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Cosmological Models in a Modified Theory of Gravity

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Abstract

A new class of spatially homogeneous LRS Bianchi type - I cosmological models filled with perfect fluid in the framework of *f (R,T)* gravity proposed by Harko et al. (2011) has been obtained with an appropriate choice of a function *f (T)*. In this paper we have considered different cases and presented anisotropic and isotropic cosmological models. The obtained and presented Zeldovich fluid cosmological models are quite different from the model obtained by Adhav (2012). Some important features of the models, thus obtained, have been discussed and it is established that the additional condition, special law of variation of Hubble parameter proposed by Bermann (1983), taken by Adhav (2012) in this theory is superfluous.

Keywords: LRS, Bianchi Type-I, space-time, f (R,T) gravity, perfect fluid, general relativity.

1. Introduction

In recent years, there has been a lot of interest in alternative theories of gravitation. In view of the late time acceleration of the universe and the existence of the dark matter and dark energy, very recently, modified theories of gravity have been developed. Noteworthy amongst them are *f (R)* theory of gravity formulated by Nojiri and Odintsov (2003a) and *f (R,T)* theory of gravity proposed by Harko et al. (2011). Carroll et al. (2004) explained the presence of a late time cosmic acceleration of the universe in *f(R)* gravity. Nojiri and Odintsov (2003b) demonstrated that phantom scalar in many respects looks like strange effective quantum field theory by

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introducing a non-minimal coupling of phantom field with gravity. Kucukakca et al. (2012) have discussed LRS Bianchi type-I universes exhibiting no ether symmetry in the scalar tensor Brans-Dicke theory.

Recently, Harko et al. (2011) developed *f (R,T)* modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar *R* and of the trace *T* of the stress-energy tensor. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor.

The *f (R,T)* gravity model depends on a source term, representing the variation of the matter stress energy tensor with respect to the metric. A general expression for this source term is obtained as a function of the matter Lagrangian L_m so that each choice of L_m would generate a specific set of field equations. Some particular models corresponding to specific choices of the function *f (R,T)* are also presented, they have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of a function *f (T)*. In the present model the covariant divergence of the stress energy tensor is nonzero. Hence the motion of test particles is non-geodesic and an extra acceleration due to the coupling between matter and geometry is always present.

In *f (R,T)*gravity, the field equations are obtained from the Hilbert-Einstein type variation principle. The action principle for this modified theory of gravity is given by

$$S = \frac{1}{16\pi G}\int f(R,T)\sqrt{-g}\,d^4x + \int L_m\sqrt{-g}\,d^4x \qquad (1.1)$$

where *f (R,T)* is an arbitrary function of the Ricci scalar *R* and of the trace *T* of the stress energy tensor of matter and L_m is the matter Lagrangian.

The stress energy tensor of matter is

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g})}{\partial g^{ij}} L_m, \Theta_{ij} = -2T_{ij} - pg_{ij}, \tag{1.2}$$

Using gravitational units (by taking *G & c* as unity) the corresponding field equations of *f (R,T)* gravity are obtained by varying the action principle (1.1) with respect to g_{ij} as

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_R(R,T) = \\ 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \end{aligned} \tag{1.3}$$

where $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ & $\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$

Here ∇_i is the covariant derivative and T_{ij} is usual matter energy-momentum tensor derived from the Lagrangian L_m. It can be observed that when *f (R,T) = f (R)*, then (1.3) reduce to field equations of *f (R)* gravity.

It is mentioned here that these field equations depend on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T). \end{cases}$$

In this paper we are focused to the first class, i.e.

$$f(R,T) = R + 2f(T). \tag{1.4}$$

where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. The simplicity of the field equations and relative ease of solutions made Bianchi space times useful in constructing models of spatially homogeneous and anisotropic cosmologies. The anomalies found in the cosmic microwave background (CMB) and large scale structure observations stimulated a growing interest in anisotropic cosmological models of the universe. Kumar and Singh (2008) investigated perfect fluid solutions using Bianchi type-I space- time in scalar-tensor theory. Paul et al. (2009) obtained FRW models in *f (R)* gravity while Sharif and Shamir (2009, 2010) have studied the solutions of Bianchi type-I and V space-times in the framework of *f (R)* gravity. Shamir (2010) studied the exact vacuum solutions of Bianchi type I, III and Kantowski-Sachs space-times in the metric version of *f (R)* gravity. Ahmad Sheykhi (2012) has discussed Magnetic strings in *f (R)* gravity and Ihsan Yılmaz et al. (2012) have discussed Quark and strange quark matter in *f (R)* gravity for Bianchi type *I* and *V* space-times. Rao et al. (2011) have discussed anisotropic Bianchi type-I universe with cosmic strings and bulk viscosity in a scalar-tensor theory of gravitation. Rao et al. (2012a, b, c) have obtained various Bianchi type-I cosmological models in several theories of gravitation.

