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Non-static Plane Symmetric Zeldovich Universe in a Modified Theory of Gravity

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Abstract

Non static plane symmetric perfect fluid distribution in the framework of $f(R,T)$ gravity proposed by Harko et al. (2011) has been obtained with an appropriate choice of a function $f(R,T)$ and in general relativity. It is observed that only Zel'dovich universes exist. Some important features of the models, thus obtained, have been discussed. We noticed that the involvement of new function $f(R,T)$ doesn't affect the geometry of the space-time but slightly changes the matter distribution.

Keywords: non-static plane symmetric metric, $f(R,T)$ gravity, perfect fluid, general relativity.

1. Introduction

In recent years, there has been a lot of interest in alternative theories of gravitation. In view of the late time acceleration of the universe and the existence of the dark matter and dark energy, very recently, modified theories of gravity have been developed. Noteworthy amongst them are $f(R)$ theory of gravity formulated by Nojiri and Odintsov (2003a) and $f(R,T)$ theory of gravity proposed by Harko et al. (2011). Carroll et al. (2004) explained the presence of a late time cosmic acceleration of the universe in $f(R)$ gravity. Nojiri and Odintsov (2003b) demonstrated that phantom scalar in many respects looks like strange effective quantum field theory by introducing a non-minimal coupling of phantom field with gravity. Recently, Harko et al. (2011) developed $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an

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arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor.

The $f(R,T)$ gravity model depends on a source term, representing the variation of the matter stress energy tensor with respect to the metric. A general expression for this source term is obtained as a function of the matter Lagrangian L_m so that each choice of L_m would generate a specific set of field equations. Some particular models corresponding to specific choices of the function $f(R,T)$ are also presented; they have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of a function $f(T)$. In the present model the covariant divergence of the stress energy tensor is nonzero. Hence the motion of test particles is non-geodesic and an extra acceleration due to the coupling between matter and geometry is always present.

In $f(R,T)$ gravity, the field equations are obtained from the Hilbert-Einstein type variation principle.

The action principle for this modified theory of gravity is given by

$$S = \frac{1}{16\pi G}\int f(R,T)\sqrt{-g}\,d^4x + \int L_m\sqrt{-g}\,d^4x \tag{1.1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor of matter and L_m is the matter Lagrangian.

The stress energy tensor of matter is

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g})}{\partial g^{ij}}L_m, \Theta_{ij} = -2T_{ij} - pg_{ij}, \tag{1.2}$$

Using gravitational units (by taking G & c as unity) the corresponding field equations of $f(R,T)$ gravity are obtained by varying the action principle (1.1) with respect to g_{ij} as

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_R(R,T) = \\ 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \end{aligned} \tag{1.3}$$

where $f_R = \frac{\delta\ f(R,T)}{\delta R}$, $f_T = \frac{\delta\ f(R,T)}{\delta T}$ & $\Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$

Here ∇_i is the covariant derivative and T_{ij} is usual matter energy-momentum tensor derived from the Lagrangian L_m .

It can be observed that when $f(R,T) = f(R)$ then (1.3) reduce to field equations of $f(R)$ gravity.

It is mentioned here that these field equations depend on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. (2011) gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R) f_3(T). \end{cases}$$

In this paper we are focused to the first class, i.e.

$$f(R,T) = R + 2f(T). \tag{1.4}$$

where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter.

Paul et al. (2009) obtained FRW models in $f(R)$ gravity while Sharif and Shamir (2009, 2010) have studied the solutions of Bianchi type-I and V space-times in the framework of $f(R)$ gravity. Ahmad Sheykhi (2012) has discussed Magnetic strings in $f(R)$ gravity and Ihsan Yılmaz et al. (2012) have discussed Quark and strange quark matter in $f(R)$ gravity for Bianchi type *I* and *V* space-times. Reddy et al. (2012a, b) have obtained Kaluza klein cosmological model in the presence of perfect fluid source and Bianchi type-III cosmological model in $f(R,T)$ gravity using the assumption of law of variation for the Hubble parameter proposed by Bermann (1983). Chaubey and Shukla (2013) have obtained a new class of Bianchi cosmological models in $f(R,T)$ gravity. Reddy and Santhi Kumar (2013) have presented some anisotropic cosmological models in this theory. Rao et al. (2013) have obtained LRS Bianchi type-I with perfect fluid in a modified theory of gravity and established that the additional condition, special law of variation for the Hubble parameter proposed by Bermann (1983), taken by Adhav (2012) is superfluous. Recently Rao and Neelima (2013a, b) have discussed perfect fluid Einstein-Rosen and Bianchi type-VI_0 universes in $f(R,T)$ gravity respectively.

