Article

LRS Bianchi Type-III Massive String Cosmological Models in Scalar Theory of Gravitation

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Abstract

The present study deals with locally rotationally symmetric (LRS) Bianchi type III cosmological models representing massive string. The energy-momentum tensor for such string as formulated by Letelier [10] is used to construct massive string cosmological models. Exact solutions of the field equations are obtained with the help of: (i) proportionality relationship between rest energy density and tension density of strings; and (ii) a relationship between the metric coefficients. We have derived some models depending on different values of m. It is observed that in early stage of the evolution of the universe, the universe is dominated by strings. Our models are in accelerating phase which is consistent to the recent observations of Type Ia supernovae .The properties of the models are discussed at the end.

Keywords: massive strings, scalar-tensor, cosmology, gravitation.

1. Introduction

Modified theories of gravity have been the subject of study for the last few decades. As an alternative to Einstein's theory of gravitation, Sen and Dunn [1] have proposed a new scalar-tensor theory of gravitation in which both the scalar and tensor fields have intrinsic geometrical significance. The scalar field in this theory is characterized by the function $\phi = \phi(x^i)$ where x^i are coordinates in the four – dimensional Lyra manifold and the tensor field is identified with the metric tensor g_{ij} of the manifold. The field equations given by Sen and Dunn[1] for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2} g_{ij} R = \omega \phi^{-2} (\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k}) - \phi^{-2} T_{ij} \quad (1)$$

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where $\omega = \frac{3}{2}$, R_{ij} and R are respectively the usual Ricci-tensor and Riemaan-curvature scalar (in our units $C = 8\pi G = 1$).

In recent years, there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe (Kibble [2]). Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (Zel'dovich et al. [3, 4]; Kibble [5]; Everett [6]; Vilenkin [7]). It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies (Zel'dovich [8]). These cosmic strings have stress-energy and couple to the gravitational field. Therefore it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings was done by Letelier [9] who considered the massive strings to be formed by geometric strings with particle attached along its extension. Letelier [10] first used this idea in obtaining cosmological solutions in Bianchi I and Kantowski-Sachs space-times. Stachel [11] has studied massive string. Bali et al. [12-18] have obtained Bianchi types-I, III and IX string cosmological models in general relativity. Yadav et al. [19] have studied some Bianchi type- I viscous fluid string cosmological models with magnetic field. Recently Wang [20-23] has also discussed LRS Bianchi type-I and Bianchi type-III cosmological models for a cloud string with bulk viscosity.

The energy momentum tensor for a cloud of strings is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \,. \tag{2}$$

Here ρ, the proper energy density and λ, the string tension density and are related by $\rho = \rho_p + \lambda$, where ρ_p is the particle density of the configuration. The velocity u^i describes the 4 – velocity which has components (1, 0, 0, 0) for a cloud of particles and x^i represents the direction of string which will satisfy

$$u^i u_i = -x^i x_i = 1 \text{ and } u^i x_i = 0 \tag{3}$$

The large scale matter distribution in the observable universe, largely manifested in the form of discrete structures, does not exhibit homogeneity of a high order.

In contrast, the cosmic background radiation (CMB), which is significant in the microwave region, is extremely homogeneous, however, recent space investigations detect anisotropy in the CMB. The observations from cosmic background explorers differential microwave radiometer (COBE-DMR) have detected and measured CMB anisotropies in different angular scales. These anisotropies are supposed to hide in their fold the entire history of cosmic evolution dating back to the recombination epoch and are being considered as indicative of the geometry and the content of the universe. More about CMB anisotropy is expected to be uncovered by the investigations of Wilkinson Microwave Anisotropy Probe (WMAP). There is widespread consensus among cosmologists that CMB anisotropies in small angular scales have the key to the formation of discrete structures. Our interest is in observed CMB anisotropies in the large angular scales and we intend to attribute it to the anisotropy of the spatial cosmic geometry. Bianchi type space-times exhibit spatial homogeneity and anisotropy. Yadav et al. [24] have obtained cylindrically symmetric inhomogeneous universe with a cloud of strings. Baysal et al. [25] have investigated the behaviour of a string in the cylindrically symmetric inhomogeneous universe. Yavuz et al. [26] have examined charged strange quark matter attached to the string cloud in the spherically symmetric space-time admitting one-parameter group of conformal motion. Kaluza-Klein cosmological solutions are obtained by Yilmaz [27]. Singh & Singh [28], and Singh [29, 30] have studied string cosmological models in different symetry and Bianch space-times. Reddy [31, 32], Reddy et al. [33-35], Rao et al. [36-39], Pradhan [40, 41], and Pradhan et al. [42-44] have studied string cosmological models in different contexts. Recently, Tripathi et al. [45, 46] have obtained cosmic strings with bulk viscosity. Venkateswarlu and Pavan Kumar [47] have studied various aspects of string cosmologies in alternative theories of gravitation. Recently Venkateswarlu et. al [48] have studied the Bianchi Type –I cosmic strings in this theory.

