Article

Bianchi Type-VI$_0$ String Cosmological Models in a New Scalar-Tensor Theory of Gravitation

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Abstract

The field equations in scalar-tensor theory of gravitation are derived in the presence of cosmic strings .This model is used as a source of Bianchi type VI$_0$ cosmological model. To get a determinate model, we assume that the expansion (θ) in the model is proportional to the shear (σ).It is found that the cosmic string do exist with the scalar field. Some physical properties of the model are also discussed.

Keywords: Bianchi Type-VI$_0$, strings, scalar field, scalar tensor theory, gravitation.

1. INTRODUCTION

In recent years there has been lot of interest in the study of cosmic strings. Cosmic strings have received considerable attention as they are believed to have served in the structure formation in the early stages of the universe. Cosmic strings may have been created during phase transitions in the early era [1] and they act as a source of gravitational field [2]. It is also believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures of the universe. So far a considerable amount of work has been done on cosmic strings and string cosmological models by Krori et al.[3,4],Tikekar and Patel [5],Bali et al[6] .

Sen and Dunn [7] have proposed a new scalar-tensor theory of gravitation in which both the scalar and tensor fields have intrinsic geometrical significance. The scalar field in this theory is characterized by the function $\phi = \phi(x^i)$ where x^i are coordinates in the four – dimensional Lyra manifold and the tensor field is identified with the metric tensor g_{ij} of the manifold. The field equations given by Sen and Dunn[7] for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2} g_{ij} R = \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}) - \phi^{-2}T_{ij} \tag{1}$$

where $\omega = \frac{3}{2}$, R_{ij} and R are respectively the usual Ricci-tensor and Riemaan-curvature scalar (in our units $C = 8\pi G = 1$). Where C is the speed of light in free space. The scalar field ϕ incorporates the varying nature of Newtonian Gravitational constant.

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Latelier [2] and Stachel [8] have developed general relativistic treatment of syrings. Latelier[9] obtained relativistic cosmological model in Bianchi type I and Kantowski-sachs space-times adopting the energy momentum tensor as given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j . \qquad (2)$$

Here ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the tension density of the strings and $\rho = \rho_p + \lambda$, ρ_p being the energy density of the particles. The velocity u^i describes the 4 – velocity which has components (1, 0, 0, 0) for a cloud of particles and x^i represents the direction of string which will satisfy

$$u^i u_i = -x^i x_i = 1 \text{ and } u^i x_i = 0 . \qquad (3)$$

The scalar field ϕ is function of the cosmic time t only .And x^i to be along X-axis, so that $x^i = (0, \frac{1}{A}, 0, 0)$

The strings that form the cloud are massive strings instead of geometric strings. Each massive string is formed by a geometric string with particles attached along its extension. Hence, the string that form the cloud are generalization of Takabayasi's relativistic model of strings (called p-string). This is simplest model wherein we have particles and strings together. In principle we can eliminate the strings and end up with a cloud of particles. This is a desirable property of a model of a string cloud to be used in cosmology since strings are not observed at the present time of evolution of the universe.Pradhan et al. [10-12], Venkateswarlu et al.[13], Yadav et al. [14-16], and Venkateswarlu and Pavan Kumar [17] are some of the authors who have studied various aspects of string cosmologies in general relativistic theory as well as in alternative theories of gravitation. Recently Venkateswarlu et al [18] have studied the Bianchi Type –I cosmic strings in this theory.

In this paper, we intended to study the Bianchi type-VI_0 in the context of cosmic strings in a new scalar-tensor theory of gravitation proposed by Sen and Dunn[7]. Section 2 contains Bianchi type –VI_0 metric and the field equations of this theory. In section 3, the solutions of the field equations are obtained in the context of cosmic strings and also discussed some properties of the models obtained. Conclusions are given in last section.

2. METRIC AND FIELD EQUATIONS

We consider the Bianchi type –VI_0 metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2\alpha x}dy^2 + C^2(t)e^{-2x}dz^2 \qquad (4)$$

where A, B, C are functions of time only .

