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Some New LRS Bianchi Type-I Bulk Viscous Cosmological Models with Decaying Λ-Term

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Abstract

Locally rotationally symmetric Bianchi type-I cosmological models are examined in presence of bulk viscous fluid source and variable Λ-term. Consequences of four phenomenological decay of Λ have been discussed which are consistent with recent observations. Exact solutions of Einstein's field equations are obtained by assuming a special law of variation for Hubble parameter, which yields a constant value of deceleration parameter. Some cosmological parameters are discussed.

Keywords: Bianchi space-time, bulk viscosity, constant deceleration parameter, cosmological models, variable Λ-term.

1. Introduction

Cosmological observations of the type-Ia supernovae (SN Ia) [1, 2], cosmic microwave background radiation (CMBR) [3-9], large-scale structure (LSS) [10, 11] have confirmed that we live in a universe [12, 13] which is expanding with acceleration. The present accelerating phase of the expansion of the universe stands as one of the most challenging open problems in modern cosmology and astrophysics. This acceleration is characterized by the negative pressure and the positive energy density and hence violates strong energy condition which is popularly known as dark energy. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment suggests 73% content of the universe in the form of dark energy, 23% in the form of non-baryonic dark matter and the rest 4% in the form of usual baryonic matter as well as radiation.

Among many possible alternatives, the simplest candidate for dark energy is the vacuum energy which is mathematically equivalent to the cosmological constant (Λ). Due to the problem associated with its energy scale, it needs to be severely fine-tuned. There is a huge difference between the cosmological constant inferred from observation and the vacuum energy density predicted by quantum field theories. In an attempt to solve this problem, variable Λ was introduced such that Λ was large in the early universe and then decayed with evolution [14]. Cosmological scenarios with time varying Λ were proposed by several researchers. A number of

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models with different decay laws for the variation of cosmological term were investigated during last two decades [15-23].

In most of the cosmological models, the source of the gravitational field is assumed to be a perfect fluid. But these models do not satisfactorily explain the early stages of evolution. Dissipative effects involving both the bulk and shear viscosity play a significant role in the early evolution of the universe. A combination of cosmic fluid with bulk dissipative pressure can generate accelerated expansion [24]. Influence of viscosity on the nature of the initial singularity and on the formation of galaxies have been investigated by Murphy [24] and Collins and Stewart [25]. By taking viscous effects into account, the coincidence problem can also be solved [26, 27]. Bulk viscosity leading to an accelerated phase of the universe today has been studied by Fabries et al. [28]. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors [29-36].

The universe which we observe today is neither homogeneous nor isotropic. These symmetries can only be approximate. There are theoretical arguments [37, 38] and recent experimental data regarding CMBR anisotropies which support the existence of an anisotropic phase that approaches isotropic one [39]. Spatially homogeneous and anisotropic cosmological models which provide a richer structure, both geometrically and physically, than the FRW model play significant role in the description of early universe. For simplification and description of the large scale behaviour of the actual universe, LRS Bianchi-I space-time have been widely studied [40-45]. These kinds of models are interesting because they are equivalent to flat FRW universe [46]. For studying the possible effects of anisotropy in the early universe on present day observation, many researchers [47-52] have investigated Bianchi type-I models from different point of view.

Recently, Singh and Baghel [53] and Singh et al. [69] have studied the phenomenological decay of Λ in the background of Bianchi type-V and Bianchi type-I universe respectively. Motivated by these works, in the present paper, we have examined the possibility of the following four cases of phenomenological decay of Λ in the background of LRS Bianchi type-I space-time with bulk viscous fluid source and time dependent Λ-term:

Case 1: $\Lambda \sim H^2$

Case 2: $\Lambda \sim H$

Case 3: $\Lambda \sim \rho$

Case 4: $\Lambda \sim S^{-2}$

Here $H,\ \rho,\ S$ are respectively the Hubble parameter, energy density and average scale factor of the Bianchi type-I space-time. The dynamical laws for decay of Λ have been widely studied by many researchers [15, 17, 54-58].