The present day universe is satisfactorily described by homogeneous and isotropic models given by the FRW space-time. The universe in a smaller scale is neither homogeneous nor isotropic nor do we expect the Universe in its early stages to have these properties. Homogeneous and anisotropic cosmological models have been widely studied in the framework of General Relativity in the search of a realistic picture of the universe in its early stages. Although these are more restricted than the inhomogeneous models which explain a number of observed phenomena quite satisfactorily. Bianchi type-III space-time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of universe. A spatially homogeneous Bianchi model necessarily has a three-dimensional group, which acts simply transitively on space-like three-dimensional orbits. Here we confine ourselves to a locally rotationally symmetric (LRS) model of Bianchi type-III. The strings that form the cloud are massive strings instead of geometric strings. Each massive string is formed by a geometric string with particles attached along its extension. Hence, the string that form the cloud are generalization of Takabayasi's relativistic model of strings (called p-string). This is simplest model wherein we have particles and strings together.

In this paper, we intended to study the Bianchi type-III in the context of cosmic strings in a new scalar-tensor theory of gravitation proposed by Sen and Dunn[1]. Section 2 contains Bianchi type –III metric and the field equations of this theory. In section 3, the solutions of the field equations are obtained in the context of cosmic strings and also discussed some properties of the models obtained. Conclusions are given in last section.

2. Metric and Field equations

We consider the Bianchi type –III metric

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2 \tag{4}$$

where $A,\ B, C$ are functions of time only ,and α is a constant. We now consider x^i to be along z – axis so that we have $x^i = (0,0,0,\frac{1}{C})$.

The field equation (1) for the metric (4) are given by

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C} = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{5}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\ \frac{\dot{C}}{C} = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\alpha^2}{A^2} = \phi^{-2}\lambda+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{7}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\ \frac{\dot{C}}{C}-\frac{\alpha^2}{A^2} = \phi^{-2}\rho-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{8}$$

$$\alpha\ (\frac{\dot{A}}{A}-\frac{\dot{B}}{B}) = 0 \tag{9}$$

where the overhead "dot" denotes ordinary differentiation with respect to *t*. For $\alpha = 0$, we recover the solutions of Bianchi type-I model [48] in this theory.

From (9), we have

$$A = \mu B \tag{10}$$

where μ is an arbitrary. Using equation (10), equations (6)-(8) reduce to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C} = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{11}$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\alpha^2}{\mu^2B^2} = \phi^{-2}\lambda+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{12}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{\alpha^2}{\mu^2B^2} = \phi^{-2}\rho-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{13}$$

3. Solution of the field equations

The field equations (11) – (13) are a system of three equations with five unknown parameters B, C, ϕ, ρ and λ. In order to obtain explicit solutions of the system, we must impose two additional conditions. We assume that

(i) $\rho = \beta\lambda$, where β is a proportionality constant which gives rise to the following three cases:

(a) For a cloud of geometric strings (or) Nambu strings, we have $\beta = 1$

(b) For massive strings we have $\beta = -1$

(c) For $\beta = (1+\xi)$, $\xi \geq 0$, we get p-string (or) Takabayaski strings

and (ii) a functional relationship [49] between the metric functions B and C of the form

$$C = B^m \tag{14}$$

where m is a constant.

