Article

Bianchi Type-IX Universe with Anisotropic Dark Energy in Lyra Geometry

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Abstract

The exact solutions of the Einstein field equations for dark energy (DE) in Bianchi type-IX metric under the assumption on anisotropy of the fluid are obtained for exponential volumetric expansion within the frame work of Lyra manifold for uniform and time varying displacement field. The isotropy of the fluid and space is examined.

Keywords: anisotropic dark energy, Bianchi type-IX, Lyra geometry.

1. Introduction

In the last decade, most remarkable observational discoveries have shown that our universe is currently accelerating. High precession data from type Ia (SNe Ia), cosmic microwave background radiation (CMBR) data and large scale structure which seems to hint the universe is presently dominated by an unknown form of energy known as DE [1, 4, 5, 6, 18, 22, 28].The astronomical observations indicate that, the universe is spatially flat and is accelerating which is composed of 73% DE, 23% dark matter (DM) and 4% baryonic matter. It is believed that, the DE has large negative pressure that leads to accelerated expansion of the universe. The discovery that the expansion of universe is accelerating [25] has promoted the search for new types of matter that can behave like a cosmological constant [17, 24, 30]. This type of matter is called quintessence. Another candidate for DE density has been used by Kamenchik *et al.* [2] called as Chaplygin gas. Many relativists [10, 15, 19, 20, 23] have obtained cosmological models with anisotropic DE in different theories of gravitation.

The study of Bianchi type-IX universe is important because familiar solutions like Robertson-Walker universe, the de Sitter universe, the Taub-Nut solutions etc., are of Bianchi type-IX space-times. Chakraborty [29], Raj Bali and Dave [25], Raj Bali and Yadav [26] studied Bianchi type-IX string as well as viscous fluid models in general relativity. Pradhan [3] have studied some homogeneous Bianchi type-IX viscous fluid cosmological models with varying Λ. Bianchi type-IX stiff fluid tilted cosmological models with bulk viscosity have been investigated by Bali and Kumawat [27]. Rahaman *et al.* [12] have studied Bianchi type-IX string cosmological model in Lyra geometry. Recently, Ghate and Sontakke [14, 15] have studied Bianchi type-IX cosmological models with anisotropic DE and DE models in Brans-Dicke theory of gravitation.

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Einstein(1916) proposed his theory of general relativity which provides a geometrical description of gravitation. Many physicists attempted to generalize the idea of geometrizing the gravitation to include a geometrical description of electromagnetism. One of the first attempts was made by Weyl [16] who proposed a more general theory by formulating a new kind of gauge theory involving metric tensor to geometrize gravitation and electromagnetism. But Weyl theory was criticized due to non-integrability of length of vector under parallel displacement.
Later, Lyra [13] suggested a modification of Riemannian geometry by introducing a gauge function into the structureless manifold which removes the non-integrability condition. This modified geometry is known as Lyra geometry. Subsequently, Sen [11] formulated a new scalar-tensor theory of gravitation and constructed an analogue of the Einstein's field equations based on Lyra geometry. He investigated that the static model with finite density in Lyra manifold is similar to the static model in Einstein's general relativity. Halford [31] has shown that the constant displacement vector field in Lyra geometry plays the role of cosmological constant in general relativity. He has also shown that the scalar-tensor treatment based in Lyra geometry predicts the same effects, within observational limits, as in Einstein's theory (Halford, [32]).
In the present paper, Bianchi type-IX cosmological model with anisotropic dark energy in Lyra's geometry for uniform and time varying displacement field has been studied. The geometrical and physical aspects of the model have also been studied.

2. Metric and Field Equations

Bianchi type-IX metric is given by

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + (b^2\sin^2 y + a^2\cos^2 y)dz^2 - 2a^2\cos y dxdz, \tag{1}$$

where the scale factors a and b are functions of cosmic time *t* only.

The energy-momentum tensor of anisotropic fluid is

$$T_j^i = diag[T_0^0, T_1^1, T_2^2, T_3^3].$$

By parametrizing it, we get

$$\begin{aligned} T_j^i &= diag[-\rho,\ p_x,\ p_y,\ p_z] \\ &= diag[-1, \omega_x, \omega_y, \omega_z]\,\rho \\ &= diag[-1, \omega, (\omega+\delta), (\omega+\delta)]\,\rho, \end{aligned} \tag{2}$$

where ρ is the energy density of the fluid; p_x, p_y, p_z are the pressures and $\omega_x, \omega_y, \omega_z$ are the directional equation of state (EoS) parameters of the fluid.
Now, parametrizing the deviation from isotropy by setting $\omega_x = \omega_y = \omega_z = \omega$, then introducing skewness parameter δ which is the deviation from ω on *y* and *z* axis. Here ω and δ are not necessarily constants and can be functions of the cosmic time t.