2. The Metric, Field Equations and Solutions

The metric of the LRS Bianchi type-I space-time is taken as

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)(dy^2 + dz^2) \tag{1}$$

The energy momentum tensor for bulk viscous fluid source is given

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p} g_{ij}, \tag{2}$$

with

$$\bar{p} = p - \zeta u^i_{;i,} \tag{3}$$

where ρ is the energy density of matter, p is the isotropic pressure, ζ is the coefficient of bulk viscosity and $u_{i,}$ the four-velocity vector of the fluid satisfying the condition $u_i u^i = 1$. The semi–colon stands for covariant differentiation. The Einstein's field equations with variable cosmological term Λ(t) are given by

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G T_i^j + \Lambda g_i^j \tag{4}$$

The field equations (4) for the metric (1) and energy momentum tensor lead to the following system of equations

$$8\pi G\bar{p} - \Lambda = -2\frac{\ddot{b}}{b} - \frac{\dot{b}^2}{b^2} \tag{5}$$

$$8\pi G\bar{p} - \Lambda = -\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} - \frac{\dot{a}\dot{b}}{ab} \tag{6}$$

$$8\pi G\rho + \Lambda = 2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} \tag{7}$$

Here, and also in what follows, an overhead dot designates ordinary differentiation with respect to t.

The energy conservation equation $T^{ij}_{;j} = 0$, leads to the following expression

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) + \frac{\dot{\Lambda}}{8\pi G} = 0 \tag{8}$$

When Λ is a constant, equation (8) reduces to the equation of continuity. From equation (8), one can conclude that a decaying Λ term transfers energy to the matter component as

$\rho_v = \frac{\Lambda}{8\pi G}$ where ρ_v is the vacuum energy density. For specification of ζ, we assume that the fluid obeys the equation of state of the form

$$p = \gamma\rho, \quad 0 \le \gamma \le 1 \tag{9}$$

We define the following parameters to be used in solving Einstein's field equations for the metric (1).

The average scale factor (S) for LRS Bianchi type-I model is defined as

$$S = (ab^2)^{\frac{1}{3}} \tag{10}$$

The spatial volume (V) is given by

$$V = S^3 = ab^2 \tag{11}$$

The mean Hubble parameter (H) for LRS Bianchi type-I space-time is defined by

$$H = \frac{\dot{S}}{S} = \frac{1}{3}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) \tag{12}$$

The deceleration parameter q is given by

$$q = -\frac{\ddot{S}}{SH^2} \tag{13}$$

We define the kinematical quantities such as expansion scalar (θ) and shear scalar (σ) as follows:

$$\theta = u^i_{;i,} \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{14}$$

where σ_{ij} is the shear tensor given by

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}h^\alpha_j + u_{j;\alpha}h^\alpha_i\right) - \frac{1}{3}\theta h_{ij} \tag{15}$$

Here the projection tensor h_{ij} has the form

$$h_{ij} = g_{ij} + u_i u_j \tag{16}$$

From equations (5)-(7), (11) and (12), we get

$$\frac{\dot{a}}{a} = \frac{\dot{S}}{S} + \frac{2}{3}\frac{k_1}{S^3} \tag{17}$$

$$\frac{\dot{b}}{b} = \frac{\dot{S}}{S} - \frac{1}{3}\frac{k_1}{S^3} \tag{18}$$

where k_1 is the constant of integration. After integrating equations (17) and (18), we obtain

$$a = lSexp\left(\frac{2k_1}{3}\int\frac{dt}{S^3}\right) \tag{19}$$

$$b = mSexp\left(\frac{-k_1}{3}\int\frac{dt}{S^3}\right) \tag{20}$$

where l and m are constants of integration such that $l = m = 1$. For LRS Bianchi I universe, expansion scalar (θ) and shear scalar (σ) take the form

$$\theta = 3\frac{\dot{S}}{S} \tag{21}$$

$$\sigma = \frac{k_1}{\sqrt{3}S^3} \tag{22}$$

Equations (5)-(7) and (8) can be written in terms of $H,\ \sigma$ and q as

$$8\pi G\overline{p} - \Lambda = H^2(2q-1) - \sigma^2 \tag{23}$$

$$8\pi G\rho + \Lambda = 3H^2 - \sigma^2 \tag{24}$$

$$\dot{\rho} + 3(\rho + \bar{p})H + \frac{\dot{\Lambda}}{8\pi G} = 0 \tag{25}$$

Equation (24) gives

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2} \tag{26}$$

Therefore $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Here we observe that a positive Λ puts restriction on the upper limit of anisotropy whereas a negative Λ contributes to the anisotropy. Equations (23) and (24) yield

