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Bianchi Type-IX Dark Energy Model in Brans-Dicke Theory of Gravitation

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Abstract

We have discussed the cosmological model in Brans-Dicke theory (Phys. Rev. 124:925, 1961) for a Bianchi Type-IX space-time filled with dark energy and obtained exact solutions by applying variation law for generalized Hubble's parameter given by Bermann (Nuovo Cimento 74:182, 1983). The physical and geometrical properties of the cosmological model are also discussed.

Keywords: Dark Energy, Bianchi Type-IX, Brans-Dicke theory.

1. Introduction

Over the past decade, one of the most remarkable discoveries is that our universe is currently accelerating. The high redshift supernovae Ia was first observed by Astier, P. *et al.* [6], Perlmutter *et al.* [32], Riess *et al.* [46, 47], which later confirmed from the cosmic microwave background radiation by Bennett *et al.* [13, 14], Bemui *et al.* [12] and Spergel *et al.* [59] and from the large scale structure by Abazajian *et al.* [1, 2, 3], Hawkins *et al.* [23], Tegmark *et al.* [61] and Verde *et al.* [64]. Recent astronomical observations, based on Friedmann cosmological model, indicate that our universe is flat and currently consists of approximately 73% DE, 23% dark matter, and 4% baryon matter and radiation. The nature of DE is unknown, and many radically different models have been proposed, such as, quintessence by Caldwell [19], Frieman *et al.* [22], Liddle and Scherrer [26], Peebles [30], Ratra and Peebles [41], Steinhardt *et al.* [60], Turner and White [62] and Wetterich [66], Chaplygin gas by Kamenshchik [24], phantom energy by Caldwell [20] and DE in brane worlds by Cai [18], Sahni [50], Yang and Wang [67] among many others. (see the review articles Sahni [48, 49], Yang [67], Carroll [21], Padmanabhan [29], Peebles [31])

Many relativists have taken keen interest in studying Bianchi type-IX universe because familiar solutions like Robertson Walker universe, the de-sitter universe, the Taub-NUT solutions etc.

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Bianchi type-IX cosmological models include closed FRW models. These models allow not only expansion but also rotation and shear and in general are anisotropic. The dynamical effects of Bianchi type-IX models are studied by Waller [65]. Bali and Dave [7] have investigated Bianchi type-IX string cosmological model in general relativity. Bali and Kumawat [8] have investigated Bianchi type-IX stiff fluid tilted cosmological model with bulk viscosity. Tyagi and Sharma [63] investigated Bianchi-IX string cosmological model for perfect fluid distribution in general relativity. Bali and Yadav [9] studied Bianchi type-IX string as well as viscous models. Reddy *et al.* [43] studied Bianchi type-II, VIII and IX models in scale covariant theory of gravitation. Shanthi and Rao [53] studied Bianchi type-VIII and IX models in Lyttleton-Bondi universe. Also Rao and Sanyasi Raju [34, 35] have studied Bianchi type-VIII and IX models in zero mass scalar fields and self creation cosmology. Rahaman *et al.* [33] have investigated Bianchi type-IX string cosmological model in scalar-tensor theory formulated by Sen [51] based on Lyra [27] manifold. Rao *et al.* [38, 39, 40] have studied Bianchi type-II, VIII and IX string cosmological models, perfect fluid cosmological models in Saez-Ballester scalar-tensor theory of gravitation and string cosmological models in general relativity as well as self creation theory of gravitation respectively.

Brans and Dicke [17] theory of gravitation is well known modified version of Einstein's general theory of gravitation. It is a scalar tensor theory in which the gravitational interaction is mediated by a scalar field ϕ as well as the tensor field g_{ij} of Einstein's theory. In this theory the scalar field ϕ has the dimension of the inverse of the gravitational constant. In recent years, there has been a renewed interest of the gravitational constant. Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. The work of Singh and Rai [57] gives a detailed discussion of Brans-Dicke cosmological models. In particular, spatially homogeneous Bianchi models in Brans-Dicke theory in the presence of perfect fluid with or without radiation are quite important to discuss the early stages of evolution of the universe. Nariai [28], Belinskii and Khalatnikov [11], Reddy and Rao [42], Banerjee and Santos [10], Shri Ram [55], Shri Ram and Singh [56], Berman *et al.* [16], Reddy [44], Reddy and Naidu [45], Adhav *et al.* [4] and Rao *et al.* [36, 37] are some of the authors who have investigated several aspects of this Brans-Dicke theory.

