}{\varphi}\rho \tag{2.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \frac{-8\pi(\omega+\delta)\rho}{\varphi} \tag{2.8}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{1}{B^2} = \frac{8\pi}{\varphi}\rho \tag{2.9}$$

$$\ddot{\varphi} + \left(\frac{\dot{A}}{A} = \frac{2\dot{B}}{B}\right)\dot{\varphi} = \frac{8\pi\lambda}{3}(1-2\delta-3\omega)\rho \tag{2.10}$$

Here overhead dot of the field variables represents the ordinary differentiation with respect to cosmic time.

3. Solutions of the Field Equations

The system of the equations (2.7) – (2.10) containing six unknowns $A, B, \rho, \omega, \delta, \varphi$ and four linearly independent equations. Therefore in order to obtain an explicit solution we have to assume two additional constraints. We solve above system of equations by assuming special law of variations of Hubble's parameter proposed by Berman (1983) which yields constant deceleration parameter of the models of the universe. For an accelerating universe we consider the constant as negative.
Let us consider

$$q = \frac{-R\ddot{R}}{\dot{R}^2} = cons\tan t \tag{3.1}$$

For an accelerating model of the universe we take negative constant.

Equation (3.1) gives

$$R=\left((1-q)^{\frac{1}{(1-q)}}\right)(at+b)^{\frac{1}{(1-q)}} \tag{3.2}$$

provided $a\neq 0$, b are constants of integrations

Another constraint we have to assume that the expansion scalar θ in the model is proportional to the shear scalar which leads to the condition $A=R^n$ as

$$A=R^n=(1-q)^{\frac{n}{(1-q)}}(at+b)^{\frac{n}{(1-q)}} \tag{3.3}$$

where $n\neq 0$ is constant.

Also from the given space time we have

$$R^3=AB^2 \tag{3.4}$$

Equations (3.2),(3.3) and (3.4) gives

$$B=(1-q)^{\frac{3-n}{2(1-q)}}(at+b)^{\frac{3-n}{2(1-q)}} \tag{3.5}$$

Using equations (2.9) (3.3),(3.5) with eq^n. (2.7) gives two values for EoS parameter, which are

$$\omega=-\frac{(5-3n+4q)}{3(n+1)} \tag{3.6}$$

and

$$\omega=-1 \tag{3.7}$$

But $\omega=-1$ is not admissible value to get the exact solution of the field equations because we cannot determine the Barber scalar φ and hence it is rejected.

By using eq^{n}s.(3.3),(3.5),(3.6) with eq^n. (2.8) gives two different values for the skewness parameter δ which are

$$\delta=\frac{2(2+q)(1-n)}{(3-n)(1+n)}\ ,\qquad n\neq\pm 1 \tag{3.8}$$

and

$$\delta=\frac{(5-3n+4q)}{(3-n)(1+n)} \tag{3.9}$$

In this case also the value of skewness parameter given by the eq^n. (3.9) is not admissible because it contradicts to get the exact solution of the field equations and hence rejected. After subtracting eq^n. (2.7) from eq^n. (2.9) and using it with eq^n. (3.6),(3.8) in eq^n. (2.10) we get

$$(at+b)^2\ddot{\varphi}=\frac{3a}{(1-q)}(at+b)\dot{\varphi}-\lambda\frac{a^2\left(n^2-2n+4q+5\right)\left(-2n^2+7n-2q-1\right)}{2(3-n)(3n-2q-1)(1-q)^2}\varphi=0\,. \quad (3.10)$$

On integration for eq^n. (3.10) we get the solutions for φ as

$$\varphi=c_1\left(at+b\right)^{m_1}+c_2\left(at+b\right)^{m_2} \quad (3.11)$$

Thus our two basic solutions for φ are

$$\varphi_1=c_1\left(at+b\right)^{m_1} \quad (3.12)$$

and

$$\varphi_2=c_2\left(at+b\right)^{m_2} \quad (3.13)$$

where

$$m_1=\frac{-k_1+\sqrt{\left(k_1^{\,2}-4k_2\right)}}{2},\ m_2=\frac{-k_1-\sqrt{\left(k_1^{\,2}-4k_2\right)}}{2}$$

$$k_1=\frac{(2+q)}{(1-q)},\ k_2=\frac{-\lambda\left(n^2-2n+4q+5\right)\left(-2n^2+7n-2q-1\right)}{2(3-n)(3n-2q-1)(1-q)^2}$$

Using eq^n. (3.3),(3.5),(3.11) after little manipulation eq^n.(2.9) yields

$$\rho=\frac{1}{8\pi}\left[\frac{3a^2(3-n)(n+1)}{4(1-q)^2(at+b)^2}+\frac{1}{(1-q)^{\frac{(3-n)}{(1-q)}}(at+b)^{\frac{(3-n)}{(1-q)}}}\right]\left[c_1\left(at+b\right)^{m_1}+c_2\left(at+b\right)^{m_2}\right] \quad (3.14)$$

The Exact Kantowski-Sach cosmological model in Baber's second self creation theory takes the for

$$ds^2=dt^2-\left[(1-q)(at+b)\right]^{\frac{2n}{(1-q)}}dr^2-\left[(1-q)(at+b)\right]^{\frac{(3-n)}{(1-q)}}\left(d\theta^2+\sin^2\theta d\phi^2\right) \quad (3.15)$$