$$\frac{d\theta}{dt} = -4\pi G(\rho + 3p) - 2\sigma^2 - \frac{\theta^2}{3} + 12\pi G\zeta\theta + \Lambda \tag{27}$$

which is the Raychaudhuri equation. For $\Lambda \leq 0$ and $\zeta = 0$, we get decelerating phase of the universe provided the strong energy condition [59] holds. Then we have

$$\frac{d\theta}{dt} \leq -\frac{\theta^2}{3} \tag{28}$$

After integrating equation (28), we get

$$\frac{1}{\theta} \geq \frac{1}{\theta_0} + \frac{t}{3} \tag{29}$$

where θ_0 is the initial value of θ. If $\theta_0 < 0$, θ will diverge ($\theta \to -\infty$) for $t < \frac{3}{|\theta_0|}$. Equation (27) shows that the viscosity and positive Λ contributes in slowing down the rate of decrease of expansion scalar θ. Equation (22) gives

$$\dot{\sigma} = -3\sigma H \tag{30}$$

This implies that σ decreases in an evolving universe and it becomes negligible for infinitely large values of S. Equations (23) and (24) give

$$\frac{\ddot{S}}{S} = -\frac{4}{3}\pi G(\rho + 3p) - \frac{2}{3}\sigma^2 + 4\pi G\zeta\theta + \frac{\Lambda}{3} \tag{31}$$

We note that the positive Λ term and bulk viscosity ζ account for the accelerating phase of the universe. From equation (24), we obtain

$$3\frac{\dot{S}^2}{S^2} = \sigma^2 + 8\pi G\rho + \Lambda \tag{32}$$

Here we see that for $\Lambda \geq 0$ we get an ever-expanding universe and for $\Lambda < 0$ we get an universe that expands and then re-contracts. Equation (8) gives

$$S^{-3(\gamma+1)}\frac{d}{dt}\{\rho S^{3(\gamma+1)}\} = 9\zeta H^2 - \frac{\dot{\Lambda}}{8\pi G} \tag{33}$$

We apply the special law of variation for mean Hubble parameter that yields a constant value of deceleration parameter. Since the line-element (1) is completely characterized by Hubble parameter H, therefore we assume a relation between Hubble parameter H and average scale factor S given by

$$H = kS^{-n} \tag{34}$$

where $k > 0$ and $n \geq 0$ are constants. This type of relation has already been discussed by Berman [60], Berman and Gomide [61] in case of FRW models. Such relation gives a constant value of deceleration parameter. Later on, several authors [62-67] have studied FRW and Bianchi type models by using the special law for Hubble parameter that yields constant value of deceleration parameter. From equation (34), we get

$$q = n - 1 \tag{35}$$

which is a constant. The sign of q indicates whether the model inflates or not. The positive sign of q i.e. $n > 1$ correspond to standard decelerating model whereas the negative sign of q i.e. $0 \leq n < 1$ indicates an accelerating universe. For $n = 1$ we get an anisotropic Milne universe [68]. After integrating (34), we obtain

$$S = (nkt + t_1)^{\frac{1}{n}} \text{ for } n \neq 0 \tag{36}$$

$$S = exp\{k(t - t_0)\} \text{ for } n = 0 \tag{37}$$

where $t_1$and t_0 are constants of integration. From equation (36) on using equations (19) and (20), we get

$$a = (nkt + t_1)^{\frac{1}{n}} exp\left\{\frac{2k_1(nkt + t_1)^{\frac{n-3}{n}}}{3k(n-3)}\right\} \tag{38}$$

$$b = (nkt + t_1)^{\frac{1}{n}} exp\left\{\frac{-k_1(nkt + t_1)^{\frac{n-3}{n}}}{3k(n-3)}\right\} \tag{39}$$

After suitable transformation, the line-element (1) for this solution takes the form

$$ds^2 = dT^2 - (nkT)^{\frac{2}{n}} exp\left\{\frac{4k_1(nkT)^{\frac{n-3}{n}}}{3k(n-3)}\right\} dX^2 - (nkT)^{\frac{2}{n}} exp\left\{\frac{-2k_1(nkT)^{\frac{n-3}{n}}}{3k(n-3)}\right\} (dY^2 + dZ^2) \tag{40}$$