The field equation (1) for the metric (4) are given by

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{B\,C} + \frac{1}{A^2} = \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 \qquad (5)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{A\,C}-\frac{1}{A^2} = \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 \tag{6}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{A\,B}-\frac{1}{A^2} = \phi^{-2}\lambda+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 \tag{7}$$

$$\frac{A_4B_4}{A\,B}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{A\,C}-\frac{\alpha^2}{A^2} = \phi^{-2}\rho-\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 \tag{8}$$

$$\frac{1}{A}\left(\frac{A_4}{A}-\frac{B_4}{B}\right)=0 \tag{9}$$

Where the subscript 4 denotes ordinary differentiation with respect to *t*.

From (9)

$$B=\mu C \tag{10}$$

Using (10),(5)-(8) reduces to

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{A\,C}-\frac{1}{A^2} = \frac{\omega}{2}\left(\frac{\varphi_4}{\varphi}\right)^2 \tag{11}$$

$$2\frac{C_{44}}{C}+\frac{C_4^{\,2}}{C^2}+\frac{1}{A^2} = \varphi^{-2}\lambda+\frac{\omega}{2}\left(\frac{\varphi_4}{\varphi}\right)^2 \tag{12}$$

$$\frac{C_4^{\,2}}{C^2}+2\frac{A_4C_4}{AC}-\frac{1}{A^2} = \varphi^{-2}\rho-\frac{\omega}{2}\left(\frac{\varphi_4}{\varphi}\right)^2 \tag{13}$$

3. SOLUTION OF THE FIELD EQUATIONS

The field equations (11) – (13) are a system of three equations with five unknown parameters A,C,ϕ, ρ and λ. We need two additional conditions to get a deterministic solution of the above system of equations. Thus we present the solutions of the field equations in the following physically meaningful cases:

We assume the following two conditions:

(i) The simplest relation between ρ *and* λ is the proportionality relation, written as $\rho=\beta\lambda$ Where β is a proportionality constant which gives rise to the following three cases:

(a) For $\beta=1$, we get geometric strings(or) Nambu strings

(b) For $\beta=(1+\xi)$, $\xi\geq 0$,we get p-string or Takabayaski strings

and (ii) The scalar expansion θ and the shear scalar σ are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad \text{and} \quad \sigma^2 = \frac{1}{3}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2$$

The motivation for assuming $\frac{\sigma}{\theta}$ =constant,is explained as: Referring to Thorne [19] , the observations of velocity redshift relation for extra galactic sources suggest that the Hubble expansion of the universe is isotropic to within 30% (Kantowski and Sachs[20]; Kristian and Sachs[21]). More precisely, the redshift studies place the limit $\frac{\sigma}{H} \leq 0.30$ where σ is the shear and H the Hubble constant. Collins et al.[22] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous hyper surface satisfies the condition $\frac{\sigma}{\theta} = p$ (constant). This condition leads to

$$A = d\, C^m \tag{14}$$

where d is integrating constant and $m = \frac{1-2\sqrt{3}p}{1+\sqrt{3}p}$. and $0 < m < 1$. where m is real constant.

3.1 GEOMETRIC STRINGS (OR) NAMBU STRINGS($\rho = \lambda$) i.e., WHEN $\beta = 1$

Now the field equations (11)-(13) together with (14) reduces to

$$\frac{\ddot{A}}{A} + \frac{4}{m}\frac{\dot{A}^2}{A^2} - \frac{2}{A^2} = 0 \tag{15}$$

which on integration yields

$$A(t) = [\sqrt{m}\, t + c_1] \ . \tag{16}$$

Thus the general solution of the field equations (11) to (13) is given by

$$\begin{aligned} A(t) &= [\sqrt{m}\, t + c_1] \\ B(t) &= \mu[\sqrt{m}\, t + c_1]^{\frac{1}{m}} \\ C(t) &= [\sqrt{m}\, t + c_1]^{\frac{1}{m}} \end{aligned} \tag{17}$$

where μ, c_1 are integrating constants. The constant μ can be set equal to 1 without loss of generality.