In this paper we discussed Bianchi type-IX space-time filled with DE in Brans-Dicke theory of gravitation. This work is organized as follows: In Section 2, the model and field equations have been presented. The field equations have been solved in Section 3 by using deceleration parameter. The physical and geometrical properties of the model have been discussed in Section 4. In the last Section 5, concluding remarks have been expressed.

2. Metric and Field Equations

Bianchi type-IX metric is considered in the form,

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + \left(b^2 \sin^2 y + a^2 \cos^2 y\right)dz^2 - 2a^2 \cos y dx dz, \tag{1}$$

where a, b are scale factors and are functions of cosmic time t.

Brans-Dicke [1] field equations for combined scalar and tensor fields are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{;ij} - g_{ij}\phi^{;k}_{;k}\right), \tag{2}$$

and

$$\phi^{;k}_{;k} = 8\pi\phi^{-1}(3+2\omega)^{-1}T, \tag{3}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, T_{ij} is the energy tensor of the matter and ω is the dimensionless coupling constant. The continuity equation reads as

$$T^{ij}_{;j} = 0. \tag{4}$$

Here comma and semi-colon denote partial and covariant differentiation respectively.

The energy momentum tensor of the fluid which is taken as

$$T_i^{\,j} = \left[T_0^0, T_1^1, T_2^2, T_3^3\right] \tag{5}$$

The simplest generalization of EoS parameter of perfect fluid is to determine it separately on each spatial axis by preserving diagonal form of the energy momentum tensor in a consistent way with the considered metric. Hence one can parameterize energy momentum tensor as follows:

$$T_i^{\,j} = \left[-\rho, p_x, p_y, p_z\right]$$

$$T_i^{\,j} = \left[-1, \omega_x, \omega_y, \omega_z\right]\rho$$

$$T_i^{\,j} = \left[-1, \omega, \omega+\delta, \omega+\delta\right]\rho \tag{6}$$

Here ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and ω_x, ω_y and ω_z are the directional EoS parameters along the *x, y* and *z* axes respectively, ω is the deviation free EoS parameter of the fluid.

In the co-moving coordinate system, the field equations (2)–(4) for the metric (1) and with the help of energy momentum tensor (6) can be written as

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{a^2}{4b^4} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + 2\frac{\dot{b}}{b}\frac{\dot{\phi}}{\phi} = \frac{8\pi}{\phi}\rho, \tag{7}$$

$$2\frac{\ddot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{3a^2}{4b^4} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + 2\frac{\dot{b}}{b}\frac{\dot{\phi}}{\phi} = -\frac{8\pi}{\phi}\omega\rho, \tag{8}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}\dot{b}}{ab}+\frac{a^2}{4b^4}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}+\frac{\ddot{\phi}}{\phi}+\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}+\frac{\dot{b}}{b}\frac{\dot{\phi}}{\phi}=-\frac{8\pi}{\phi}(\omega+\delta)\rho, \tag{9}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)=-\frac{8\pi T}{\phi(3+2\omega)}, \tag{10}$$

where T is the trace of matter energy momentum tensor T_i^j and the over dot (.) denotes the differentiation with respect to t.

3. Solutions of the Field Equations

The field equations (7)–(10) constitute a system of four independent equations with six unknowns parameters $a, b, \omega, \rho, \delta, \phi$. Therefore, some additional constraint equations relating these parameters are required to obtain explicit solutions to the system of equations. So we take two additional conditions relating these parameters to get the explicit solutions of the system. The above system of highly non-linear equations are solved with the help of special law of variation of Hubble's parameter proposed by Berman [15] which yields constant deceleration parameter of the models of the universe.

