**Article**

# Bianchi Type–V String Cosmological Model with Variable Deceleration Parameter

Kanika Das*&Tazmin Sultana

Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India

## Abstract

We consider in this paper a four dimensional Bianchi Type-V string cosmological model. The exact solutions of Einstein's field equations have been obtained by considering time dependent deceleration parameter and by choosing the scale factor $a(t) = [\sinh(\xi t)]^{1/n}$, where n is a positive constant. The physical behavior of the Universe is studied and it is observed that our model is evolving from decelerating phase to accelerating phase. Also it is found that cosmic strings do not exist in Bianchi type-V cosmology.

**Keywords**: Cosmic String, Bianchi type-V universe, Deceleration Parameter, Cosmology.

## 1. Introduction

Recently, Considerable work has been done in string cosmology. Einstein's theory of gravity has been the subject of intense study for its success in explaining the observed accelerated expansion of the universe at late times. Bianchi type cosmological models are important because these are homogeneous and anisotropic. The origin of the universe is one of the greatest cosmological mysteries even today. The exact physical situation at early stage of the formation of our universe is still unknown. The concept of string theory was developed to describe events of the early stage of the evolution of the universe. The present day observations of the universe indicate the existence of a large scale network of strings in the early universe (Kibble 1976, 1980). In recent years there has been a lot of interest in the study of cosmic strings. Cosmic strings have received considerable attention as they are believed to have served in the structure formation in the early stages of the universe. Kibble (1976) showed that cosmic strings may have been created during phase transitions in the early era and they act as a source of gravitational field (Letelier 1983).The study of cosmic strings in relativistic framework was intiated by Stachel (1990) and Letelier (1979). Krori et.al (1990, 1994), Raj Bali and Shuchi Dave (2001), Bhattacharjee and Baruah (2001), Rahaman et.al.(2003), Reddy (2003) are some of the authors who have studied various aspects of string cosmologies in general relativistic theory as well as alternative theories of gravitation. Krori et al. (1994) have shown that in the

*Correspondence Author: Kanika Das, Department of Mathematics, Gauhati University, India. E-mail: daskanika2@gmail.com

context of general relativity cosmic strings do not occur in Bianchi type V cosmology. Also Adhav et al. (2009) have obtained the same findings as Krorietal.about Bianchi type V cosmology.

We present, in this paper, an exact solution of Bianchi type V string cosmological model by assuming a special type of scale factor and a variable deceleration parameter. It is observed that cosmic string do not occur in Bianchi type V model and the universe showing a transition from an early decelerating phase to a recent accelerating phase.

This paper is organized as: In section 2, the metric and the field equations are presented. In section 3, we deal with an exact solution of the field equations with cloud of strings. Section 4, describes some physical and geometrical properties of the models. Finally conclusions are presented in section 5.

## 2. The Metric and the Field Equations

The line element for the spatially homogeneous and anisotropic Bianchi-V space-time is given by

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x}\left(B^2 dy^2 + C^2 dz^2\right). \tag{1}$$

where $A(t), B(t)$ and $C(t)$ are the scale factors in different spatial directions and $\alpha$ is a constant.

We define $a = (ABC)^{1/3}$ as the average scale factor of the space-time (1) so that the average Hubble's parameter read as

$$H = \frac{\dot{a}}{a}. \tag{2}$$

where the overhead dot denotes derivatives with respect to cosmic time t.

The energy momentum tensor $T^i_j$ for a cloud of massive strings and the distribution of perfect fluid is taken as

$$T^i_j = (\rho + p)v^i v_j + p g^i_j - \lambda x^i x_j. \tag{3}$$

where $p$ is the isotropic pressure; $\rho$ is the proper energy density for a cloud of strings with particle attached to them; $\lambda$ is the string tension density, $v^i = (0,0,0,1)$ is the four velocity of the particles and $x^i$ is a unit space-like vector representing the direction of the string. The vectors $v^i$ and $x^i$ satisfy the conditions

$$v^i v_j = -x^i x_j = -1, \; v^i x_i = 0 . \tag{4}$$

Choosing $x^i$ parallel to $\frac{\partial}{\partial x}$ we have

$$x^i = (A^{-1}, 0,0,0) . \tag{5}$$

If the particle density of the configuration is denoted by$\rho_p$, then

$$\rho = \rho_p + \lambda \tag{6}$$

The Einstein's field equations (in gravitational units $c = 1, 8\pi G = 1$) are as follows

$$R^i_j - \frac{1}{2} g^i_j R = T^i_j , \tag{7}$$

The Einstein's field equations (7) for the line-element (1) lead to the following system of equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -p + \lambda , \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -p , \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -p , \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = \rho , \tag{11}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 . \tag{12}$$

The energy conservation equation $T^i_{j,j} = 0$leads to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \lambda \frac{\dot{A}}{A} = 0 . \tag{13}$$

which is obtained from the field equation.