From equation (11), the scalar field is given by

$$\phi = \phi_0(\sqrt{m}t + c_2)\ ^{k}{}_{1} \tag{18}$$

where $k_1 = \left[\frac{2(1-m)}{\omega m^2}\right]^{\frac{1}{2}}$ and If we choose a suitable value of constant m ,i.e $0 < m < 1$. We can find that from equation (18) the scalar field is real and valid.

The Bianchi type –VI_0 metric reduces to the form

$$ds^2 = -dt^2 + \left[\left(\sqrt{m}t + c_1\right)\right]^2 dx^2 + e^{2x}\left[\left(\sqrt{m}t + c_1\right)^{\frac{2}{m}}\right]dy^2 + \left[\left(\sqrt{m}t + c_1\right)^{\frac{2}{m}}\right]dz^2 \qquad (19)$$

3.1.1The Geometric And Physical Significance Of Model

The string energy density ρ, tension density λ are given by

$$\rho = \lambda = \phi_0^{\ 2}\,\frac{2}{m}\left(\sqrt{m}\,t + c_1\right)^{2k_1-2} \qquad (20)$$

Here the string energy density ρ and tension density λ satisfy the energy conditions. The kinematical parameters viz., the scalar expansion θ, the shear scalar σ, spatial volume V and the deceleration parameter q are given by

$$\theta = \frac{(2+m)}{(\sqrt{m}t + c_1)}$$

$$\sigma = \left[\frac{1}{\sqrt{3}}\frac{(m-1)}{(\sqrt{m}t + c_1)}\right] \qquad (21)$$

$$V = \sqrt{-g} = \left[(\sqrt{m}\,t + c_1)\right]^{\frac{(2+m)}{m}}$$

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \frac{2(m-1)}{(2+m)}$$

and $$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\frac{(1-m)}{(2+m)}$$

The model (19) starts with a big bang at t=0 .The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta} = \text{constant}$,hence the model does not approach isotropy. Since $\rho, \lambda, \theta, \sigma$ tend to infinity and $V^3 \to 0$ at initial epoch t=0, therefore ,the model (l9) for geometric has Line –singularity .Usually the model decelerate when $q < 0$ and inflates when $q > 0$ .Here for the condition $0 < m < 1$,the model (19) gives accelerating model of the universe. The model becomes isotropic if $m = 1$.It is interesting to note that that $q \to 0,\ \sigma \to 0$ .Hence in this case ,the geometry of the universe (19) reduces to

$$ds^2 = -dt^2 + [(t+c_1)]^2 dx^2 + e^{2x}\left[(t+c_1)^2\right]dy^2 + \left[(t+c_1)^2\right]dz^2 \tag{22}$$

3.2 P-STRING (OR) TAKABAYASKI STRINGS $\rho = (1+\xi)\lambda$ i.e., WHEN $\beta = (1+\xi)$

Here the equation of state $\rho = (1+\xi)\lambda$, where $\xi(>0)$, a constant and it is small for string dominant era while large for particle dominant era. Now the field equations (11) - (13) together with (14) reduces to

$$\frac{\ddot{C}}{C} + (1+m)\frac{\dot{C}}{C^2} = \frac{(4+2\xi)}{C^{2m}(\xi m + 2m - \xi)} \tag{23}$$

Equation (23) further reduces to

$$\left(\frac{dC}{dt}\right)^2 = \frac{(8+4\xi)}{C^{2m-2}(4\xi m + 8m - 4\xi)} + c_2 \tag{24}$$

Where c_2 is an integral constant. To obtain a closed form solution for equation (28).Without any loss of generality we assume that $c_2=0$. Thus the solution of the above equation (24) is

$$C(t) = \left(m\sqrt{r}t + c_3 m\right) \tag{25}$$

Therefore the solution of the field equation (11)-(13) can be expressed as

$$A(t) = \left(m\sqrt{r}t + c_3 m\right)^m$$

$$B(t) = \mu\left(m\sqrt{r}t + c_3 m\right) \tag{26}$$

$$C(t) = \left(m\sqrt{r}t + c_3 m\right)$$

where $r = \dfrac{(8+4\xi)}{(4\xi m + 8m - 4\xi)}$ and c_3 is an integrating constant.

