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Perfect Fluid Dark Energy Cosmological Models in Saez-Ballester & Einstein Theory of Gravitation

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Abstract

Considered a spatially homogeneous anisotropic Bianchi type-V dark energy cosmological model filled with perfect fluid in the framework of Saez-Ballester (Phys. Lett. A 113: 467, 1986) as well as Einstein's theory of gravitation. Assumed that the two sources interact minimally and therefore their energy momentum tensors are conserved separately. Some important physical and geometrical features of the models, thus obtained, have been discussed.

Keywords: Bianchi Type-V, Saez-Ballester, Scalar-tensor theory, Perfect fluid and Dark energy.

1. Introduction

Saez and Ballester (1986) formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies.

The field equations given by Saez- Ballester (1986) for the combined scalar and tensor fields (using geometrized units with $c = 1$, $8\pi G = 1$) are

$$G_{ij} - \omega\phi^r\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1.1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^r\phi^{i}_{\ ,i} + r\phi^{r-1}\phi_{,k}\phi^{,k} = 0 \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R the scalar curvature,

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ω, r are constants and T_{ij} is the stress energy tensor of the matter.

The energy conservation equation is

$$T^{ij}{}_{;j} = 0 \tag{1.3}$$

The study of cosmological models in the framework of scalar –tensor theories has been the active area of research for the last few decades. In particular, Rao et al. (2007) and Rao et al. (2008a, 2008b) are some of the authors who have investigated several aspects of the cosmological models in Saez-Ballester (1986) scalar-tensor theory. Naidu et al. (2012 a, b, c, d) have discussed various aspects of Bianchi space times in Saez-Ballester (1986) scalar-tensor theory.

Recent observations of type Ia supernovae (SN Ia) (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 2004), galaxy redshift surveys (Fedeli et al. 2009), cosmic microwave background radiation (CMBR) data (Caldwell and Doran 2004, Huang et al. 2006) and large scale structure (Daniel et al. 2008) strongly suggest that the observable universe is undergoing an accelerated expansion. Observations also suggest that there had been a transition of the universe from the earlier deceleration phase to the recent acceleration phase (Caldwell et al. 2006). The cause of this sudden transition and the source of the accelerated expansion are still unknown. Measurements of CMBR anisotropies, most recently by the WMAP satellite, indicate that the universe is very close to flat. For a flat universe, its energy density must be equal to a certain critical density, which demands a huge contribution from some unknown energy stuff. Thus, the observational effects like the cosmic acceleration, sudden transition, flatness of universe and many more need explanation. It is generally believed that some sort of 'dark energy' (DE) is pervading the whole universe. It is a hypothetical form of energy that permeates all of space and tends to increase the rate of expansion of the universe (Peebles and Ratra 2003). The most recent WMAP observations indicate that DE accounts for 72% of the total mass energy of the universe (Hinshaw et al. 2009). However, the nature of DE is still a mystery.

Many cosmologists believe that the simplest candidate for the DE is the cosmological constant (Λ) or vacuum energy since it fits the observational data well. During the cosmological evolution, the Λ- term has the constant energy density and pressure $p^{(de)} = -\rho^{(de)}$, where the superscript (de) stands for DE. However, one has the reason to dislike the cosmological constant since it always suffers from the theoretical problems such as the "fine-tuning" and "cosmic coincidence" puzzles (Copeland et al. 2006). That is why, the different forms of dynamically changing DE with an effective equation of state (EoS), $\omega^{(de)} = p^{(de)} / \rho^{(de)} < -1/3$, have been proposed in the literature. Other possible forms of DE include quintessence $(\omega^{(de)} > -1)$ (Steinhardt et al. 1999), phantom $(\omega^{(de)} < -1)$ (Caldwell 2002) etc. While the possibility $\omega^{(de)} << -1$ is ruled out by current cosmological data from SN Ia (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) (Riess et al. 2004; Astier et al. 2006), CMBR (WMAP, BOOMERANG) (Eisentein et al. 2005; MacTavish et al. 2006) and large scale structure (Sloan Digital Sky Survey) (Komatsu et al. 2009) data, the dynamically evolving DE crossing the phantom divide line (PDL) $(\omega^{(de)} = -1)$ is mildly favored. SN Ia data combined with CMBR anisotropy and galaxy clustering statistics suggest that $-1.33 < \omega^{(de)} < -0.79$ (see, Tegmark et al. 2004).