The field equations in Lyra's manifold as obtained by Sen [11] are $(8\pi G = 1 \,\&\, c = 1)$

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij}\phi_m \phi^m = -T_{ij}\,, \tag{3}$$

where $g_{ij}u^i u^j = 1$, $u^j = (1, 0, 0, 0)$ is the four velocity vector; ϕ_j is the displacement vector; R_{ij} is the Ricci tensor; R is the Ricci scalar and T_{ij} is the energy-momentum tensor.

In a co-moving coordinate system, above field equations (3), for the metric (1) using equation (2) yield

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{4b^4} - \frac{3}{4}\beta^2 = \rho \tag{4}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{3}{4}\frac{a^2}{b^4} + \frac{3}{4}\beta^2 = -\omega\rho \tag{5}$$

$$\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{a^2}{4b^4} + \frac{3}{4}\beta^2 = -(\omega + \delta)\rho\,, \tag{6}$$

where the overhead dot $(\dot{\ })$ denotes derivative with respect to cosmic time *t*.

3. Isotropization & the Solutions:

The spatial volume is given by

$$V = R^3 = ab^2\,, \tag{7}$$

where *R* is the mean scale factor.

Subtracting equation (5) from equation (6), we get

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = \frac{\lambda}{V} e^{\int \frac{\delta\rho - \frac{b^2 - a^2}{b^4}}{\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right)} dt}\,, \tag{8}$$

where λ is a constant of integration.

In order to solve the above equation (8), we use the condition

$$\delta = \frac{\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right) + \frac{b^2 - a^2}{b^4}}{\rho}. \tag{9}$$

Using equation (9) in equation (8) therein , we get

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = \frac{\lambda}{V} e^{t} . \tag{10}$$

To solve the system of equations completely we use law of variation for the Hubble parameter which yields the constant value of deceleration parameter proposed by [7] for FRW metric and then by [8] and [9] for Bianchi type space-times.
According to this law, the variation of the mean Hubble parameter for the metric (1) is given by

$$H = k\left(ab^{2}\right)^{-\frac{m}{3}} , \tag{11}$$

where $k > 0$ and $m \geq 0$ are constants.
Here, in particular, we consider the model for $m = 0$ only i.e. we consider the model for exponential expansion.
The directional Hubble parameters along the directions of *x*, *y* and *z* axis respectively for the Bianchi type-IX metric are

$$H_x = \frac{\dot{a}}{a} \text{ and } H_y = H_z = \frac{\dot{b}}{b} . \tag{12}$$

The mean Hubble parameter is given as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right). \tag{13}$$

The volumetric deceleration parameter is

$$q = -\frac{R\ddot{R}}{\dot{R}^{2}} . \tag{14}$$

On integrating, after equatings (11) and (13), we get

$$V = ab^{2} = c_1 e^{3kt} \text{ for } m = 0 , \tag{15}$$

where c_1 is a positive constant of integration.

Using (11) in equation (15) therein for $m = 0$, we get

$$H = k . \tag{16}$$

Using equations (15) and (7) in equation (14) we get, constant values for the deceleration parameter for mean scale factor as:

$$q = -1 \text{ for } m = 0 . \tag{17}$$

For this model $q = -1$ which implies the fastest rate of expansion of the universe. The deceleration parameter of the universe is in the range $-1 \leq q \leq 0$ and the present day universe is undergoing accelerated expansion [1,4,6].

4. Model for Exponential Expansion i.e. for *m = 0 (q = -1)*:

Using equation (15) in the equation (10) therein, we get

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = \frac{\lambda}{V} e^{(1-3k)t} .$$

This on integration gives

$$a = c_3 b \exp\left[\frac{\lambda}{c_1(1-3k)} e^{(1-3k)t} \right], \tag{18}$$

where λ is a constant of integration.