We consider that the constant deceleration parameter

$$q=-\frac{R\,\ddot{R}}{\dot{R}^2}=\text{constant}, \tag{11}$$

where the scale factor R is given by

$$R=\left(ab^2\right)^{\frac{1}{3}}. \tag{12}$$

Here the constant is taken as negative. (*i.e.*, it is accelerating model of the universe)

The solution of equations (11) and (12), gives

$$R=(\alpha t+\beta)^{\frac{1}{1+q}}, \tag{13}$$

where $\alpha \neq 0$, and β are constants of integration.
This condition implies that the condition of expansion is $1+q>0$.

We consider that the scalar expansion θ is proportional to the shear scalar σ (Refer [5, 52, 54, 58]), i. e., $\theta \propto \sigma$ which leads to

$$a=b^m, \tag{14}$$

where $m \neq 1$ is a positive constant.

Solving the field equations (7)-(9) with the help of equations (12), (13) and (14), we get the solution for metric coefficients as:

$$a = t^{\frac{3m}{(1+q)(m+2)}}, \tag{15}$$

$$b = t^{\frac{3}{(1+q)(m+2)}}. \tag{16}$$

Taking the Gauss function

$$\phi = \xi t^n, \tag{17}$$

where ξ is a constant.

Thus the Bianchi type-IX DE cosmological model in Brans-Dicke theory of gravitation can be written as

$$ds^2 = -dt^2 + t^{\frac{6m}{(1+q)(m+2)}} dx^2 + t^{\frac{6}{(1+q)(m+2)}} dy^2 + \left(t^{\frac{6}{(1+q)(m+2)}} \sin^2 y + t^{\frac{6m}{(1+q)(m+2)}} \cos^2 y \right) dz^2 - 2t^{\frac{6m}{(1+q)(m+2)}} \cos y dx dz . \tag{18}$$

4. Some Physical Properties of the Model

Equation (18) represents Bianchi type-IX DE cosmological model in Brans-Dicke theory of gravitation. The physical quantities that are important in cosmology are spatial volume V, Hubble parameter H, expansion scalar θ, shear scalar σ^2 and deceleration parameter q, which have the following expressions for the model (18):

$$\text{Spatial volume, } V = ab^2 = t^{\frac{3}{1+q}} . \tag{19}$$

$$\text{Hubble's parameter, } H = \frac{\alpha}{(1+q)t} . \tag{20}$$

$$\text{Expansion scalar, } \theta = \frac{3\alpha}{(1+q)t} . \tag{21}$$

$$\text{Shear scalar, } \sigma^2 = \frac{1}{2}\left[\frac{6\alpha^2 (m-1)^2}{(1+q)^2 (m+2)^2 \; t^2}\right] . \tag{22}$$

$$\text{Deceleration parameter, } q = -1 . \tag{23}$$

Energy density,

$$\rho = \frac{1}{8\pi\phi^{-1}}\left[\frac{9\alpha^2(m-1)^2}{(1+q)^2(m+2)^2\ t^2} - \frac{1}{4}t^{\frac{6(m-2)}{(1+q)(m+2)}} + t^{\frac{-6}{(1+q)(m+2)}} - \frac{\omega n^2}{2t^2} + \frac{3n\alpha}{(1+q)t^2}\right]. \qquad (24)$$

EoS parameter,

$$\omega = \frac{-1}{8\pi\rho\phi^{-1}}\left[\begin{aligned}&\frac{3\alpha^2(5-2m-2qm-4q)}{(1+q)^2(m+2)^2\ t^2} - \frac{3}{4}t^{\frac{6(m-2)}{(1+q)(m+2)}} + t^{\frac{-6}{(1+q)(m+2)}} + \frac{\omega n^2}{2t^2}\\ &-\frac{n(n-1)}{t^2} + \frac{6n\alpha}{(1+q)(m+2)t^2}\end{aligned}\right]. \qquad (25)$$