Most of the models with constant DP have been studied by considering perfect fluid or ordinary matter in the universe. But the ordinary matter is not enough to describe the dynamics of an accelerating universe as mentioned earlier. This motivates the researchers to consider the models of the universe filled with some exotic type of matter such as the DE along with the usual perfect fluid. Recently, some dark energy models with constant DP have been investigated by Kumar and Singh (2007), Akarsu and Kilinc (2010 a, b & c), Yadav (2010), Yadav and Yadav (2010), Pradhan et al. (2011) and Rao et al. (2012a, b). Reddy et al. (2012) have discussed five dimensional dark energy model in Saez-Ballester (1986) scalar-tensor theory. Recently, Rao and Neelima (2013) have discussed LRS Bianchi type – I dark energy cosmological models in general scalar tensor theory of gravitation.

In this paper, we will discuss minimally interacting perfect fluid and dark energy Bianchi type – V space-times in a scalar-tensor theory of gravitation proposed by Saez and Ballester (1986) and Einstein's theory of gravitation.

2. Metric and Field equations

We consider spatially homogeneous Bianchi type-V metric in the form

$$ds^2 = -dt^2 + {a_1}^2 dx^2 + e^{-2mx}({a_2}^2 dy^2 + {a_3}^2 dz^2) \tag{2.1}$$

where a_1, a_2 and a_3 are functions of t only.

The Energy momentum tensor is given by

$$T_i^j = T_j^{(m)i} + T_j^{(de)i} \tag{2.2}$$

where $T_j^{(m)i}$ and $T_j^{(de)i}$ are the energy momentum tensors of ordinary matter and DE, respectively and are given by

$$\begin{aligned} T_j^{(m)i} &= diag[-\rho^{(m)}, p^{(m)}, p^{(m)}, p^{(m)}] \\ &= diag[-1, w^{(m)}, w^{(m)}, w^{(m)}]\rho^{(m)} \end{aligned} \tag{2.3}$$

$$\begin{aligned} T_j^{(de)i} &= diag[-\rho^{(de)}, p^{(de)}, p^{(de)}, p^{(de)}] \\ &= diag[-1, w^{(de)}, w^{(de)}, w^{(de)}]\rho^{(de)} \end{aligned} \tag{2.4}$$

where $\rho^{(m)}$ and $p^{(m)}$ are the energy density and pressure of the perfect fluid component or ordinary baryonic matter while $w^{(m)} = p^{(m)}/\rho^{(m)}$ is its EoS parameter. Similarly, $\rho^{(de)}$ and $p^{(de)}$ are the energy density and pressure of the DE component while $w^{(de)} = p^{(de)}/\rho^{(de)}$ is the corresponding EoS parameter.

Now with the help of (2.2) to (2.4), the field equations (1.1) for the metric (2.1) can be written as

$$\frac{m^2}{a_1^{\,2}} - \frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\omega}{2}\phi^r\dot{\phi}^2 = -w^{(m)}\rho^{(m)} - w^{(de)}\rho^{(de)} \tag{2.5}$$

$$\frac{m^2}{a_1^{\,2}} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\omega}{2}\phi^r\dot{\phi}^2 = -w^{(m)}\rho^{(m)} - w^{(de)}\rho^{(de)} \tag{2.6}$$

$$\frac{m^2}{a_1^{\,2}} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\omega}{2}\phi^r\dot{\phi}^2 = -w^{(m)}\rho^{(m)} - w^{(de)}\rho^{(de)} \tag{2.7}$$

$$\frac{3m^2}{a_1^{\,2}} - \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{\omega}{2}\phi^r\dot{\phi}^2 = \rho^{(m)} + \rho^{(de)} \tag{2.8}$$

$$2m\frac{\dot{a}_1}{a_1} - m\frac{\dot{a}_2}{a_2} - m\frac{\dot{a}_3}{a_3} = 0 \tag{2.9}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)+\frac{r}{2\phi}\dot{\phi}^2=0 \qquad (2.10)$$

The conservation equation yields

$$\dot{\rho}^{(m)}+3(1+w^{(m)})\rho^{(m)}H+\dot{\rho}^{(de)}+3(1+w^{(de)})\rho^{(de)}H=0 \qquad (2.11)$$

where H is the mean Hubble parameter.

3. Solutions of the Field equations

In order to solve the field equations completely, we assume that the perfect fluid and DE components interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately.