Using equation (18) in equation (15), we get the value of scale factors as

$$a = \left(\frac{c_1}{c_3^2} \right)^{1/3} \exp\left[kt + \frac{2\lambda}{3c_1(1-3k)} e^{(1-3k)t} \right] \tag{19}$$

$$b = \left(\frac{c_1}{c_3} \right)^{1/3} \exp\left[kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t} \right]. \tag{20}$$

The directional Hubble parameters are

$$H_x = k + \frac{2\lambda}{3c_1} e^{(1-3k)t} \tag{21}$$

$$H_y = H_z = k - \frac{\lambda}{3c_1} e^{(1-3k)t} . \tag{22}$$

The anisotropic parameter of the expansion (Δ) is defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_i - H}{H} \right),$$

where $H_i \ (i = 1,2,3)$ represents the directional Hubble parameters along *x*, *y* and *z* directions respectively.

Case (I): Uniform displacement field i.e. when $\beta = \beta_0$, (constant).
Here, we assume the vector displacement field ϕ_μ to be the time like constant vector $\phi_\mu = (\beta, 0, 0, 0)$, where $\beta = \beta_0$ is a constant.
The anisotropic parameter of the expansion (Δ) is defined as

$$\Delta = \frac{2}{9} \frac{\lambda^2}{k^2 c_1^2} e^{2(1-3k)t} . \tag{23}$$

The expansion scalar θ is given by

$$\theta = 3H = 3k. \tag{24}$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}\Delta H^2 = \frac{\lambda^2}{3c_1^2} e^{2(1-3k)t}. \tag{25}$$

Using equations (21) and (22) in equation (4), we obtain the energy density for the model as

$$\rho = 3k^2 - \frac{\lambda^2}{3c_1^2} e^{2(1-3k)t} + \left(\frac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} + \frac{1}{4}(c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - \frac{3}{4}\beta_0^2. \tag{26}$$

Using equations (21), (22) and (26) in equation (9), we obtain the deviation parameter as

$$\delta = -\frac{\frac{\lambda}{3c_1} e^{(1-3k)t} + \left(\frac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - (c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)}}{3k^2 - \frac{\lambda^2}{3c_1^2} e^{2(1-3k)t} + \left(\frac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} + \frac{1}{4}(c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - \frac{3}{4}\beta_0^2}. \tag{27}$$

Using equations (21), (22), (26) and (27) in equation (5), the deviation-free parameter is obtained as

$$\omega = -\frac{3k^2 + \frac{\lambda^2}{3c_1^2} e^{2(1-3k)t} + \frac{2\lambda}{3c_1} e^{(1-3k)t} + \left(\frac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - \frac{3}{4}(c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} + \frac{3}{4}\beta_0^2}{3k^2 - \frac{\lambda^2}{3c_1^2} e^{2(1-3k)t} + \left(\frac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} + \frac{1}{4}(c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - \frac{3}{4}\beta_0^2}. \tag{28}$$

The anisotropy of the expansion (Δ) is not promoted by the anisotropy of the fluid and decreases to null exponentially as t increases provided $k > 1/3$. The space approaches to isotropy in this model since $\Delta \to 0$ as $t \to \infty$. Also the spatial volume $V \to \infty$ and $\rho \to \left(3k^2 - \frac{3}{4}\beta_0^2\right) > 0$ as $t \to \infty$ for $k > 1/3$. Here λ and β_0 contribute to the energy density of the fluid ρ negatively. The energy density of the fluid ρ, the deviation-free EoS parameter ω and the deviation parameter δ are dynamical. For $k > 1/3$ and as $t \to \infty$, the anisotropic fluid isotropizes since $\delta \to 0$ and the model exhibits like phantom energy model having EoS parameter $\omega = -\dfrac{3k^2 + \frac{3}{4}\beta_0^2}{3k^2 - \frac{3}{4}\beta_0^2}$ which is equivalent to $\omega < -1$ provided that $k^2 > \frac{1}{4}\beta_0^2$. In this case (model), we get $\omega < -1$ as $\beta \to 0$ i.e. for very very small positive value of β_0, the model approaches [near to] the cosmological constant (Λ). Also, for $k = 1/3$, our model does not survive.

Case (II): Time varying displacement field i.e. when $\beta = \beta_0 t^{\alpha}$.
Here, we assume the vector displacement field ϕ_μ to be the time varying vector $\phi_\mu = (\beta(t), 0, 0, 0)$, where $\beta = \beta_0 t^{\alpha}$.