3.1 Geometric strings (or) Nambu strings($\rho = \lambda$) i.e, when $\beta = 1$

Now the field equations (11)-(13) together with (14) admit the exact solution

$$\begin{aligned} A(t) &= \mu[(m+2)(c_1 t + c_2)]^{\frac{1}{(m+2)}} \\ B(t) &= [(m+2)(c_1 t + c_2)]^{\frac{1}{(m+2)}} \\ C(t) &= [(m+2)(c_1 t + c_2)]^{\frac{m}{(m+2)}} \end{aligned} \tag{15}$$

where μ, c_1, and c_2 are integrating constants. The constant μ can be set equal to 1 without loss of generality.

From equation (11), the scalar field is given by

$$\phi = \phi_0 (c_1 t + c_2)^{k_1} \tag{16}$$

where $\mathbf{k_1} = \left[-\frac{\mathbf{2(2m+1)}}{\boldsymbol{\omega}\,\mathbf{(m+2)^2}} \right]^{\frac{1}{2}}$, $\mathbf{m < \frac{-1}{2}}$.

The Bianchi type – III model in this case reduces to the form

$$ds^2 = -dt^2 + \left[((m+2)(c_1 t + c_2)\right]^{\frac{2}{m+2}} \left(dx^2 + e^{-2\alpha x} dy^2\right) + \left[((m+2)(c_1 t + c_2)\right]^{\frac{2m}{m+2}} dz^2 \tag{17}$$

The string energy density ρ and tension density λ are given by

$$\lambda = \rho = -\phi_0^{\,2} \frac{\alpha^2}{(m+2)^{\frac{2}{(m+2)}} (c_1 t + c_2)^{\frac{2}{(m+2)} - 2k_1}} \tag{18}$$

it is noticed that the scalar field do not co-exist with geometric strings in this theory. Hence Bianchi type- III geometric strings do not exist in Sen-Dunn theory of gravitation.

3.2 Massive strings ($\rho + \lambda = 0$) i.e, when $\beta = -1$

In this case, the equations (11)-(13) together with (14) yields

$$\frac{\ddot{B}}{B} + (m+1)\frac{\dot{B}^2}{B^2} = \frac{\alpha^2}{\mu B^2} \tag{19}$$

which admits the solution

$$B(t) = \left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right). \tag{20}$$

Now the general solution in this case can be expressed as

$$A(t) = \left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right).$$

$$B(t) = \left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right). \tag{21}$$

$$C(t) = \left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right)^m.$$

where c_3 is an integrating constant.

From equation (11), the scalar field is given by

$$\phi = \phi_0 \left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right)^{k_2} \tag{22}$$

where $k_2 = \sqrt{\dfrac{2m^2}{\omega}}$.

Thus the model for massive strings in Sen-Dunn theory of gravitation takes the form

$$ds^2 = -dt^2 + \left(dx^2 + e^{-2\alpha x} dy^2\right)\left[\left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right)^2\right] + \left[\left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right)^{2m}\right] dz^2 \tag{23}$$

and the corresponding scalar field is given by equation (22).

The string energy density ρ and tension density λ are

$$\rho = -\lambda = \phi_0^{\,2} \alpha^2 m \left(\frac{\alpha t}{\sqrt{(m+1)}} + c_3\right)^{2k_2 - 2} \tag{24}$$

and the particle density ρ_p is given by

$$\rho_p = 2\phi_0^{\,2} \alpha^2 m \left(\frac{\alpha t}{\sqrt{(m+1)}} + c_2\right)^{2k_2 - 2}$$

The dominant energy conditions implies that $\rho > 0$ and $\rho^2 \geq \lambda^2$. These energy conditions do not restrict the sign of λ, accordingly the expression given by equation (23) satisfies all these conditions. Accordingly to Refs. [1] and [50] ,when $\frac{\rho_p}{|\lambda|} > 1$,in the process of evolutions, the universe is dominated by massive strings ,when $\frac{\rho_p}{|\lambda|} < 1$,the universe is dominated by the strings. Here $\frac{\rho_p}{|\lambda|} = 2$. Since $\frac{\rho_p}{|\lambda|} > 1$, we may conclude that the particles dominate over the strings in this model. We note that either for ,we obtain for $m > \sqrt{3}$ and $t \to \infty$ or for $m < \sqrt{3}$ and $t \to 0$,we obtain $\frac{\rho_p}{|\lambda|} > 1$.Thus in these cases ,the universe is dominated by massive strings throughout the whole process of evolution of the universe at early as well as late time.