The energy conservation equation $T^{(m)ij}_{;j}=0$ of the perfect fluid leads to

$$\dot{\rho}^{(m)}+3(1+w^{(m)})\rho^{(m)}H=0 \qquad (3.1)$$

where as the energy conservation equation $T^{(de)ij}_{;j}=0$ of the DE component yields

$$\dot{\rho}^{(de)}+3(1+w^{(de)})\rho^{(de)}H=0 \qquad (3.2)$$

Following Akarsu and Kilinc (2010), we assume that the EoS parameter of the perfect fluid to be a constant, that is,

$$w^{(m)}=p^{(m)}/\rho^{(m)}=constant \qquad (3.3)$$

while $w^{(de)}$ has been allowed to be a function of time since the current cosmological data from SN Ia, CMB and large scale structures mildly favor dynamically evolving DE crossing the phantom divide line (PDL).

Since we are looking for a model explaining an expanding universe with acceleration, we also assume that the anisotropic distribution of DE to ensure the present accelerating universe.

From equation (2.9), we get $a_1^2=c_1a_2a_3$ without loss of generality, by taking $c_1=1$, we get

$$a_1^2=a_2a_3 \qquad (3.4)$$

Now, we assume that the shear scalar *(σ)* of the model is proportional to the expansion scalar *(θ)*, which leads to

$$a_2 = a_3^{\,n}, \tag{3.5}$$

where a_2 & a_3 are the metric potentials and *n* is a positive constant.

From equations (2.6), (2.7) & (3.5), we get

$$\frac{\ddot{a}_3}{\dot{a}_3} + \left(\frac{3n+1}{2}\right)\frac{\dot{a}_3}{a_3} = 0 \tag{3.6}$$

From equation (3.6), we get

$$a_3 = (C_1 t + C_2)^{\frac{2}{3(n+1)}} \tag{3.7}$$

where $C_1 = \frac{3(n+1)}{2}k_1$ & $C_2 = \frac{3(n+1)}{2}k_2$

From equations (3.5) & (3.7), we get

$$a_2 = (C_1 t + C_2)^{\frac{2n}{3(n+1)}} \tag{3.8}$$

From equations (3.4), (3.7) & (3.8) we get

$$a_1 = (C_1 t + C_2)^{\frac{1}{3}} \tag{3.9}$$

The rate of expansion in the direction of x, y and z are given by

$$H_x = \frac{\dot{a}_1}{a_1} = \frac{C_1}{3(C_1 t + C_2)}$$

$$H_y = \frac{\dot{a}_2}{a_2} = \frac{2nC_1}{3(n+1)(C_1 t + C_2)} \tag{3.10}$$

$$H_z = \frac{\dot{a}_3}{a_3} = \frac{2C_1}{3(n+1)(C_1 t + C_2)}$$

The mean Hubble's parameter (H), expansion scalar (θ) and shear scalar (σ^2) are given by

$$H = \frac{C_1}{3(C_1 t + C_2)} \tag{3.11}$$

$$\theta = \frac{C_1}{(C_1 t + C_2)} \tag{3.12}$$

$$\sigma^2 = \frac{(n-1)^2 C_1{}^2}{9(n+1)^2 (C_1 t + C_2)^2} \tag{3.13}$$

The spatial volume (V), mean anisotropy parameter (A_m) and DP (q) are found to be

$$V = (C_1 t + C_2) \tag{3.14}$$

$$A_m = \frac{2(n-1)^2}{3(n+1)^2} \tag{3.15}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 2 \tag{3.16}$$

From equations (3.12) & (3.13), we get

$$\frac{\sigma}{\theta} = \frac{(n-1)}{3(n+1)} \tag{3.17}$$

It is important to note here that the proportionality relation between shear and expansion leads to the positive deceleration parameter (q) with isotropic distribution of DE in Bianchi type-V space time. Since we are looking for a model explaining an expanding universe with acceleration, we assume the anisotropic distribution of DE to ensure the present acceleration of universe. Also since we are looking for a physically viable model of the universe which is consistent with the observations, we introduce the skewness parameter $\delta\ (t)$ on y-axis and $\gamma\ (t)$ on z-axis.