4. Some physical parameter

The directional Hubble parameters $H_1 = H_r$, $H_2 = H_\theta$ ($H_2 = H_3$) are given by

$$H_1 = H_r = \frac{na}{(1-q)(at+b)} \tag{4.1}$$

$$H_2 = H_\theta = \frac{(3-n)a}{2(1-q)(at+b)} \tag{4.2}$$

The mean generalized Hubble's parameter H for the model (3.15) is given

$$H = \frac{a}{(1-q)(at+b)} \tag{4.3}$$

The spatial volume

$$V = (1-q)^{\frac{1}{(1-q)}} (at+b)^{\frac{1}{(1-q)}} \tag{4.4}$$

The expansion scalar θ and the shear scalar σ^2 are respectively given by

$$\theta = \frac{3a}{(1-q)(at+b)} \tag{4.5}$$

$$\sigma^2 = \frac{2a^2(2n-3)^2}{3(1-q)^2(at+b)^2}. \tag{4.6}$$

The mean anisotropy parameter of the expansion is defined as

$$A_m \equiv \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{(n^2-2n+1)}{2} \tag{4.7}$$

4.1. Physical behavior of the model

In the present paper it has been observed that the EoS parameter ω and the skewness parameter δ are found to be constants. The directional Hubble parameters as well as mean generalized Hubble parameter H, expansion scalar θ, shear scalar σ, the spatial volume V all these parameters are the functions of cosmic time. As cosmic time tends to infinite these parameter tends to zero, but these parameter diverges when cosmic time is $-b/a$ except the spatial volume.

The spatial volume of this model is zero when cosmic time is $-b/a$ and since the deceleration parameter is negative it increases with increase of time. Thus the derived model starts expanding with big bang singularity at $t=-b/a$ but it can be shifted to $t=0$ by choosing $b=0$.Thus since the deceleration parameter is negative as per the proposed assumption the model has a point type singularity at $t=-b/a$ The mean anisotropy parameter A_m is constant throughout the evolution of the universe. Since $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}\neq 0$ and it is independent of cosmic time. Thus the present model does not approach the isotropy for large value of cosmic time t.

5. Conclusion

Here we have studied the anisotropic Exact Kantowski-Sach cosmological model in Barber's second self creation theory of gravitation. The exact solution of the field equation is obtained using the condition that expansion scalar θ is proportional to the shear scalar σ. We have discussed some physical properties of the model. The derived model will help the workers for understanding the structure of the universe at the early stages of evolution of the universe because the recent observations of SN Ia reveal that the present universe is aceelerating.

References

01) Brans,C.,Dicke,R.H. : Phys. Rev. 124,925 (1961)
02) Barber,G.A.,Gen.Relativ. Gravit. 14, 117 (1982)
03) Brans, C.,Gen.Relativ.Gravit.19,949 (1987)
04) Reiss, A.G.,et al., Astron.J.116,1009 (1998)
05) Perlmutter S.,et al., Astrophys.J.517, 565 (1999)
06) Spergel, D.N.,et al., Astrophys. J. Suppl.Ser 148,175 (2007)
07) Spergel D.N.,et al., Astrophys. J.Suppl. Ser. 170,377 (2007)
08) Ress, A.G.,et al., Astron.J. 607,665 (2004)
09) Dobado, A.,Maroto,A.L.,Astrophys.Space Sci. 320,167 (2009)
10) Caldwell,R.R.,Phys.Lett. B. 545,23 (2002)
11) Ratra,B. Peebles,J.Phys.Rev.D37,321(1988)
12) Kamenshchik,A.,Moschella U.,Pasquier V.,Phys. Lett. B.511,265(2001)
13) Padmanabhan T.,Phys. Rev. D66,021301(2002)
14) Sen,A., High Energy Phys.04,048 (2002)
15) Carroll,S.M.,Hoffman M., Phys.Rev. D68, 023509 (2003)
16) Sahni V., Starobinsky A.A. Int. J.Mod. Phys.D15,2105 (2006)
17) Sahni V.,Shafielooa A.,Starobinsky A.A.Phys.Rev. D78,103502 (2008)
18) Kujat J.,et al., Astrophys.J 572,1(2002)
19) Bartelman M.,et al., New Astron.Rev.49,199 (2005)
20) Jimenez R.,New Astron.Rev.47, 761 (2003)
21) Das A.,et al.,Phys. Rev . D72,043528 (2005)
22) Steinhardt,P.J. et al.,Phys. Rev.D.59,123504 (1999)

23) Rahaman,F.,Bhui,B.C.,Astrophys. Space sci. 301,47 (2006)
24) Soleng,H.H. : Astrophys.space sci. 139,13 (1987)
25) Reddy,D.R.K.Venkateshwarlu,R.: Astrophys. Space sci. 155,135 (1989)
26) Pawar, D.D., Bayaskar S. N., Deshmukh A.G.: Rom. Journ. Phys., Vol. 56, Nos. 5–6, P. 842–848, (2011)