Article

A Higher-Dimensional Charged Singularity Free-Solution

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Abstract

We present in this paper a singularity-free solution for a static charged fluid sphere in higher dimensions. The solution satisfies the physical conditions inside the sphere. It reduces to Krori-Barua solution if the number of dimension becomes four.

Keywords: Higher dimension, singularity-free, charged fluid sphere, Einstein-Maxwell equation.

1. Introduction

Considerable work has been done on charged fluid spheres. With regard to singularity we would like to mention that Efinger (1965), Kyle and Martin (1967) and Wilson (1967) have found relativistic internal solutions for static charged spheres in general relativity, but none of these solutions is absolutely free from singularities. It is observed that in Efinger's solution the metric has a singularity at the origin $(r = 0)$. Solution due to Kyle and Martin and Wilson do not have singularities at $r = 0$.However it is argued by Junevicus(1976) that in both the above cases the metrics may have singularities at points other than the origin so that restrictions have to be imposed on the sphere to avoid them. According to him the fluid sphere solutions of Kyle and Martin (1967), Wilson (1967), Kramer and Neugebauer (1971) and Krori-Barua (1975) are of special interest since, with the imposition of suitable conditions, they are completely free of metric singularities and satisfy physical considerations. KB (1975) constructed static, spherically symmetric solutions of the Einstein-Maxwell equations based on a particular choice of the metric coefficients g_{00} and g_{11} in curvature coordinates. A special feature of KB solutions is that they are singularity free. Spheres of charged dust have been investigated by Bonner (1965), Bonner and Wickramasuriya (1975), and Raychoudhuri (1975).

The unification of gravity with other fundamental forces in nature is still a challenging problem today. Most recent efforts in this line demand the space-time dimensions to be more than four. Also modern developments of super-string theory and Yang Mills super gravity theory in its field theory limit need higher-dimensional space-time. For these reasons in recent years there has been

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considerable interest in theories with higher-dimensional space-time, in which extra dimensions are constructed to a very small size, apparently beyond our ability for measurement. The cosmological dimensional reduction process was proposed by Chodos and Detweiller (1980) and it is useful for more than four dimensions. There are perceptions in some quarters that the Universe might have passed through a phase of higher-dimensions. Already a number of important solutions of Einstein's equations in higher dimensions have been obtained. Various authors (Chatterjee 1990, Shen 1990, Chatterjee and Bhui 1990, Chatterjee and Banerjee 1993, Sil and Chatterjee 1993, Krori et.al.,1989,Weyl 1918, constructed higher-dimensional cosmological model in general theory of relativity. A model of higher-dimension was proposed by Kaluza (1921) and Klein (1926) who tried to unify gravity with electromagnetic interaction by introducing an extra dimension which is an extension of Einstein's general relativity in five-dimension. The latest development of super gravity and superstring theory has created interest among scientists to consider higher dimensional space-time for the study of the early universe.

Our present work deals with a spherically symmetric static charged fluid sphere in higher dimensions. The solutions obtained are completely singularity-free and satisfies physical considerations. They are suitable for early universe. The present day observations do not rule out the possible existence of large scale networks of string in the early universe. Our work is an extension of Krori-Barua (1975) solution in N-dimension. KB solutions are obtained if we put $N=4$.

This paper is organised as follows: Section 2 deals with field equations and their solution. Section 3 deals with the central boundary conditions. The paper ends with a brief discussion in section 4.

2. Field equations and their solutions

We take the Krori-Barua (1975) metric given by

$$ds^2 = -e^{\lambda(r)}dr^2 + e^{\nu(r)}dt^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{1}$$

with $\lambda(r) = Ar^2$, $\nu(r) = Br^2 + C$ where A, B and C are arbitrary constants to be determined on physical grounds and $d\Omega^2$ is a line element on a unit $(N-2)$ sphere.

The Einstein's-Maxwell field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi (T_{\mu\nu} + E_{\mu\nu}) \tag{2}$$

where $E^{\mu}{}_{\nu} = \frac{1}{4\pi}\left[-F^{\mu\alpha}F_{\nu\alpha} + \frac{1}{2}\delta^{\mu}{}_{\nu}F^{\alpha\beta}F_{\alpha\beta} \right]$ (3)

and $T_{\mu\nu} = (\rho g_{00} - p g_{AB})$ (4)

where $A, B = 2,3,.........(N-1)$ and ρ and p correspond to the energy density and pressure respectively.