The scalar field is given by

$$\phi = \phi_1\left(m\sqrt{r}t + c_3 m\right)^{k_2} \tag{27}$$

where $k_2 = \left(\dfrac{2(2-4m+3\xi-2m\xi}{\omega m^2(2+\xi)}\right)^{\frac{1}{2}}$. The model, for takabayasi strings in this theory,reduces to

$$ds^2 = -dt^2 + \left[\left(m\sqrt{r}t + c_3 m\right)\right]^{2m} dx^2 + e^{2x}\left[\left(m\sqrt{r}t + c_3 m\right)^2\right]dy^2 + \left[\left(m\sqrt{r}t + c_3 m\right)^2\right]dz^2 \tag{28}$$

together with the scalar field given by equation (27).

3.2.1 The Geometric And Physical Significance Of Model:

The string energy density ρ and tension density λ are given by

$$\rho = (1+\xi)\lambda = \phi_1^{\,2} \frac{16(1+\xi)}{(4\xi m + 8m - 4\xi)} \left(m\sqrt{r}t + c_3 m\right)^{2k_2-2} \tag{29}$$

The realistic energy condition $\rho \geq 0$ and $\rho_p \geq 0$ can be satisfied.

The scalar expansion θ, the shear scalar σ, spatial volume V and the deceleration parameter q are given by

$$\text{Scalar expansion } \theta = \frac{(2+m)\sqrt{r}}{(m\sqrt{r}t + c_3 m)}$$

$$\text{Shear scalar } \sigma = \left[\frac{1}{\sqrt{3}} \frac{(1-m)\sqrt{r}}{(m\sqrt{r}t + c_3 m)} \right]$$

$$\text{Spatial volume } V = \sqrt{-g} = \left(m\sqrt{r}t + c_3 m\right)^{(2+m)} \tag{30}$$

$$\text{Deceleration parameter } q = \frac{-a\ddot{a}}{\dot{a}^{2}} = \frac{(m-1)}{(2+m)}$$

$$\text{and } \frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \frac{(1-m)}{(2+m)}.$$

The model (28) starts with a big bang at t=0.The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta} =$ Constant, the anisotropy is maintained throughout .Usually the model decelerate when q>0 and inflates when q<0. Here for 0<m < 1, the model inflates .The values of the deceleration parameter separates decelerating (q > 0) from accelerating (q < 0) periods in the evolution of the Universe. Determination of the deceleration parameter from the count magnitude relation for galaxies is a difficult task due to evolutionary effects. The present value q of the deceleration parameter obtained from observations [23] are $-1.27 \leq q \leq 2$. Studies of galaxy counts from red shift surveys provide a value of $q = 0.1$, with an upper limit of $q \leq 0.75$ [23]. Recent observations by Perlmutter et al [24,25] and Riess et al [26] show that the deceleration parameter of the Universe is in the range $-1 \leq q \leq 0$ and the present day Universe is undergoing accelerated expansion. It may be noted that though the current observations of SNe Ia and the CMBR favour accelerating models (q < 0), they do not altogether rule out the existence of the decelerating phase in the early history of our Universe which are also consistent with these observations [27].

4. CONCLUSIONS

The string cloud cosmological solutions for spatially homogeneous and anisotropic Bianchi type VI_0 model are derived from Sen-Dunn theory field equations. The string is coupled to the usual gravitational field. In order to solve the Sen-Dunn field equations we have used a more general equation of state for the proper energy density and string tension density from which the prevailing pure geometric cosmic strings and the p-strings or Takabayasi string solutions can be easily inferred. It is shown that for plausible geometric srting solution in Sen-Dunn theory, the power index 'm' should be in the range (0,1). It is known that the present upper limit of the anisotropy ratio $\frac{\sigma}{\theta}$ is 10^{-3} obtained from indirect arguments concerning the isotropy of the primordial blackbody radiation [22]. The ratio $\frac{\sigma}{\theta}$ of our models is considerably greater than its present value for all m except m=1 and m=-2. This fact indicates that our solution represents the early stages of evolution of the universe [21]. For m=1, it may be noted that the model become isotropic in all cases. The solutions obtained are quite new and certainly exhibit some interesting facts of the scalar-tensor theory of gravitation proposed by Sen-Dunn.

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