The dot (.) denotes ordinary differentiating with respect to t.

## 3. Solution of Field Equations

Integrating Equation (12) and taking the constant of integration as unity in B or C, without loss of generality, we obtain

$$A^2 = BC \tag{14}$$

Subtracting Equation (9) from Equation (10) and taking the second integral, we get the following relation

$$\frac{B}{C} = d_1 \exp\left(k_1 \int \frac{dt}{ABC}\right) \tag{15}$$

where $d_1$ and $k_1$ are constants of integration.

Equations (8)-(12) are five independent equations in six unknowns $A, B, C, p, \rho$ and $\lambda$. For the complete determination of the system, we need one extra condition.

Following Pradhan et.al (2012), we assume the law of variation of scale factor as increasing function of time

$$a = (\sinh(\xi t))^{1/n} \tag{16}$$

where n is a positive constant and $\xi$ is an arbitrary constant.

Now the spatial volume V of the model is read as

$$V = a^3 = (\sinh(\xi t))^{3/n} \tag{17}$$

Equations (14), (16) and (17) lead to

$$A(t) = (\sinh(\xi t))^{1/n} \tag{18}$$

Inserting equation (18) into (14) and (15), we get

$$B = (\sinh(\xi t))^{1/n} \sqrt{d_1} \exp\left(\frac{k_1}{2} \int \frac{dt}{(\sinh(\xi t))^{3/n}}\right) \tag{19}$$

$$C = \left(\sinh(\xi t)\right)^{1/n} \frac{1}{\sqrt{d_1}} \exp\left(-\frac{k_1}{2}\int \frac{dt}{\left(\sinh(\xi t)\right)^{3/n}}\right) \tag{20}$$

# 4. Some Physical and Geometrical Properties

The isotropic pressure$(p)$, proper energy density$(\rho)$, string tension $(\lambda)$ and particle density $(\rho_p)$ are given by

$$p = \alpha^2 \left(\sinh(\xi t)\right)^{-2/n} - 3\left(\frac{\xi}{n}\right)^2 \left(\coth(\xi t)\right)^2 + 2\frac{\xi^2}{n}\left(\operatorname{cosech}(\xi t)\right)^2 - \frac{k_1^{\ 2}}{4}\left(\sinh(\xi t)\right)^{-6/n} \tag{21}$$

$$\rho = 3\left(\frac{\xi}{n}\right)^2 \left(\coth(\xi t)\right)^2 - \frac{k_1^{\ 2}}{4}\left(\sinh(\xi t)\right)^{-6/n} - 3\alpha^2\left(\sinh(\xi t)\right)^{-2/n} \tag{22}$$

$$\lambda = 0 \tag{23}$$

$$\rho_p = \rho \tag{24}$$

Equation (23) shows that cosmic strings do not occur in Bianchi type-V space-time with average scale factor $a = (\sinh(\xi t))^{1/n}$.

The average Hubble's parameter $(H)$, expansion scalar $(\theta)$, anisotropy parameter ( $A_m$ ) and shear scalar( $\sigma$ )of the model are given by

$$H = \frac{\dot{a}}{a} = \frac{\xi}{n}\coth(\xi t) \tag{25}$$

$$\theta = 3H = 3\frac{\xi}{n}\coth(\xi t) \tag{26}$$

$$A_m = \frac{1}{9H^2}\left[\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)^2+\left(\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right)^2\right]$$
$$= \frac{1}{6}\left(\frac{n}{\xi}\right)^2 k_1(\tanh(\xi t))^2(\sinh(\xi t))^{-6/n} \tag{27}$$

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{1}{4}k_1{}^2(\sinh(\xi t))^{-6/n} \tag{28}$$

The value of DP $(q)$ is found to be

$$q = -\frac{\ddot{a}}{aH^2} = n\left(1-(\tanh(\xi t))^2\right)-1 \tag{29}$$

We observe that $q>0$ for $n>1$ and $q<0$ for $n<1$. Thus it is evident that for $0<n\le 1$, our model is in accelerating phase but for $n>1$, our model is evolving from decelerating phase to accelerating phase.