Reddy et al. (2012a, b) have obtained Kaluza klein cosmological model in the presence of perfect fluid source and Bianchi type-III cosmological model in *f (R,T)* gravity using the assumption of law of variation for the Hubble parameter proposed by Bermann (1983). Chaubey and Shukla (2013) have obtained a new class of Bianchi cosmological models in $f(R,T)$ gravity. Reddy and Santhi Kumar (2013) have presented some anisotropic cosmological models in this theory. Recently Rao and Neelima (2013a, b) have discussed perfect fluid Einstein-Rosen and Bianchi type-VIo universes in $f(R,T)$ gravity.

We noticed that the field equations of Adhav (2012) need some corrections. Hence, in this paper, we have revisited and corrected the field equations and also presented a new class of spatially homogeneous anisotropic LRS Bianchi type - I as well as isotropic cosmological models filled with perfect fluid in the framework of $f(R,T)$ gravity where $f(R,T) = R + 2f(T)$. This model is very important in the discussion of large scale structure, to identify early stages and finally to study the evolution of the universe.

2. Metric and Energy Momentum Tensor

The LRS Bianchi type-I line element can be taken as

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2) \tag{2.1}$$

where A and B are functions of the cosmic time t only .

The field equations in $f(R,T)$ gravity for the function

$$f(R,T) = R + 2f(T)$$

when the matter source is perfect fluid are given by

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} + f(T)g_{ij} \tag{2.2}$$

where the prime indicates derivative with respect to the argument 'T'.

The matter tensor for perfect fluid is

$$\Theta^i_j \equiv -2T^i_j - \delta^i_j p = (\rho, -p, -p, -p),$$
$$where\, T^i_j = (\rho + p)u_i u^j - \delta^i_j p \tag{2.3}$$

Then the field equations (2.2) can be written as

$$R^i_j - \frac{1}{2}\delta^i_j R = 8\pi T^i_j + 2f'(T)T^i_j + [2pf'(T) + f(T)]\delta^i_j \tag{2.4}$$

Now we choose the function *f (T)* as the trace of the stress energy tensor of the matter so that $f(T)=\mu T$, where μ is an arbitrary constant.

3. Solutions of Field equations

Now with the help of (2.3), the field equations (2.4) for the metric (2.1) can be written as

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}=(8\pi+3\mu)p-\rho\mu \tag{3.1}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=(8\pi+3\mu)p-\rho\mu \tag{3.2}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}=-(8\pi+3\mu)\rho+p\mu \tag{3.3}$$

Here the over head dot denotes differentiation with respect to't'.

The field equations (3.1) to (3.3) are only three independent equations with four unknowns A, B, ρ and p. So, in order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials A and B, i.e.,

$$A=B^n \tag{3.4}$$

where n is an arbitrary constant.

From equations (3.1), (3.2) & (3.4), we get

$$(n-1)\frac{\dot{B}}{B}\left[\frac{\ddot{B}}{\dot{B}}+(n+1)\frac{\dot{B}}{B}\right]=0 \tag{3.5}$$

From the equation (3.5), we will consider the following two important cases

$$(1)\frac{\dot{B}}{B}\neq 0,\ (n-1)\neq 0\ and\ \left[\frac{\ddot{B}}{\dot{B}}+(n+1)\frac{\dot{B}}{B}\right]=0$$

$$(2)\frac{\dot{B}}{B}\neq 0,\ (n-1)=0\ and\ \left[\frac{\ddot{B}}{\dot{B}}+(n+1)\frac{\dot{B}}{B}\right]=0$$

ANISOTROPIC COSMOLOGICAL MODEL IN f (R,T) GRAVITY:

We will get anisotropic cosmological model in *f (R,T)* gravity,

$$if\ \frac{\dot{B}}{B}\neq 0,\ (n-1)\neq 0\ \&\ \left[\frac{\ddot{B}}{\dot{B}}+(n+1)\frac{\dot{B}}{B}\right]=0. \tag{3.6}$$

From (3.6), we get

$$B=\left[(n+2)(C_1t+C_2)\right]^{1/(n+2)}\ ,\ \ n\neq -2 \tag{3.7}$$

From (3.4) & (3.7), we get

$$A=\left[(n+2)(C_1t+C_2)\right]^{n/(n+2)} \tag{3.8}$$

where $C_1\neq 0$ & C_2 are integrating constants.