The scalar expansion θ, the shear scalar σ, spatial volume V and the deceleration parameter q are given by

$$\theta = \frac{(2+m)\alpha}{(\sqrt{\alpha}t + c_3\sqrt{m+1})}$$

$$\sigma = \left[\frac{1}{\sqrt{3}}\frac{(1-m)\alpha}{(\sqrt{\alpha}t + c_3\sqrt{m+1})}\right] \tag{25}$$

$$V = \sqrt{-g} = \left[\left(\frac{\alpha t}{\sqrt{m+1}} + c_3\right)\right]^{(2+m)}$$

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \frac{(1-m)}{(2+m)}$$

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\frac{(1-m)}{(2+m)}$$

From the above results it can be seen that the spatial volume is zero at $t = 0$ and t increases with the increase of time. This shows that the universe starts evolving with zero volume at $t = 0$ and expands with cosmic time 't'. As $t \to \infty$,the scale factors A(t),B(t) and C(t) tend to infinity. The energy density becomes zero as $t \to \infty$.This shows that the universe is expanding with the increase of cosmic time but the of expansion and shear scalar decreases to zero and tend to isotropic. It is observed that the scalar field becomes constant and $\lambda = 0 = \rho$ when $m = 0$. For $m = \sqrt{3}$, $\lambda = \rho =$ Constant . Thus we have a general relativistic vacuum solution in this case. The model becomes isotropic if $m = 1$. The cosmological evolution of Bianchi type –III space time is expansionary, with all the three scale factors monotonically increasing function of time. It is interesting to note that $q \to 0$, $\sigma \to 0$ when $m = 1$ and the metric (4) takes the form

$$ds^2 = -dt^2 + \left(dx^2 + e^{-2\alpha x}dy^2 + dz^2\right)\left[\left(\frac{\alpha t}{\sqrt{2}} + c_3\right)^2\right] . \tag{26}$$

3.3 p-string (or) Takabayaski strings $\rho = (1+\xi)\lambda$ i.e, when $\beta = (1+\xi)$

In this case we consider the equations (11)-(13) together with (14) reduces to

$$\frac{\ddot{B}}{B} + (m+1)\frac{\dot{B}^2}{B^2} = \frac{\alpha^2\xi}{B^2(\xi - \xi m - 2m)} \tag{27}$$

which on integration yields

$$B(t) = (\frac{\alpha\sqrt{\xi}}{\sqrt{r}}t + c_4) \, . \tag{28}$$

Therefore the solution of the field equation (11)-(13) can be expressed as

$$A(t) = (\frac{\alpha\sqrt{\xi}}{\sqrt{r}}t + c_4) \; .$$

$$B(t) = (\frac{\alpha\sqrt{\xi}}{\sqrt{r}}t + c_4) \tag{29}$$

$$C(t) = (\frac{\alpha\sqrt{\xi}}{\sqrt{r}}t + c_4)^{m} \; .$$

where $r = (1+m)(\xi - \xi m - 2m)$ and c_3 is an integrating constant.

The scalar field is given by

$$\phi = \phi_0 \, (\frac{\alpha\sqrt{\xi}}{\sqrt{r}}t + c_4)^{k_3} \tag{30}$$

where $k_3 = \sqrt{\frac{2m^2}{\omega}}$.

The model, for Takabayasi strings in this theory, reduces to

$$ds^2 = -dt^2 + \left[\left(\frac{\alpha\sqrt{\xi}}{r}t + c_4\right)^2\right]\left(dx^2 + e^{-2\alpha x}dy^2\right) + \left[\left(\frac{\alpha\sqrt{\xi}}{\sqrt{r}}t + c_4\right)^{2m}\right]dz^2 \tag{31}$$

together with the scalar field given by equation (30).