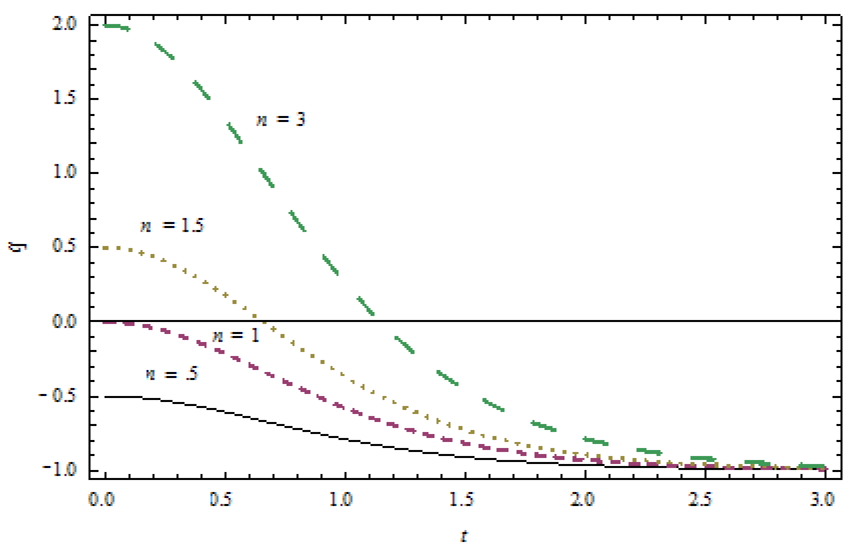


Fig.1 The plot of DP (q) vs. time (t)

Figure 1, shows the variation of the deceleration parameter q against time t which gives the behavior of q for different values ofn.

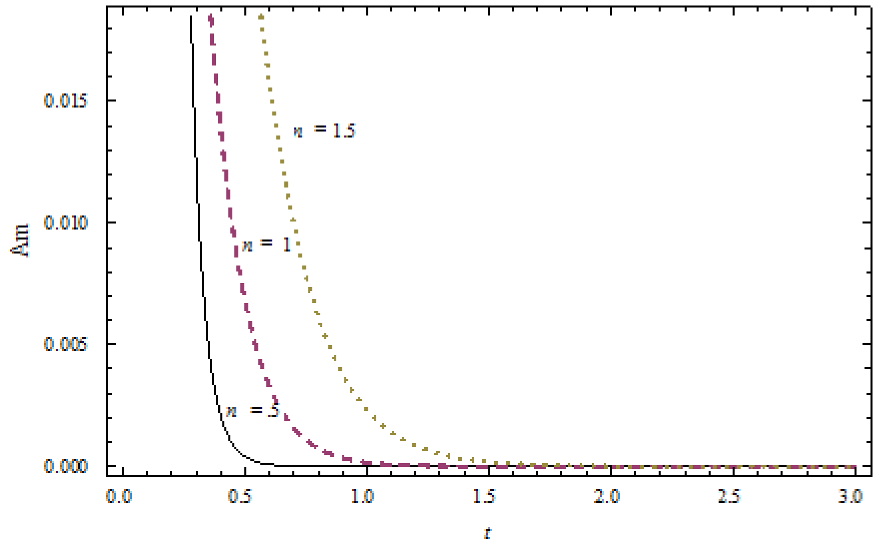


Fig.2 The plot of anisotropic parameter $A_m$ vs. time (t)

Figure 2 shows the variation of parameter $A_m$ versus cosmic time. It shows that $A_m$ decreases with time and tends to zero for sufficiently large times. Thus the anisotropic behavior of the universe dies out at later times and the observed isotropy of the universe can be derived by the model at the present epoch.