From equations (3.1), (3.3), (3.7) & (3.8), we get the energy density and the pressure as

$$\rho=p=\frac{-1}{(8\pi+2\mu)}\left[\frac{(2n+1){C_1}^2}{(n+2)(C_1t+C_2)^2}\right] \tag{3.9}$$

If $-2<n<\frac{-1}{2}$, then $p>0$. It shows that perfect fluid may behave like Zeldovich fluid distribution and the energy conditions $\rho>0$, $p>0$ are satisfied.

If $n > -2$, then $p < 0$. It shows that perfect fluid may behave like Phantom type dark energy. So we can conclude that perfect fluid may be source of early dark energy due to negative pressure, since energy conservations are violated.

Stiff fluid creates more interest in cosmology because the speed of light is equal to speed of sound and its governing equations have the same characteristics as those of gravitational field (Zeldovich 1970). The casual limit for ideal gas has also the form $\rho = p$ (Zeldovich and Novikov,1971). Also this state describes several important cases, e.g., radiation, relativistic degenerate Fermi gas and probably very dense baryon matter (Zeldovich and Novikov, 1971; Walecka, 1974). Furthermore, if the fluid satisfies the equation of state $\rho = p$ and if in addition its motion is irrotational, then such a source has the same stress energy tensor as that of a massless scalar field. Cosmological models with stiff fluid equation of state have been studied by many authors.

The great importance of cosmological models where the matter content is represented by a stiff matter perfect fluid was recognized since its introduction by Zeldovich. In order to understand better the importance of this perfect fluid for cosmology, one has to compute its energy density. Several authors observed that there may have existed a phase earlier than that of radiation, in our Universe, which was dominated by stiff matter. Due to that importance, many physicists have started to consider the implications of the presence of a stiff matter perfect fluid in isotropic and anisotropic cosmological models. It may also play an important role in the spectrum of relic gravity waves created during inflation. Since there may have existed a phase earlier than that of radiation which was dominated by stiff matter some physicists considered quantum cosmological models with this kind of matter. Hence the stiff equation of state of the model doesn't restrict its astrophysical/cosmological applicability.

The overall density parameter Ω is given by

$$\Omega = \frac{-3(2n+1)}{(n+2)^2(8\pi + 2\mu)} \tag{3.10}$$

The metric (2.1) can now be written as

$$ds^2 = dt^2 - \left[(n+2)(C_1 t + C_2)\right]^{2n/(n+2)} dx^2 - \left[(n+2)(C_1 t + C_2)\right]^{2/(n+2)} (dy^2 + dz^2) \qquad (3.11)$$

Thus the metric (3.11) together with (3.9) constitutes a homogeneous and anisotropic LRS Bianchi type-I Zeldovich fluid cosmological model in *f (R,T)* gravity.

ISOTROPIC COSMOLOGICAL MODEL IN f (R,T) GRAVITY:

If $\frac{\dot{B}}{B} \neq 0, (n-1) = 0 \, \& \left[\frac{\ddot{B}}{\dot{B}} + (n+1)\frac{\dot{B}}{B}\right] = 0$, we will get isotropic cosmological model in

f (R,T) gravity,

If $(n-1) = 0$, we get $\left[\frac{\ddot{B}}{\dot{B}} + 2\frac{\dot{B}}{B}\right] = 0$ (3.12)

From (3.12), we get

$$A = B = [3(C_1 t + C_2)]^{\frac{1}{3}} \qquad (3.13)$$

where $C_1 \neq 0$ & C_2 are integrating constants.

From (3.1), (3.3) & (3.13), we get the energy density and the pressure as

$$\rho = p = \frac{-1}{(8\pi + 2\mu)}\left[\frac{C_1^{\,2}}{3(C_1 t + C_2)^2}\right] \qquad (3.14)$$

This shows that perfect fluid may behave like Phantom type dark energy. So we can conclude that perfect fluid may be source of early dark energy due to negative pressure since energy conservations are violated. Also since μ is an arbitrary constant, it is always possible to assign suitable value to μ to make pressure positive.

The overall density parameter Ω is given by

$$\Omega = \frac{-1}{(8\pi + 2\mu)} \tag{3.15}$$

The metric (2.1) can now be written as

$$ds^2 = dt^2 - [3(C_1 t + C_2)]^{2/3}(dx^2 + dy^2 + dz^2) \tag{3.16}$$

Thus the metric (3.16) together with (3.14) constitutes a homogeneous and isotropic Zeldovich fluid cosmological model in *f (R,T)* gravity.