Motivated by the above investigations, we study non static plane symmetric cosmological models with perfect fluid matter source in $f(R,T)$ gravity, where $f(R,T) = R + 2f(T)$. This model is very important in the discussion of large scale structure, to identify early stages and finally to study the evolution of the universe.

2. Metric and Energy Momentum Tensor

We consider a Riemannian space-time described by the line element

$$ds^2 = e^{2h}(dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2) \tag{2.1}$$

where r, θ, z are the usual cylindrical polar coordinates and h & s are functions of t alone. It is well known that this line element is plane symmetric.

From (1.3) & (1.4), we get the gravitational field equations as

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} + f(T)g_{ij} \tag{2.2}$$

where the overhead prime indicates differentiation with respect to the argument .

The matter tensor for perfect fluid is

$$\begin{aligned} &\Theta^i_j \equiv -2T^i_j - \delta^i_j p = (\rho, -p, -p, -p), \\ &where\, T^i_j = (\rho + p) u_i u^j - \delta^i_j p \end{aligned} \tag{2.3}$$

Then the field equations (2.2) can be written as

$$R^i_j - \frac{1}{2}\delta^i_j R = 8\pi T^i_j + 2f'(T)T^i_j + [2pf'(T) + f(T)]\delta^i_j \tag{2.4}$$

Now we choose the function $f(T)$ as the trace of the stress energy tensor of the matter so that $f(T) = \mu T$, where μ is an arbitrary constant.

3. Solutions of the field equations:

Now with the help of (2.3), the field equations (2.4) for the metric (2.1) can be written as

$$e^{-2h}\left(2\ddot{h}+\dot{h}^2+\frac{2\dot{h}\dot{s}}{s}+\frac{\ddot{s}}{s}\right)=(8\pi+3\mu)p-\mu\rho \tag{3.1}$$

$$e^{-2h}\left(2\ddot{h}+\dot{h}^2\right)=(8\pi+3\mu)p-\mu\rho \tag{3.2}$$

$$e^{-2h}\left(\frac{2\dot{h}\dot{s}}{s}+3\dot{h}^2\right)=-(8\pi+3\mu)\rho+\mu\, p \tag{3.3}$$

Here the over head dot denotes differentiation with respect to t. The field equations (3.1) to (3.3) are only three independent equations with four unknowns h, s, ρ and p.

From (3.1) & (3.2), we get

$$2\dot{h}+\frac{\ddot{s}}{\dot{s}}=0 \tag{3.4}$$

From (3.4), we get

$$e^{h}=(k_1 t+k_2)^m \tag{3.5}$$

$$s=k_4(k_1 t+k_2)^{1-2m},\ m\neq\frac{1}{2}$$
$$where\ \ k_4=\frac{k_3}{(1-2m)k_1} \tag{3.6}$$

and k_1, k_2, k_3 & m are arbitrary constants.

The metric (2.1), in this case, can be written as

$$ds^2=(k_1 t+k_2)^{2m}(dt^2-dr^2-r^2 d\theta^2-k_4^{\,2}(k_1 t+k_2)^{2(1-2m)}dz^2) \tag{3.7}$$

From equations (3.1) to (3.3), we get the energy density and pressure as

$$\rho=p=\frac{m(m-2)k_1^{\,2}}{2(\mu+4\pi)(k_1 t+k_2)^{2(1+m)}} \tag{3.8}$$

From (3.8) we can conclude that only Zel'dovich universe exists for non static plane symmetric metric. Stiff fluid creates more interest in cosmology because the speed of light is equal to speed of sound and its governing equations have the same characteristics as those of gravitational field

(Zel'dovich 1970). The casual limit for ideal gas has also the form $\rho = p$ (Zel'dovich and Novikov, 1971). Also this state describes several important cases, e.g., radiation, relativistic degenerate Fermi gas and probably very dense baryon matter (Zel'dovich and Novikov, 1971; Walecka, 1974). Furthermore, if the fluid satisfies the equation of state $\rho = p$ and if in addition its motion is irrotational, then such a source has the same stress energy tensor as that of a mass less scalar field. Cosmological models with stiff fluid equation of state have been studied by many authors.

The great importance of cosmological models where the matter content is represented by a stiff matter perfect fluid was recognized since its introduction by Zel'dovich. In order to understand better the importance of this perfect fluid for cosmology, one has to compute its energy density. Several authors observed that there may have existed a phase earlier than that of radiation, in our universe, which was dominated by stiff matter. Due to that importance, many physicists have started to consider the implications of the presence of a stiff matter perfect fluid in FRW cosmological models. It may also play an important role in the spectrum of relic gravity waves created during inflation. Since there may have existed a phase earlier than that of radiation which was dominated by stiff matter some physicists considered quantum cosmological models with this kind of matter. Hence the stiff equation of state of the model doesn't restrict its astrophysical/cosmological applicability.