We consider $\alpha = -1$ *i.e.*

$$\beta = \frac{\beta_0}{t}. \tag{29}$$

The anisotropic parameter of the expansion (Δ) is defined as

$$\Delta = \frac{2}{9}\frac{\lambda^2}{k^2 c_1^2} e^{2(1-3k)t}. \tag{30}$$

Using equation (29) in the equations (26), (27) and (28) we obtain, the energy density, deviation parameter and the deviation-free parameter for the model respectively as follows:

$$\rho = 3k^2 - \frac{\lambda^2}{3c_1^2} e^{2(1-3k)t} + \left(\frac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} + \frac{1}{4}(c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - \frac{3}{4}\frac{\beta_0^2}{t^2}. \tag{31}$$

$$\delta = -\frac{\dfrac{\lambda}{3c_1} e^{(1-3k)t} + \left(\dfrac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - (c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)}}{3k^2 - \dfrac{\lambda^2}{3c_1^2} e^{2(1-3k)t} + \left(\dfrac{c_3}{c_1}\right)^{2/3} e^{-2\left(kt - \frac{\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} + \dfrac{1}{4}(c_1)^{-2/3} e^{-2\left(kt - \frac{4\lambda}{3c_1(1-3k)} e^{(1-3k)t}\right)} - \dfrac{3}{4}\dfrac{\beta_0^2}{t^2}}. \tag{32}$$

$$\omega=-\frac{3k^2+\frac{\lambda^2}{3c_1^2}e^{2(1-3k)t}+\frac{2\lambda}{3c_1}e^{(1-3k)t}+\left(\frac{c_3}{c_1}\right)^{2/3}e^{-2\left(kt-\frac{\lambda}{3c_1(1-3k)}e^{(1-3k)t}\right)}-\frac{3}{4}(c_1)^{-2/3}e^{-2\left(kt-\frac{4\lambda}{3c_1(1-3k)}e^{(1-3k)t}\right)}+\frac{3}{4}\frac{\beta_0^2}{t^2}}{3k^2-\frac{\lambda^2}{3c_1^2}e^{2(1-3k)t}+\left(\frac{c_3}{c_1}\right)^{2/3}e^{-2\left(kt-\frac{\lambda}{3c_1(1-3k)}e^{(1-3k)t}\right)}+\frac{1}{4}(c_1)^{-2/3}e^{-2\left(kt-\frac{4\lambda}{3c_1(1-3k)}e^{(1-3k)t}\right)}-\frac{3}{4}\frac{\beta_0^2}{t^2}}. \tag{33}$$

Here, the anisotropy of the expansion (Δ) is not promoted by the anisotropy of the fluid and decreases to null exponentially as t increases provided $k > 1/3$. The space approaches to isotropy in this model since $\Delta \to 0$ as $t \to \infty$. The spatial volume $V \to \infty$ and $\rho = 3k^2 > 0$ as $t \to \infty$ for $k > 1/3$. Here λ and β_0 contribute to the energy density of the fluid ρ negatively. The energy density of the fluid ρ, the deviation-free EoS parameter ω and the deviation parameter δ are dynamical. For $k > 1/3$ and as $t \to \infty$, the anisotropic fluid isotropizes and mimics to the vacuum energy which is mathematically equivalent to the cosmologial constant (Λ)i.e. we get $\delta \to 0$, $\omega \to -1$ and $\rho \to 3k^2$ which matches with the result obtained by Akarsu *et al.* [23] and Ghate *et al.* [15]. Also, in this case for $k = 1/3$, our model does not survive.

5. Conclusion :

The exact solutions of the Einstein field equations for dark energy (DE) in Bianchi type-IX metric under the assumption on anisotropy of the fluid are obtained for exponential volumetric expansion within the frame work of Lyra manifold for uniform and time varying displacement field. In cases I and II, we described the model for uniform displacement field and time varying displacement field. It is observed that the universe can approach to isotropy monotonically even in the presence of an anisotropic fluid in the model. The anisotropy of the fluid also isotropizes at later times and evolves into the well-known cosmological constant in the model for exponential volumetric expansion. Also, for $k = 1/3$ our model does not survive. In case-I, the model evolves to the phantom energy and in case-II, it evolves to the cosmological constant (Λ). Thus, even if we observe an isotropic expansion in the present universe we still cannot rule out possibility of DE with an anisotropic EoS in Lyra geometry. Also, the displacement field β plays a crucial role in the dynamics of the universe. Therefore, we cannot rule out the possibility of an anisotropic nature of the DE at least in the framework of Lyra geometry. These observations are worth to pay attention in astrophysical observations. It is interesting to note that our model resembles analogous with the investigations of Adhav [19].

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