Thus the above field equations (2.5)-(2.11) may be re-written as

$$\frac{m^2}{a_1{}^2} - \frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\omega}{2}\phi^r \dot{\phi}^2 = -w^{(m)}\rho^{(m)} - w^{(de)}\rho^{(de)} \tag{3.18}$$

$$\frac{m^2}{a_1{}^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \frac{\omega}{2}\phi^r \dot{\phi}^2 = -w^{(m)}\rho^{(m)} - (w^{(de)} + \delta)\rho^{(de)} \tag{3.19}$$

$$\frac{m^2}{a_1{}^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\omega}{2}\phi^r \dot{\phi}^2 = -w^{(m)}\rho^{(m)} - (w^{(de)} + \gamma)\rho^{(de)} \tag{3.20}$$

$$\frac{3m^2}{a_1{}^2}-\frac{\dot a_1\dot a_2}{a_1a_2}-\frac{\dot a_2\dot a_3}{a_2a_3}-\frac{\dot a_3\dot a_1}{a_3a_1}-\frac{\omega}{2}\phi^r\dot\phi^2=\rho^{(m)}+\rho^{(de)} \tag{3.21}$$

$$2m\frac{\dot a_1}{a_1}-m\frac{\dot a_2}{a_2}-m\frac{\dot a_3}{a_3}=0 \tag{3.22}$$

$$\ddot\phi+\dot\phi\left(\frac{\dot a_1}{a_1}+\frac{\dot a_2}{a_2}+\frac{\dot a_3}{a_3}\right)+\frac{r}{2\phi}\dot\phi^2=0 \tag{3.23}$$

The conservation equation yields

$$\dot\rho^{(de)}+3(1+w^{(de)})\rho^{(de)}H+\rho^{(de)}(\delta H_y+2\gamma H_z)=0 \tag{3.24}$$

The third term of (3.24) arises due to the deviation from $w^{(de)}$ while the first two terms of (3.24) are deviation free part of $T_j^{(de)i}$. According to (3.24), the behavior of $\rho^{(de)}$ is controlled by the deviation free part of EoS parameter of DE but deviation will affect $\rho^{(de)}$ indirectly, since as can be seen later, they affect the value of EoS parameter. But we are looking for a physically viable model of the universe consistent with the observations. Hence we constrained the skewness parameters $\delta\ (t)$ and $\gamma\ (t)$ by assuming the special dynamics which are consistent with (3.24).

The dynamics of skewness parameter on y-axis δ and z-axis γ are given by

$$\delta=\frac{2mHH_z}{\rho^{(de)}} \tag{3.25}$$

$$\gamma=\frac{-mHH_y}{\rho^{(de)}} \tag{3.26}$$

where m is the dimensionless constant that parameterizes the amplitude of the deviation from $w^{(de)}$ and can be given real values.

From equations (3.19), (3.20), (3.25) & (3.26), we get

$$\frac{\ddot a_3}{a_3}+\left(\frac{(n^2-1)+2n(n-1)+m(n+1)(n+2)}{2(n-1)}\right)\frac{\dot a_3{}^2}{a_3{}^2}=0 \tag{3.27}$$

From equation (3.27), we get

$$a_3 = (C_1 t + C_2)^{\frac{2(n-1)}{C_3}} \tag{3.28}$$

where $C_1 = \frac{C_3}{2(n-1)} k_1$, $C_2 = \frac{C_3}{2(n-1)} k_2$,

$$C_3 = 3(n^2 - 1) + m(n+1)(n+2) \ \& \ n \neq 1.$$

From equations (3.4) & (3.28), we get

$$a_2 = (C_1 t + C_2)^{\frac{2n(n-1)}{C_3}} \tag{3.29}$$

From equations (3.5), (3.28) & (3.29) we get

$$a_1 = (C_1 t + C_2)^{\frac{(n^2-1)}{C_3}} \tag{3.30}$$

From equations (3.23), (3.28) - (3.30) we get

$$\phi^{\frac{r+2}{2}} = \frac{(r+2) C_4{}^2}{2} (C_1 t + C_2)^{\frac{-6(n^2-1)}{C_3}} \tag{3.31}$$

From equation (3.1), we get the energy density of the perfect fluid

$$\rho^{(m)} = \rho_0 (C_1 t + C_2)^{-\frac{3(1+w^{(m)})(n^2-1)}{C_3}} \tag{3.32}$$

where ρ_0 is a constant of integration.