The Maxwell equation is

$$\frac{d}{dr}\left[\sqrt{-g}F^{01}\right] = 4\pi\sigma v^{0}\sqrt{-g} \tag{5}$$

And

$$v^{\mu} = \frac{\delta^{\mu}{}_{0}}{\sqrt{g_{00}}} \tag{6}$$

Thus we obtain the following field equations:

$$e^{-\lambda}\left[\frac{(N-2)}{2}\frac{\lambda'}{r} - \frac{(N-2)(N-3)}{2r^2}\right] + \frac{(N-2)(N-3)}{2r^2} = 8\pi\rho + E \tag{7}$$

$$e^{-\lambda}\left[\frac{(N-2)}{2}\frac{\nu'}{r} + \frac{(N-2)(N-3)}{2r^2}\right] - \frac{(N-2)(N-3)}{2r^2} = 8\pi p - E \tag{8}$$

$$e^{-\lambda}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{(N-3)}{2}\frac{\lambda'}{r} + \frac{(N-3)}{2}\frac{\nu'}{r} + \frac{(N-3)(N-4)}{2r^2}\right] - \frac{(N-3)(N-4)}{2r^2} = 8\pi p + E \tag{9}$$

$$4\pi\sigma = e^{\nu/2}\left[\frac{dF^{01}}{dr} + \left(\frac{\lambda' + \nu'}{2}\right)F^{01} + \frac{(N-2)}{r}F^{01}\right] \tag{10}$$

where

$$E = -F_{01}F^{01} = \frac{Q^2(r)}{r^{2(N-2)}} \tag{11}$$

with

$$Q(r)=\int_0^r 4\pi\sigma r^{N-2}e^{\lambda/r}\,dr \tag{12}$$

which represents the total electric charge within a sphere of radius r.

Krori-Barua solution [7] gives

$$\lambda(r)=Ar^2 \tag{13}$$

$$\nu(r)=Br^2+C \tag{14}$$

Using (13) and (14) we can write the field equations (7)-(9) in the form

$$e^{-Ar^2}\left[(N-2)A-\frac{(N-2)(N-3)}{2r^2}\right]+\frac{(N-2)(N-3)}{2r^2}=8\pi\rho+E \tag{15}$$

$$e^{-Ar^2}\left[(N-2)B+\frac{(N-2)(N-3)}{2r^2}\right]-\frac{(N-2)(N-3)}{2r^2}=8\pi p-E \tag{16}$$

$$e^{-Ar^2}\left[(N-2)B-(N-3)A+B(B-A)r^2+\frac{(N-3)(N-4)}{2r^2}\right]-\frac{(N-3)(N-4)}{2r^2}$$
$$=8\pi p+E \tag{17}$$

There are three equations (15)-(17) and three unknowns ρ, p and E. Solving these equations we get,

$$16\pi p=e^{-Ar^2}\left[2B(N-2)-(N-3)A+B(B-A)r^2+\frac{(N-3)^2}{r^2}\right]-\frac{(N-3)^2}{r^2} \tag{18}$$

$$16\pi\rho=e^{-Ar^2}\left[(3N-7)A-B(B-A)r^2-\frac{(N-3)^2}{r^2}\right]+\frac{(N-3)^2}{r^2} \tag{19}$$

$$2E=e^{-Ar^2}\left[-(N-3)A+B(B-A)r^2-\frac{(N-3)}{r^2}\right]+\frac{(N-3)}{r^2} \tag{20}$$

Also equations (10), (11) and (12) take the form on using (13) and (14)

$$4\pi\sigma=e^{(Br^2+C)/2}\left[\frac{dF^{01}}{dr}+\frac{(N-2)}{r}F^{01}+(A+B)rF^{01}\right] \tag{21}$$

where

$$F^{01} = \left[e^{-2Ar^2 - Br^2 - C} \left\{ \frac{B(B-A)r^2}{2} - \frac{A(N-3)}{2} - \frac{(N-3)}{2r^2} \right\} + \frac{(N-3)}{2r^2} e^{-Ar^2 - Br^2 - C} \right]^{1/2} \tag{22}$$

3. Analysis of the solutions

(i) At r=0 , we have from equations (18)-(22)

$$16\pi p_0 = 2B(N-2) - A(N-2)(N-3) \tag{23}$$

$$E_0 = 0 \tag{24}$$

$$16\pi\rho_0 = (N-1)(N-2)A \tag{25}$$

$$4\pi\sigma_0 = \frac{(N-1)}{2}\left[B^2 + (A-B)^2 + (N-4)A^2\right] \tag{26}$$

where ρ_0 , p_0, E_0 and σ_0 are the values of density, pressure, electric field and charged density respectively at r=0.