The expression for string energy density ρ and the tension density λ are given by

$$\rho = (1+\xi)\,\lambda = \phi_0{}^2 \left((\frac{\alpha\sqrt{\xi}}{\sqrt{r}}t + c_4) \right)^{(2k_3-2)} \frac{2m(1+\xi)\alpha^2}{(\xi - \xi m - 2m)} \tag{32}$$

and the kinematical parameters are given by

Scalar expansion $\theta = \dfrac{(2+m)\alpha\sqrt{\xi}}{(\alpha\sqrt{\xi}t + c_4\sqrt{r})}$

Shear scalar $\sigma = \left[\dfrac{1}{\sqrt{3}} \dfrac{(1-m)\alpha\sqrt{\xi}}{(\alpha\sqrt{\xi}t + c_4\sqrt{r})}\right]$

$$\text{Spatial volume } V = \sqrt{-g} = \left[\frac{\sqrt{\xi}\alpha t}{\sqrt{r}} + c_4\right]^{(2+m)} \tag{33}$$

Deceleration parameter $q = \dfrac{-a\ddot{a}}{\dot{a}^2} = \dfrac{(1-m)}{(2+m)}$

It may be noted that the parameters remain finite and physically significant for all $t > 0$. Hence we see that space-time admits a big-bang singularity but the rate of expansion of the universe decreases with increase of time. In this case also, we have $\phi =$ constant, and $\lambda = 0 = \rho$ when $m = 0$. Thus the p-strings or Takabayasi strings do not exist for $m = 0$. The values of the deceleration parameter separates decelerating ($q > 0$) from accelerating ($q < 0$) periods in the evolution of the Universe. Here for $0<m < 1$, the model decelerate and accelerate if $m > 1$ at any stage. Determination of the deceleration parameter from the count magnitude relation for galaxies is a difficult task due to evolutionary effects. The present value q of the deceleration parameter obtained from observations [51] are $-1.27 \le q \le 2$. Studies of galaxy counts from redshift surveys provide a value of $q = 0.1$, with an upper limit of $q \le 0.75$ [51]. Recent observations by Perlmutter et.al [52,53] and Riess et al [54] show that the deceleration parameter of the Universe is in the range $-1 \le q \le 0$ and the present day Universe is undergoing accelerated expansion. It may be noted that though the current observations of SNe Ia and the CMBR favour accelerating models ($q < 0$), they do not altogether rule out the existence of the decelerating phase in the early history of our Universe which are also consistent with these observations [55].

5. Conclusions

The field equations of a scalar-tensor theory of gravitation proposed by Sen-Dunn are solved in the presence of cosmic strings for spatially homogeneous and anisotropic Bianchi type-III model. In order to solve the field equations we have used a more general equation of state for the proper energy density and string tension density. It is observed that the Bianchi type-III geometric strings do not exist with the scalar field. Our universe starts evolving with zero volume at t=0 and expands with cosmic time(t) .Since $\dfrac{\sigma}{\theta}$ is Constant in both cases ,the models do not approach isotropy at any time. The models (26) and (31) represent realistic models. We observe that the solutions obtained are quite new and certainly exhibit some interesting facts in a scalar-tensor theory of gravitation proposed by Sen-Dunn.