From equation (37) on using equations (19) and (20), we obtain

$$a = exp\left\{k(t - t_0) - \frac{2k_1}{9k} e^{-3k(t-t_0)}\right\} \tag{41}$$

$$b = exp\left\{k(t - t_0) + \frac{k_1}{9k} e^{-3k(t-t_0)}\right\} \tag{42}$$

The line-element (1) for this solution takes the form

$$ds^2 = dT^2 - exp\left(2kT - \frac{4k_1}{9k} e^{-3kT}\right) dX^2 - exp\left(2kT + \frac{2k_1}{9k} e^{-3kT}\right) (dY^2 + dZ^2) \tag{43}$$

3. Discussion

In this section we describe the models on using different laws for the decay of Λ.

3.1

For the model (40), (when $n \neq 0$), average scale factor S, expansion scalar θ, Hubble parameter H and shear scalar σ are :

$$S = (nkT)^{\frac{1}{n}} \tag{44}$$

$$\theta = 3H = \frac{3}{nT} \tag{45}$$

$$\sigma = \frac{k_1}{\sqrt{3}} \frac{1}{(nkT)^{\frac{3}{n}}} \tag{46}$$

For $n = 0$ and $n = 3$, the model is not viable. For $n < 3$, $\frac{\sigma}{\theta} \to 0$ as $T \to \infty$. Therefore, the model approaches isotropy asymptotically.

3.1.1 Case 1:

We take

$$\Lambda = 3\beta H^2, \tag{47}$$

whereβ is a constant. Equations (9), (23) and (24) give

$$8\pi G\rho = \frac{3 - 3\beta}{n^2T^2} - \frac{k_1^2}{3(nkT)^{\frac{6}{n}}} \tag{48}$$

$$24\pi G\zeta = \frac{3(1+\gamma)(1-\beta) - 2n}{nT} + \frac{(1-\gamma)k_1^2 nT}{3(nkT)^{\frac{6}{n}}} \tag{49}$$

$$\Lambda = \frac{3\beta}{n^2T^2} \tag{50}$$

At $T = 0$, ρ, p, Λ, ζ are all infinite and they become negligible for large values of T. Therefore, for large times, the model represents a non-rotating, shearing and expanding universe having big bang star and approaches isotropy asymptotically.

3.1.2 Case 2:

We consider

$$\Lambda = \delta H, \tag{51}$$

whereδ is a positive constant. For this case, we get

$$8\pi G\rho = \frac{3}{n^2T^2} - \frac{k_1^2}{3(nkT)^{\frac{6}{n}}} - \frac{\delta}{nT} \tag{52}$$

$$24\pi G\zeta = \frac{3(1+\gamma)-2n}{nT} + \frac{(1-\gamma)k_1^2 nT}{3(nkT)^{\frac{6}{n}}} - (1+\gamma)\delta \tag{53}$$

$$\Lambda = \frac{\delta}{nT} \tag{54}$$

This model also has singularity at $T = 0$ with $\rho,\ p,\ \Lambda,\ \zeta$ all diverging and expansion becomes zero for $T \to \infty$.

3.1.3 Case 3:

We assume

$$\Lambda = 8\pi G\eta\rho, \tag{55}$$

whereη is a constant. For this choice, we get

$$8\pi G(1+\eta)\rho = \frac{3}{n^2T^2} - \frac{k_1^2}{3(nkT)^{\frac{6}{n}}} \tag{56}$$

$$24\pi G(1+\eta)\zeta = \frac{3(1+\gamma)-2n(1+\eta)}{nT} + \frac{(1-\gamma+2\eta)k_1^2 nT}{3(nkT)^{\frac{6}{n}}} \tag{57}$$

$$\left(1+\frac{1}{\eta}\right)\Lambda = \frac{3}{n^2T^2} - \frac{k_1^2}{3(nkT)^{\frac{6}{n}}} \tag{58}$$

At $T = 0$, the model starts expanding with a big bang and $\rho,\ p,\ \Lambda,\ \zeta$ all diverge and expansion becomes zero for $T \to \infty$.For large time, ρ andΛ decrease in the course of expansion.