PERFECT FLUID COSMOLOGICAL MODEL IN GENERAL RELATIVITY:

Interestingly we can observe that, if $\mu = 0$, the metric (3.11) together with (3.9) represents homogeneous and anisotropic LRS Bianchi type-I Zeldovich fluid cosmological model in general relativity. Also the metric (3.16) together with (3.14) constitutes a homogeneous and isotropic Zeldovich fluid cosmological model in general relativity.

4. Some important features of the models

The spatial volume for the anisotropic model (3.11) is

$$V = [(n+2)(C_1 t + C_2)] \tag{4.1}$$

The spatial volume for the isotropic model (3.16) is

$$V = [3(C_1 t + C_2)] \tag{4.2}$$

The expression for expansion scalar θ and the shear scalar σ for the models (3.11) & (3.16) are given by

$$\theta = u^i{}_{,i} = \frac{C_1}{(C_1 t + C_2)} \tag{4.3}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7}{18}\frac{C_1^2}{(C_1 t+C_2)^2} \tag{4.4}$$

The deceleration parameter q for the models (3.11) & (3.16) is given by

$$q = (-3\theta^{-2})(\theta_{,i}u^i + \frac{1}{3}\theta^2) = 2\,. \tag{4.5}$$

If $q<0$, the model accelerates and when $q>0$, the model decelerates in the standard way. Here the models decelerate in the standard way which is not in accordance with the present day scenario of accelerating universe. It may be noted that Bianchi models represent cosmos in its early stage of evolution. However, in spite of the fact that the universe, in this case, decelerates in the standard way it will accelerate in finite time due to cosmic re collapse where the universe in turns inflates "decelerates and then accelerates" (Nojiri and Odintsov 2003c).

The Hubble's parameter H for the models (3.11) & (3.16) is given by

$$H = \frac{C_1}{3(C_1 t+C_2)} \tag{4.6}$$

The tensor of rotation

$w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

The average anisotropy parameter for the anisotropic model (3.11) is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2(n-1)^2}{(n+2)^2}\text{ , where }\Delta H_i = H_i - H\ \ (i=1,2,3) \tag{4.7}$$

The average anisotropy parameter for the isotropic model (3.16) is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 0\text{ , where }\Delta H_i = H_i - H\ \ (i=1,2,3) \tag{4.8}$$

5. Discussion and Conclusions

In this paper, we have presented spatially homogeneous anisotropic LRS Bianchi type - I as well as isotropic cosmological models filled with perfect fluid in the framework of *f (R,T)* gravity proposed by Harko et al. (2011).

The following are the observations and conclusions.

1) The anisotropic model (3.11) has point type singularity at $t = \frac{-C_2}{C_1}$ for $n > -2$ and cigar type singularity for $-2 < n < 0$ while the isotropic cosmological model (3.16) is free from singularities.
2) For both models, the spatial volume vanishes at $t = \frac{-C_2}{C_1}$ and increases with time. This shows that at the initial epoch, the universe starts with zero volume and expands uniformly.
3) For both models, the expansion scalar θ, shear scalar σ and the Hubble parameter H decrease with the increase of time.
4) From (3.9) & (3.14), we can see that matter pressure and density will vanish with the increase of cosmic time. Hence they represent vacuum cosmological models in general relativity for large values of t.
5) These anisotropic and isotropic models not only represent Zeldovich fluid distribution but also the cosmos in its early stage of evolution.
6) For anisotropic model the perfect fluid may behave like Phantom type dark energy for $n > -2$.
7) The deceleration parameter appears with a positive sign implies that the models decelerate in the standard way which is not in accordance with the present day scenario of accelerating universe. It may be noted that Bianchi models represent cosmos in its early stage of evolution. However, in spite of the fact that the universe, in this case,

decelerates in the standard way it will accelerate in finite time due to cosmic re collapse where the universe in turns inflates "decelerates and then accelerates"

8) The models are expanding and non-rotating.
9) From (4.7), one can observe that the average anisotropy parameter $A_m \neq 0$ for $n \neq 1$, which indicates that the model (3.11) is anisotropic. The experiments show that there is a certain amount of anisotropy in the universe and hence anisotropic space-times are important. Also f(R,T) gravity is proposed to explain early inflation and late time acceleration.
10) From (4.8), we can see that the average anisotropy parameter $A_m = 0$, for $n = 1$, which indicates that the model (3.16) is always isotropic.
11) The involvement of new function $f(R,T)$ doesn't affect the geometry of the space-time but slightly changes the matter distribution.

Finally we can conclude that these cosmological models are new and different in all aspects from the model presented by Adhav (2012) and also established that the additional condition, special law of variation of Hubble parameter proposed by Bermann (1983), taken by Adhav (2012) in this theory is superfluous.

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