From equations (3.21) & (3.28) - (3.32), we get the dark energy density

$$\rho^{(de)} = \frac{3m^2}{(C_1 t + C_2)^{\frac{2(n^2-1)}{C_3}}} - \frac{2C_1 (n-1)^2 (n^2 + 4n + 1)^2}{C_3{}^2 (C_1 t + C_2)^2} - \frac{\omega C_4{}^2}{2(C_1 t + C_2)^{\frac{6(n^2-1)}{C_3}}} - \frac{\rho_0}{(C_1 t + C_2)^{\frac{3(1+w^{(m)})(n^2-1)}{C_3}}} \tag{3.33}$$

From (3.25) & (3.33), we get the skewness parameter

$$\delta(t)=[4m(n-1)^2(n+1){C_1}^2]X\,{C_3}^2[3m^2(C_1t+C_2)^{2-\frac{2(n^2-1)}{C_3}}-\frac{\omega {C_4}^2}{2}(C_1t+C_2)^{2-\frac{6(n^2-1)}{C_3}} -\frac{2C_1(n-1)^2(n^2+4n+1)^2}{{C_3}^2}-\rho_0(C_1t+C_2)^{2-\frac{3(1+w^{(m)})(n^2-1)}{C_3}}]^{-1} \tag{3.34}$$

From (3.26) & (3.33), we get the skewness parameter

$$\gamma(t)=[-2mn(n-1)^2(n+1){C_1}^2]X\,{C_3}^2[3m^2(C_1t+C_2)^{2-\frac{2(n^2-1)}{C_3}}-\frac{\omega {C_4}^2}{2}(C_1t+C_2)^{2-\frac{6(n^2-1)}{C_3}} -\frac{2C_1(n-1)^2(n^2+4n+1)^2}{{C_3}^2}-\rho_0(C_1t+C_2)^{2-\frac{3(1+w^{(m)})(n^2-1)}{C_3}}]^{-1} \tag{3.35}$$

The EoS parameter of DE is given by

$$W^{(de)}=-\begin{pmatrix} [\frac{2(n^2-1)}{C_3}-\frac{4(n-1)^2(n+2)^2}{{C_3}^2}]{C_1}^2(C_1t+C_2)^{-2}+m^2(C_1t+C_2)^{-\frac{2(n^2-1)}{C_3}} \\ +\frac{\omega {C_4}^2}{2}(C_1t+C_2)^{-\frac{6(n^2-1)}{C_3}}+\rho_0 W^{(m)}(C_1t+C_2)^{-\frac{3(1+w^{(m)})(n^2-1)}{C_3}} \end{pmatrix} X\begin{pmatrix} 3m^2(C_1t+C_2)^{-\frac{2(n^2-1)}{C_3}}-\frac{\omega {C_4}^2}{2}(C_1t+C_2)^{-\frac{6(n^2-1)}{C_3}} \\ -\frac{2C_1(n-1)^2(n^2+4n+1)^2}{{C_3}^2(C_1t+C_2)^2}-\rho_0(C_1t+C_2)^{-\frac{3(1+w^{(m)})(n^2-1)}{C_3}} \end{pmatrix}^{-1} \tag{3.36}$$

The density parameters of perfect fluid and DE are as follows:

$$\Omega^{(m)} = \frac{\rho^{(m)}}{3H^2} = \frac{\rho_0 {C_3}^2 (C_1 t + C_2)^{2-\frac{3(1+w^{(m)})(n^2-1)}{C_3}}}{3(n^2-1)^2 {C_1}^2} \tag{3.37}$$

$$\Omega^{(de)} = \frac{\rho^{(de)}}{3H^2} = \{{C_3}^2[3m^2(C_1 t + C_2)^{2-\frac{2(n^2-1)}{C_3}} - \frac{\omega {C_4}^2}{2}(C_1 t + C_2)^{2-\frac{6(n^2-1)}{C_3}} - \frac{2C_1(n-1)^2(n^2+4n+1)^2}{{C_3}^2} - \rho_0 (C_1 t + C_2)^{2-\frac{3(1+w^{(m)})(n^2-1)}{C_3}}]\} X [3(n^2-1)^2 {C_1}^2]^{-1} \tag{3.38}$$

The overall density parameter Ω is given by

$$\Omega = \Omega^{(m)} + \Omega^{(de)} = \{3m^2 {C_3}^2 (C_1 t + C_2)^{2-\frac{2(n^2-1)}{C_3}} - \frac{\omega {C_4}^2 {C_3}^2}{2}(C_1 t + C_2)^{2-\frac{6(n^2-1)}{C_3}} - 2C_1(n-1)^2(n^2+4n+1)^2\} X [2(n-1)^4 {C_1}^2]^{-1} A_m \tag{3.39}$$

where A_m is the average anisotropy parameter.