Skewness parameter,

$$\delta = -\,\omega - \frac{1}{8\pi\rho\phi^{-1}}\left[\begin{aligned}&\frac{3\alpha^2(2m^2-3m-qm^2-3qm-2q+1)}{(1+q)^2(m+2)^2\ t^2} + \frac{9m\alpha^2}{(1+q)^2(m+2)^2t^2}\\ &+\frac{1}{4}t^{\frac{6(m-2)}{(1+q)(m+2)}} + \frac{\omega n^2}{2t^2} + \frac{n(n-1)}{t^2} + \frac{3\alpha n(m+1)}{(1+q)(m+2)t^2}\end{aligned}\right]. \ (26)$$

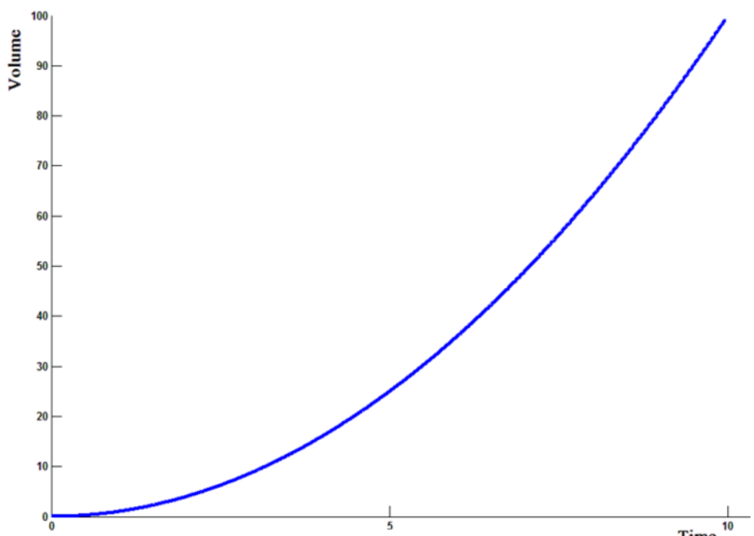

Figure: 1. Spatial Volume versus Cosmic Time

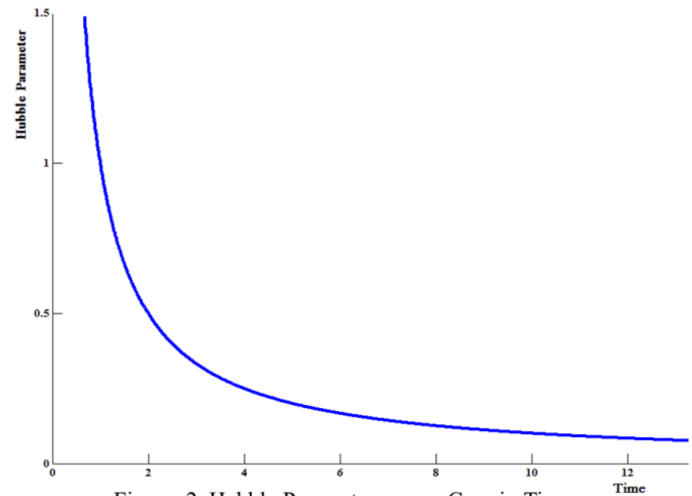

Figure: 2. Hubble Parameter versus Cosmic Time

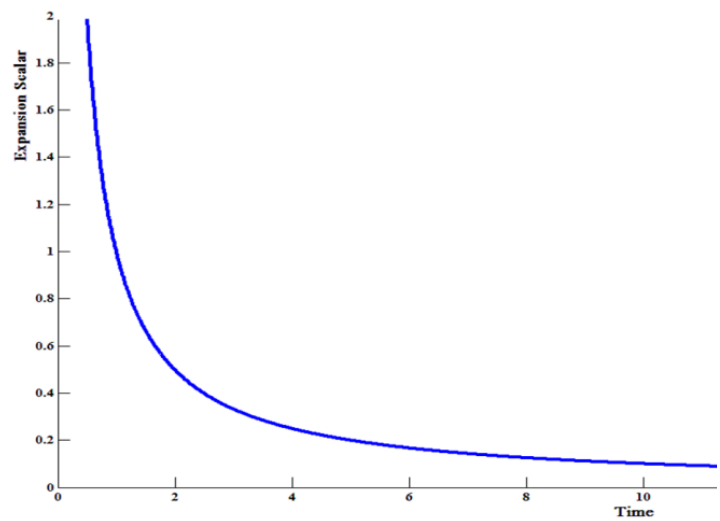

Figure: 3. Expansion Scalar versus Cosmic Time

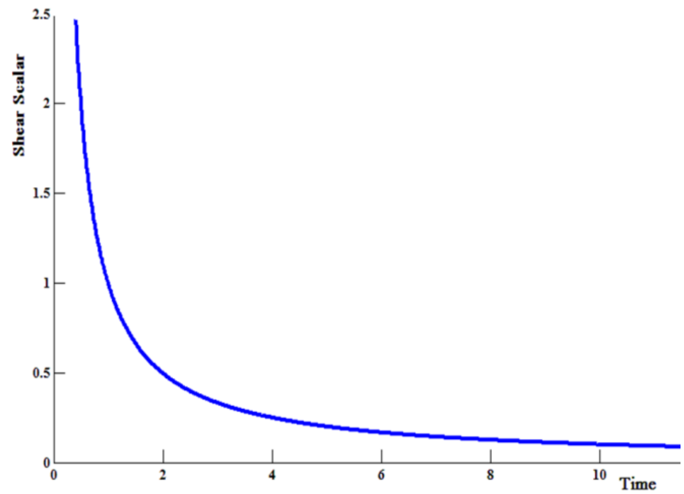

Figure: 4. Shear Scalar versus Cosmic Time

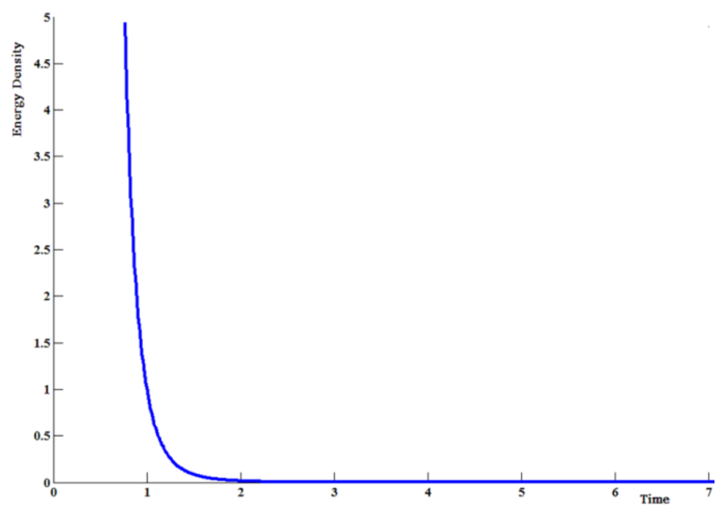

Figure: 5. Energy Density versus Cosmic Time

It may be observed that at an initial moment $(t = 0)$, the spatial volume will be zero. The expansion scalar θ and shear scalar σ^2 tend to infinity as $t \to 0$ whereas when $t \to \infty$, the spatial volume becomes infinitely large but expansion scalar and shear scalar and Hubble parameter tends to zero. Also, since $\lim_{T\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ being independent of cosmic time implies that the model does not approach isotropy for large values of *t*. The model is expanding shearing, non-rotating and has no initial singularities. From equation (25) it is observed that the EoS parameter is time dependent. Also from (17) one can observe that for $t = 0$, the scalar field vanishes while the energy density tends to infinity and for $t \to \infty$, scalar field $\phi \to \infty$ while energy density vanishes.

5. Conclusion

In this paper, we have obtained Bianchi type-IX cosmological model filled with dark energy in Brans-Dicke theory. While solving Brans-Dicke's field equations for Bianchi type-IX cosmological model, we have used a special law of variation of Hubble's parameter proposed by Bermann [15]. The model thus obtained is expanding and free from initial singularity. It is observed that the dark energy EoS parameters are time dependent. The model obtained in this paper is of considerable interest and may be useful in Brans-Dicke theory to study an accelerating model of the Universe. It is interesting to note that our model resembles with the investigations of Katore and Shaikh [25].

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