References

1.D. K Sen, K .A Dunn, J .Math.Phys.**13**, 557(1971)
2. T.W.B. Kibble, J. Phys. A: Math. Gen. 9, 1387 (1976).
3. Ya.B. Zel'dovich, I. Yu. Kobzarev, L. B. Okun, Zh. Eksp. Teor. Fiz. 67, 3 (1975).
4. Ya.B. Zel'dovich, I. Yu. Kobzarev, L. B. Okun, Sov. Phys.-JETP 40, 1 (1975).
5. T.W.B. Kibble, Phys. Rep., 67, 183 (1980).
6. A.E. Everett, Phys. Rev., 24, 858 (1981).
7. A. Vilenkin, Phys. Rev., D 24, 2082 (1981).
8. Ya.B. Zel'dovich, Mon. Not. R. Astron. Soc., 192, 663 (1980).
9. P.S. Letelier, Phys. Rev., D 20, 1294, (1979).
10. P.S. Letelier, Phys. Rev., D 28, 2414 (1983).
11. J. Stachel, Phys. Rev., D 21, 2171 (1980).
12. R. Bali, S. Dave, Pramana-J. Phys., 56, 513 (2001).
13. R. Bali, R.D. Upadhaya, Astrophys. Space Sci., 283, 97 (2003).
14. R. Bali, S. Dave, Astrophys. Space Sci., 288, 503 (2003).
15. R. Bali, D.K. Singh, Astrophys. Space Sci., 300, 387 (2005).
16. R. Bali, Anjali, Astrophys. Space Sci., 302, 201 (2006).
17. R. Bali, U.K. Pareek and A. Pradhan, Chin. Phys. Lett., 24, 2455 (2007).
18. R. Bali, A. Pradhan, Chin. Phys. Lett., 24. 585 (2007).
19. M.K. Yadav, A. Pradhan, S. K. Singh, Astrophys. Space Sci., 311, 423 (2007).
20. X.X. Wang, Astrophys. Space Sci., 293, 933 (2004).
21. X.X. Wang, Chin. Phys. Lett., 21, 1205 (2004).
22. X.X. Wang, Chin. Phys. Lett., 22, 29 (2005).
23. X.X. Wang, Chin. Phys. Lett., 23, 1702 (2006).
24. A.K. Yadav, V. K. Yadav and L. Yadav, Int. J. Theor. Phys., 48, 568 (2009).
25. H. Baysal, I. Yavuz, I. Tarhan, U. Camci, and ˙I. Yilmaz, Turk. J. Phys., 25, 283 (2001).
26. ˙I. Yavuz, ˙I. Yilmaz, and H. Baysal, Int. J. Mod. Phys. D, 8, 1365 (2006).
27. ˙I. Yilmaz, Gen. Relativ. Gravit., 38, 1397 (2006).
28. G.P. Singh, T. Singh, Gen. Relativ. Gravit., 31, 371 (1999).
29. G.P. Singh, IL Nuovo Cimento B ,110, 1463 (1995).
30. G.P. Singh, Pramana-J. Phys., 45, 189 (1995).
31. D.R.K. Reddy, Astrophys. Space Sci., 281, 365 (2003).
32. D.R.K. Reddy, Astrophys. Space Sci., 300, 381 (2005).
33. D.R.K. Reddy, Subba Rao, M. V. Rao, Astrophys. Space Sci., 305, 183 (2005).
34. D.R.K. Reddy, R.L. Naidu, Astrophys. Space Sci., 307, 395 (2007).
35. D.R.K. Reddy, R.L. Naidu, and V.U.M. Rao, Int. J. Theor. Phys., 46, 1443 (2007).
36. V.U.M.Rao, M.Vijaya Santhi, T.Vinutha, Astrophys. Space Sci., 314,73(2008).
37. V.U.M. Rao, T. Vinutha, M. Vijaya Santhi, Astrophys. Space Sci., 314, 213 (2008).
38. V.U.M. Rao, T. Vinutha, K. V. S. Sireesha, Astrophys. Space Sci., 323, 401(2009).
39. V.U.M. Rao and T. Vinutha, Astrophys. Space Sci., 325, 59 (2010).
40. A. Pradhan, Fizika B, 16, 205 (2007).
41. A. Pradhan, Commun. Theor. Phys., 51, 367 (2009).
42. A. Pradhan, K. Jotania, and A. Singh, Braz. J. Phys., 38, 167 (2008).
43. A. Pradhan, P. Mathur, Astrophys. Space Sci., 318, 255 (2008).
44. A. Pradhan, R. Singh, J.P. Shahi, Elect. J. Theor. Phys., 7, 197 (2010).
45. S.K. Tripathi, S.K. Nayak, S.K. Sahu, T. R. Routray, Astrophys. Space Sci., 323, 281 (2009).
46. S.K. Tripathi, D. Behera, T.R. Routray, Astrophys. Space Sci., 325, 93 (2010).
47.Venkateswarlu, R., P Kumar, K.: Int. J.Theor. Phys.**49**,1894(2010).

48.Venkateswarlu,R.,Satish.J., P Kumar, K.: Ele. J.Theor.Phys.,**8**,25 (2011)
49. R .Tikekar, L.K .Patel,.: 1992, *Gen. Relat. Gravit.* **24**, 394.
50. K.D.,Krori, Choudhury, T., Mahanta, C.R.:Gen. Relativ. Gravit.**22**,123(1990)
51.R Schuecker et al.:Astrophys. J. **496**, 635 (1998)
52.S Perlmutter et al. :Nature (London) **319**, 51 (1998)
53.S Perlmutter et al.: Astrophys. J. **517**, 565 (1999)
54.A G Riess et al.:Astrophys. J. **116**, 1009 (1998)
55.R G Vishwakarma.: Mon. Not. R. Soc. **345**, 545 (2003)