3.1.4 Case 4:

We take

$$\Lambda = \frac{\alpha}{S^2}, \tag{59}$$

where α is a constant. For this case, we obtain

$$8\pi G\rho = \frac{3}{n^2T^2} - \frac{k_1^2}{3(nkT)^{\frac{6}{n}}} - \frac{\alpha}{(nkT)^{\frac{2}{n}}} \tag{60}$$

$$24\pi G\zeta = \frac{3(1+\gamma)-2n}{nT} - \frac{(1+\gamma)\alpha nT}{(nkT)^{\frac{2}{n}}} + \frac{(1-\gamma)k_1^2 nT}{3(nkT)^{\frac{6}{n}}} \tag{61}$$

$$\Lambda = \frac{\alpha}{(nkT)^{\frac{2}{n}}} \tag{62}$$

This model also starts expanding with a big bang at $T = 0$ with ρ, p, Λ, ζ all infinite and expansion becomes zero for $T \to \infty$.

3.2

For the model (43), (i.e. when $n = 0$), average scale factor S, expansion scalar θ, Hubble parameter H, shear scalar σ and deceleration parameter q are :

$$S = e^{kT} \tag{63}$$

$$\theta = 3H = 3k \tag{64}$$

$$\sigma = \frac{k_1}{\sqrt{3}} e^{-3kT} \tag{65}$$

$$q = -1 \tag{66}$$

The energy density ρ and bulk viscosity ζ have the expressions :

$$8\pi G\rho = 3k^2 - \frac{k_1^2}{3} e^{-6kT} - \Lambda \tag{67}$$

$$24\pi Gk\zeta = 3(\gamma+1)k^2 + (1-\gamma)\frac{k_1^2}{3} e^{-6kT} - (\gamma+1)\Lambda \tag{68}$$

Here $\Lambda \sim H^2$ and $\Lambda \sim H$ which implies Λ is constant because H is constant. So, we get a model similar to the model considered by Singh and Baghel [67]. For the case$\Lambda = 8\pi G\eta\rho$, from equations (67) and (68), we get

$$8\pi G(1+\eta)\rho = 3k^2 - \frac{k_1^2}{3}e^{-6kT} \tag{69}$$

$$24\pi G(1+\eta)k\zeta = 3(\gamma+1)k^2 + (1-\gamma+2\eta)\frac{k_1^2}{3}e^{-6kT} \tag{70}$$

$$\left(1+\frac{1}{\eta}\right)\Lambda = 3k^2 - \frac{k_1^2}{3}e^{-6kT} \tag{71}$$

Again for $\Lambda = \frac{\alpha}{S^2}$, from equations (67) and (68), we obtain

$$8\pi G\rho = 3k^2 - \frac{k_1^2}{3}e^{-6kT} - \alpha\, e^{-2kT} \tag{72}$$

$$24\pi Gk\zeta = 3(\gamma+1)k^2 + (1-\gamma)\frac{k_1^2}{3}e^{-6kT} - (\gamma+1)\alpha e^{-2kT} \tag{73}$$

$$\Lambda = \alpha e^{-2kT} \tag{74}$$

We observe that at $T = 0$, ρ, p, Λ, ζ are all finite. So this model has no singularity. This model represents a non-rotating, shearing and accelerating universe which becomes isotropic for large times.

4. Conclusion

Locally rotationally symmetric Bianchi type-I cosmological models with bulk viscous fluid and variable cosmological term have been constructed. Exact solutions of Einstein's field equations have been obtained by assuming a special law of variation for the mean Hubble parameter, which yields a constant value of deceleration parameter. Four different cases of phenomenological decay of Λ have been discussed. We obtain two models for cases $n \neq 0$ and $n = 0$. For $n \neq 0$, the model starts with a big bang at $T = 0$ where all cosmological parameters become infinite. On the other hand, we get a non-singular and accelerating model of the universe for $n = 0$. In both the cases the universe approaches isotropy for large value of T provided for the first case $n < 3$. Thus we get a decelerating model for $n > 1$ and an accelerating model of the universe for $0 \leq n < 1$. And for $n = 1$ we get an anisotropic Milne model [68].

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