Thus the metric (3.7) together with (3.8) constitutes non static plane symmetric Zel'dovich universe in $f(R,T)$ gravity.

Perfect fluid cosmological model in general relativity

Interestingly we can observe that, if $\mu = 0,$ the metric (3.7) together with (3.8) represents an anisotropic non static plane symmetric Zel'dovich universe in general relativity. Also we can see that from equation (3.8), for $m = 2$, the matter pressure and density will vanish and hence (3.7) represents a non static vacuum cosmological model in general relativity.

4. Some important features of the model

The spatial volume for the model (3.7) is

$$V = \frac{rk_3}{(1-2m)k_1}(k_1 t + k_2)^{1+2m} \tag{4.1}$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = \frac{(1+m)k_1}{(k_1 t + k_2)} \tag{4.2}$$

and the shear σ is given by

$$\sigma^2 = \frac{7}{18}\frac{(1+m)^2 k_1{}^2}{(k_1 t + k_2)^2} \tag{4.3}$$

The deceleration parameter q is given by

$$q = \frac{2-m}{1+m}, \; m \neq -1. \tag{4.4}$$

The deceleration parameter $q > 0$ for $-1 < m < 2$ and $q < 0$ for $-\infty < m < -1$ & $m > 2.$

If $q < 0$, the model accelerates and when $q > 0$, the model decelerates in the standard way. Here the model sometimes decelerates in the standard way and later accelerates which is in accordance with the present day scenario. However, in spite of the fact that the universe, in this case, decelerates in the standard way it will accelerate in finite time due to cosmic re collapse where the universe in turns inflates “decelerates and then accelerates” (Nojiri and Odintsov 2003c).

Therefore the generalized mean Hubble parameter H is

$$H = \frac{(1+m)k_1}{3(k_1 t + k_2)}. \tag{4.5}$$

The overall density parameter Ω is given by

$$\Omega = \frac{\rho}{3H^2} = \frac{3m(m-2)}{2(1+m)^2(\mu + 4\pi)(k_1 t + k_2)^{2m}} \tag{4.6}$$

To study the nonsingular behavior of the model

$$R = 2e^{-2h}\left[3\ddot{h} + 3\dot{h}^2 + 3\dot{h}\frac{\dot{s}}{s} + \frac{\ddot{s}}{s}\right] = 2m(m-2)k_1{}^2(k_1t+k_2)^{-2(m+1)}$$

Here $R \to 0 \; as \;\; t \to \infty.$ Hence the model (3.41) is singular in the infinite future.

The tensor of rotation $w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Conclusions

In this paper, we have presented a non static plane symmetric cosmological models filled with perfect fluid in the framework of $f(R,T)$ gravity proposed by Harko et al. (2011) and also in general relativity . Bera (1969) has introduced an interesting metric and pointed out that one can imagine the field given by the metric represents a transitional model, like a vacuum pocket into which matter is introduced from the surrounding portions of an extra galactic nebula. Hence this metric is of great interest from the point of view of cosmology. We observe that at $t = \frac{-k_2}{k_1}$, the spatial volume vanishes and increases continuously with time while all other parameters diverge. This shows that at the initial epoch the universe starts with zero volume and expands continuously approaching to infinite volume. The model has point type singularity for $0 < m < \frac{1}{2}$ and has cigar type singularity for $m < 0$ & $m > \frac{1}{2}$. For $m = 2$, from (3.8), we can see that matter pressure and density will vanish and hence (3.7) represents a non static vacuum cosmological model in general relativity. Also since $q = 0$, this empty universe expands at a constant rate. The expansion scalar θ, the shear scalar σ and Hubble parameter H decrease with the increase of time. The model presented here is expanding, non-rotating. Also since $lt_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy. The model obtained here remains anisotropic throughout the evolution of the universe. Since experiments show that there is a certain amount of anisotropy in the universe and therefore anisotropic space-times are important. Also $f(R,T)$ gravity is proposed to explain early inflation and late time acceleration. Nojiri and Odintsov (2003c) have shown that for the anisotropic Kaluza-Klein space-time, the universe inflates, decelerates and then accelerates at late times. This is possible by cosmic re-collapse. The same thing happens here in this anisotropic model and the universe accelerates. So, this model is realistic. Finally we noticed that the involvement of new function $f(R,T)$ doesn't affect the geometry of the space-time but slightly changes the matter distribution.

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