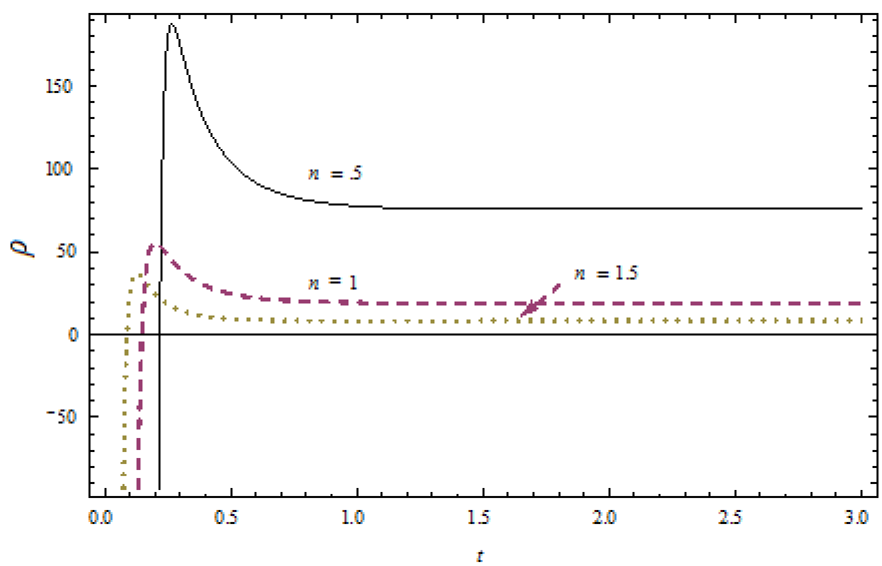


Fig.3 The plot of proper energy density $\rho$ vs. time t

Figure 3 shows the variation of proper density $\rho$ versus cosmic time. It shows that the universe starts with finite values of proper energy density($\rho$).

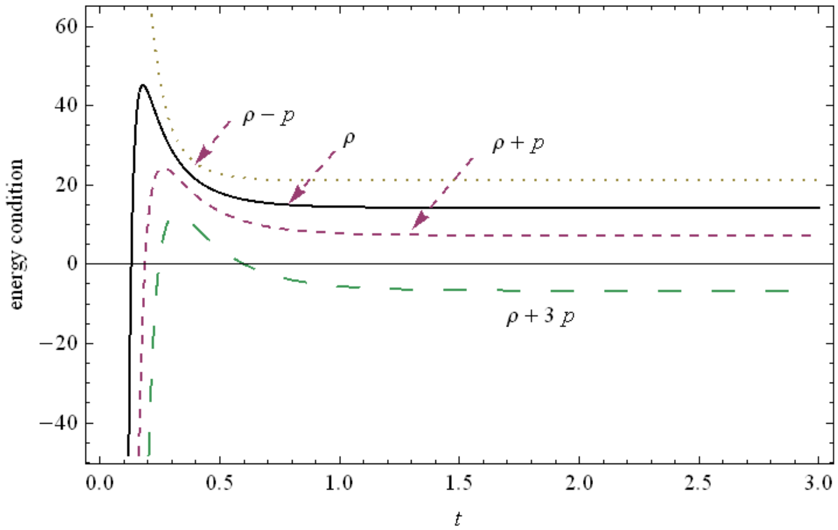


Fig.4 The energy conditions vs. time t

From Figure 4, we can conclude that $(i)\rho > 0$, $(ii)\rho + p > 0$, $(iii)\rho - p > 0$. Therefore, the weak energy condition (WEC) as well as the dominant energy condition (DEC) are satisfied in our model. We can also observed that $\rho + 3p > 0$ at initial time and at later time $\rho + 3p \leq 0$ which in turn imply that the strong energy condition (SEC) violates in the present model on later time. The violation of SEC gives anti-gravitational effect. Due to this effect, the universe gets jerk and the transition from the earlier decelerated phase to the present accelerating phase take place (Caldwell et al.2006), hence the present model is turning out as a suitable model for describing the late time acceleration of the universe.

It is observed that the above set of solutions satisfy the energy conservation equation (13) identically. Thus, the above solutions are exact solutions of Einstein's field equations (8)-(12). From equations (17) and (26), we can conclude that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario. From equations (18)-(20), we see that the spatial scale factors are zero at the initial epoch $t = 0$ and hence the model has a point type singularity (MacCallum, 1971). All the physical quantities isotropic pressure$(p)$, proper energy density$(\rho)$, Hubble's parameter $(H)$ and shear scalar $(\sigma)$ diverge at $t = 0$. Thus we may conclude that the model represents an expanding universe, which starts with a big bang and approaches to isotropy at present epoch.

## 5. Conclusions

In this paper, we have obtained an exact solution of Einstein's field equations for the anisotropic Bianchi type-V space-time with variable deceleration parameter (DP). Interestingly, cosmic strings do not occur in this Bianchi type-V cosmological model. Also the DP yield two phases of the universe. Initially since the sign of the DP is positive that yields the decelerating phase of the universe. At later times, the DP becomes negative which describes the present phase of accelerating universe. The physical properties are satisfied.

**Acknowledgments:** The authors express their profound gratitude to Prof.K.D.Krori for his constant encouragement and advice. Also the authors acknowledge the financial support of UGC, New Delhi and the Department of Mathematics, Gauhati University for providing all facilities for doing this work.

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