The metric (2.1), in this case, can be written as

$$ds^2 = -dt^2 + (C_1 t + C_2)^{\frac{2(n^2-1)}{C_3}} dx^2 + e^{-2mx}[(C_1 t + C_2)^{\frac{4n(n-1)}{C_3}} dy^2 + (C_1 t + C_2)^{\frac{4(n-1)}{C_3}} dz^2] \tag{3.40}$$

Thus the metric (3.40) together with (3.31) - (3.36) constitutes a Bianchi type-V perfect fluid dark energy cosmological model in Saez and Ballester (1986) theory of gravitation.

Bianchi type-V cosmological model in Einstein's theory of gravitation:

We can easily observe that the metric (3.40) together with (3.31) - (3.36) constitutes a Bianchi type-V perfect fluid dark energy cosmological model in Einstein's theory of gravitation with $C_4 = 0$.

4. Some other important features of the models

The spatial volume for the model (3.40) is

$$V=(C_1t+C_2)^{\frac{3(n^2-1)}{C_3}}e^{-2mx} \tag{4.1}$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta=3H=\left(\frac{3(n^2-1)}{C_3}\right)\frac{C_1}{(C_1t+C_2)} \tag{4.2}$$

and the shear σ is given by

$$\sigma^2=\frac{7}{18}\left(\frac{3(n^2-1)}{C_3}\right)^2\frac{C_1{}^2}{(C_1t+C_2)^2} \tag{4.3}$$

The deceleration parameter q is given by

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1=2+\frac{m(n+2)}{(n-1)} \tag{4.4}$$

The sign of q indicates whether the model 'inflates' or not. A positive sign of q, i.e., $m>\frac{2(1-n)}{(n+2)}$ corresponds to standard deceleration model where as a negative sign of q, i.e., $m<\frac{2(1-n)}{(n+2)}$ indicates accelerating model.

The recent observations of SN Ia, reveal that the present universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$. It is always possible to assign suitable values to m & n to make the derived model consistent with the observations.

The average anisotropy parameter

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2=\frac{2(n-1)^2}{3(n+1)^2} \tag{4.5}$$

where $\Delta H_i=H_i-H\ \ (i=1,2,3)$.

A cosmological diagnostic pair {*r*, *s*} called state finder as proposed by Sahni et al. (2003) is given by

$$r=\frac{\dddot{a}}{aH^3}=\left(1-\frac{C_3}{(n^2-1)}\right)\left(1-\frac{2C_3}{(n^2-1)}\right) \tag{4.6}$$

$$s=\frac{r-1}{3(q-\frac{1}{2})}=\frac{\left(1-\frac{C_3}{(n^2-1)}\right)\left(1-\frac{2C_3}{(n^2-1)}\right)-1}{3\left(\frac{C_3}{(n^2-1)}-\frac{3}{2}\right)} \tag{4.7}$$

The dynamics of state finder {*r*, *s*} depends on C_3 and $n^2 \neq 1$. It follows that in the derived model, one can choose the pair of state finder, which can successfully differentiate between a wide variety of DE models including cosmological constant, quintessence, phantom, quintom, the Chaplygin gas, braneworld models and interacting DE models. For example if we put $C_3 = 0$, the state finder pair will be {1, 0} which yields the ΛCDM (cosmological constant cold dark matter) model.

5. Conclusions

In this paper, we have presented spatially homogeneous anisotropic Bianchi type - V space time filled with perfect fluid and dark energy possessing dynamical energy density in Saez - Ballester (1986) scalar tensor theory of gravitation and Einstein's theory of gravitation. Studying the interaction between the ordinary matter and dark energy will open up the possibility of detecting dark energy. We observe that the spatial volume vanishes at $t=\frac{-C_2}{C_1}$. The expansion scalar θ, shear scalar σ and Hubble's parameter H decreases with the increase of time t. Thus the derived model starts expanding with big bang singularity at $t=\frac{-C_2}{C_1}$. The model has point type singularity at $t=\frac{-C_2}{C_1}$ for $n>1$ and $C_3>0$. Also the model has cigar type singularity at $t=\frac{-C_2}{C_1}$ for $n>1$ & $C_3<0$ and $n<1$ & $C_3>0$. In the derived model, the EoS parameter of dark energy $w^{(de)}$ is obtained as time varying and it is evolving with negative sign which may be attributed to the current accelerated expansion of universe. It may be noted that Bianchi type –V dark energy cosmological model filled with perfect fluid represents the cosmos in its early stage of evolution of the universe.

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