Now for ρ_0 and p_0 to be positive we must have

$$\frac{2B}{N-3} \geq A \tag{27}$$

And

$$A \geq 0 \tag{28}$$

Further for $\rho_0 \geq 3p_0$

$$A \geq \frac{3B}{(2N-5)} \tag{29}$$

Combining (27) and (29),

$$\left(\frac{2}{N-3}\right)B \geq A \geq \left(\frac{3}{2N-5}\right)B \tag{30}$$

(ii) Now we consider the conditions to be satisfied at the boundary at $r = r_1$

At the boundary at $r = r_1$, pressure becomes zero. i.e. $p_1 = 0$, (p_1 being the pressure at $r = r_1$)

From equation (18),

$$e^{-Ar_1^2}\left[2B(N-2)+B(B-A)r_1^{\,2}-(N-3)A+\frac{(N-3)^2}{r_1^{\,2}}\right]-\frac{(N-3)^2}{r_1^{\,2}}=0 \tag{31}$$

This equation has a unique solution at r_1. Also since the pressure is positive at $r = 0$, the pressure must remain positive for all $r < \mathrm{r}_1$.

(iii) $$E_1=\frac{Q^2}{r_1^{\,2(N-2)}} \tag{32}$$

where Q is the total charge of the sphere and E_1 is the value of electric field at $r = r_1$.

From equation (20) and (31), the expression for Q^2 is given by

$$e^{-Ar_1^{\,2}}\left[(N-2)B+\frac{(N-3)^2}{r_1^{\,2}}\right]=\frac{(N-3)^2}{r_1^{\,2}}-\frac{Q^2}{r_1^{\,2(N-2)}} \tag{33}$$

Also we get from equation (20)

$$2E=e^{-Ar^2}\left[\frac{1}{2}\{B^2+(A-B)^2\}r^2+\frac{A^2r^2(N-4)}{2!}+\frac{A^3r^4(N-3)}{3!}+\dots\dots\dots\right] \tag{34}$$

We see that E is positive through the sphere because the right hand side of (34) is positive.

(iv) From equation (19) and (31),for $\rho_1 \geq 0$,(ρ_1 is the density ρ at $r = r_1$) we have

$$A+B\geq 0 \tag{35}$$

Since A and B are positive constants, ρ_1 cannot be zero at $r = r_1$.

Again $\lambda_1+\nu_1=0$ .where λ_1 and ν_1 are the value of λ and ν at $r = r_1$.

From equations (13) and (14) we get,

$$Ar_1^{\,2}+Br_1^{\,2}+C=0 \tag{36}$$

This equation (36) indicates that C is negative since A, B and $r_1^{\,2}$ are all positive.

The variation of p against r, E versus r and ρ versus r are shown in the following graphs.

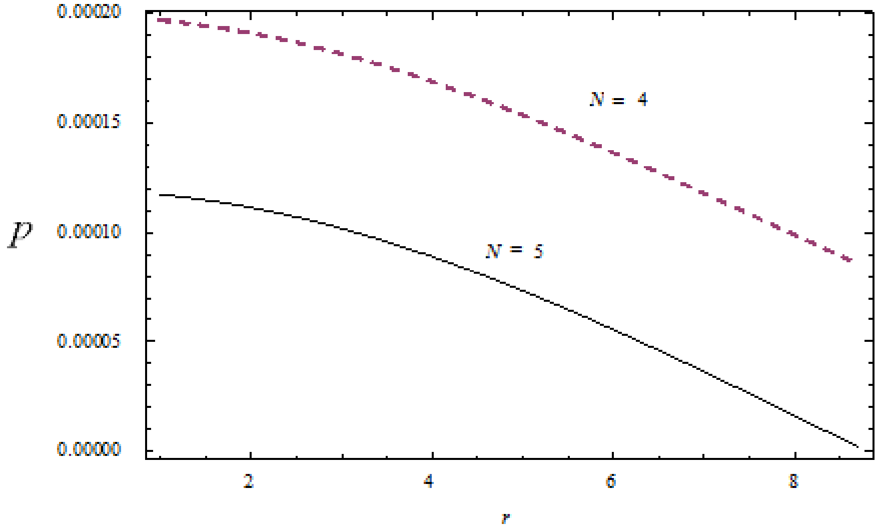

Fig.1

Fig.1 shows that the variation of p and r by using parameter $A = .006, B = .007$ for $N = 5$ and $A = .007, B = .006$ for $N = 4$.

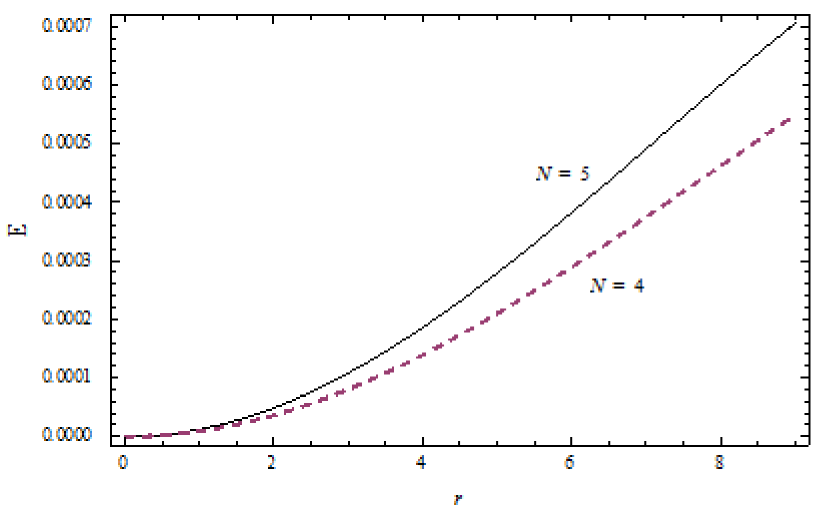

Fig. 2

Fig. 2 shows that the variation of E and r by using parameter $A = .006, B = .007$ for $N = 5$ and $A = .007, B = .006$ for $N = 4$.

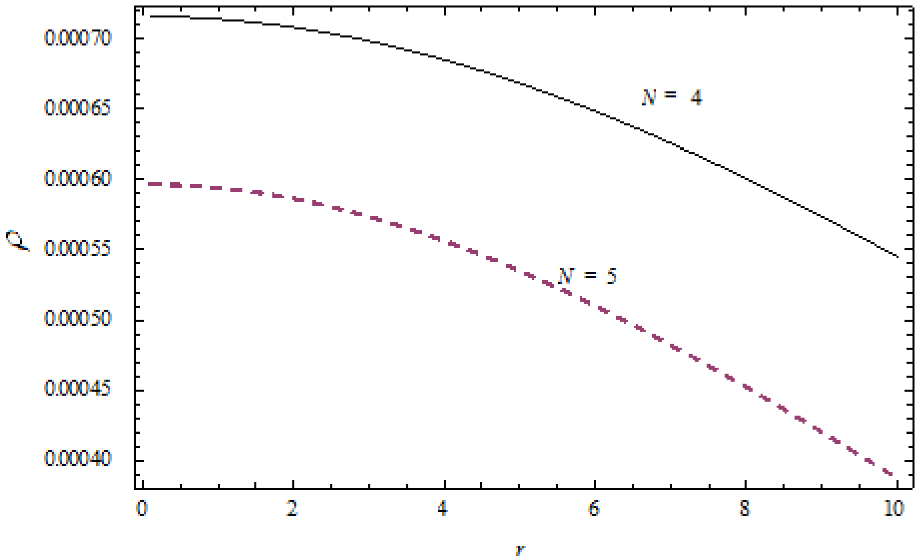

Fig.3

Fig.3 shows that the variation of ρ and r by using parameter $A = .003, B = .005$ for $N = 5$ and $A = .005, B = .003$ for $N = 4$.

4. Conclusions

We have derived in higher-dimensions, an isotropic solution in general form. The physical properties of pressure, density and electricfield are satisfied and are shown in the figures. It reduces to Krori- Barua solution if the number of dimension is four. This model may have physical significance in the early stage of the universe.

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