Article

Physics as Infinite-dimensional Geometry III: Configuration Space Spinor Structure

Matti Pitkänen[1]

Abstract

There are three separate approaches to the challenge of constructing WCW Kähler geometry and spinor structure. The first approach relies on a direct guess of Kähler function. Second approach relies on the construction of Kähler form and metric utilizing the huge symmetries of the geometry needed to guarantee the mathematical existence of Riemann connection. The third approach discussed in this article relies on the construction of spinor structure based on the hypothesis that complexified WCW gamma matrices are representable as linear combinations of fermionic oscillator operator for the second quantized free spinor fields at space-time surface and on the geometrization of super-conformal symmetries in terms of spinor structure. This implies a geometrization of fermionic statistics.

The basic philosophy is that at fundamental level the construction of WCW geometry reduces to the second quantization of the induced spinor fields using Dirac action. This assumption is parallel with the bosonic emergence stating that all gauge bosons are pairs of fermion and anti-fermion at opposite throats of wormhole contact. Vacuum function is identified as Dirac determinant and the conjecture is that it reduces to the exponent of Kähler function. In order to achieve internal consistency induced gamma matrices appearing in Dirac operator must be replaced by the modified gamma matrices defined uniquely by Kähler action and one must also assume that extremals of Kähler action are in question so that the classical space-time dynamics reduces to a consistency condition. This implies also super-symmetries and the fermionic oscillator algebra at partonic 2-surfaces has interpretation as $\mathcal{N}=\infty$ generalization of space-time super-symmetry algebra different however from standard SUSY algebra in that Majorana spinors are not needed. This algebra serves as a building brick of various super-conformal algebras involved.

The requirement that there exist deformations giving rise to conserved Noether charges requires that the preferred extremals are critical in the sense that the second variation of the Kähler action vanishes for these deformations. Thus Bohr orbit property could correspond to criticality or at least involve it.

Quantum classical correspondence demands that quantum numbers are coded to the properties of the preferred extremals given by the Dirac determinant and this requires a linear coupling to the conserved quantum charges in Cartan algebra. Effective 2-dimensionality allows a measurement interaction term only in 3-D Chern-Simons Dirac action assignable to the wormhole throats and the ends of the space-time surfaces at the boundaries of CD. This allows also to have physical propagators reducing to Dirac propagator not possible without the measurement interaction term. An essential point is that the measurement interaction corresponds formally to a gauge transformation for the induced Kähler gauge potential. If one accepts the weak form of electric-magnetic duality Kähler function reduces to a generalized Chern-Simons term and the effect of measurement interaction term to Kähler function reduces effectively to the same gauge transformation.

The basic vision is that WCW gamma matrices are expressible as super-symplectic charges at the boundaries of CD. The basic building brick of WCW is the product of infinite-D symmetric spaces assignable to the ends of the propagator line of the generalized Feynman diagram. WCW Kähler metric has in this case "kinetic" parts associated with the ends and "interaction" part between the ends. General expressions for the super-counterparts of WCW flux Hamiltonians and for the matrix elements of WCW metric in terms of their anti-commutators are proposed on basis of this picture.

Keywords: Infinite-dimensional geometry,Kähler metric, spinor structure, second quantization, symmetric space, super-conformal invariance, electric-magnetic duality.

1 Introduction

Quantum TGD should be reducible to the classical spinor geometry of the configuration space. In particular, physical states should correspond to the modes of the configuration space spinor fields. The immediate consequence is that configuration space spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the configuration space spinor structure there are some important clues.

1.1 Geometrization of fermionic statistics in terms of configuration space spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the configuration space spinor structure in the sense that the anti-commutation relations for configuration space gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

[1]Correspondence: Matti Pitkänen `http://tgd.wippiespace-com/public_html`. Address: Köydenpunojankatu 2 D 11 10940, Hanko, Finland. Email: matpitka@luukku.com.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. [48] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, configuration space spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of configuration space spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the 'orbital' degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the configuration space geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the configuration space spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the configuration space geometry. This is indeed true if the complexified configuration space gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the configuration space to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma^{\dagger}_{\overline{A}}, \gamma_B\} = iJ_{\overline{A}B} \ ,$$

   where J_{AB} denotes the matrix elements of the Kähler form of the configuration space. The presence of the Hermitian conjugation is necessary because configuration space gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly.

1.2 Modified Dirac equation for induced classical spinor fields

The basic vision is that WCW geometry reduces to the second quantization of induced spinor fields. This means that WCW gamma matrices are linear combinations of fermionic oscillator operators and the vacuum functional of the theory is identifiable as Dirac determinant. An unproven conjecture is that this determinant equals to the exponent of Kähler action for its preferred extremal.

The motivation for the modified Dirac action came from the observation that the counterpart of the ordinary Dirac equation is internally consistent only if the space-time surfaces are minimal surfaces. One can however assign to any general coordinate invariant action principle for space-time surfaces a unique modified Dirac action, which is internally consistent and super-symmetric. Space-time geometry must carry information about conserved quantum charges assignable to partonic 2-surfaces and it took considerable to to realize that this is achieved via a measurement interaction terms linear in conserved charges. It took still some time to conclude that Kähler action with a measurement interaction term is is required in order the code information about quantum numbers to the space-time geometry.

1.2.1 Preferred extremals as critical extremals

The study of the modified Dirac equation leads to a detailed view about criticality. Quantum criticality [50] fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \to K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs). The discovery that the hierarchy of Planck constants [15] realized in terms of singular covering spaces of $CD \times CP_2$ can be understood in terms of the extremely non-linear dynamics of Kähler action implying 1-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates led to a further very concrete understanding of the criticality at space-time level and its relationship to zero energy ontology [6, A1].

1.2.2 Inclusion of the measurement interaction term

One can pose several conditions on the measurement interaction term of Dirac action. The term should be linear in the measured charges which must commute and act on their eigenstates. The effective 2-dimensionality requires that the measurement interaction term is 3-dimensional and this allows only the Dirac action associated with the generalized Chern-Simons action. Measurement interaction term must define fermionic 3-D propagators along wormhole throats. This is necessary because 4-D Dirac equation is satisfied always and cannot define the fermionic propagator. For Chern-Simons term off mass shell propagation is possible since 3-D Chern-Simons Dirac equation need not to be satisfied.

1. The basic vision is that the addition of the measurement interaction term induces a $U(1)$ gauge transformation $K \rightarrow K + f + \overline{f}$ of the Kähler function of WCW. Here f is holomorphic function of WCW ("world of classical worlds") complex coordinates and arbitrary function of zero mode coordinates. Although WCW Kähler metric is not affected, Kähler function changes and this means that preferred extremal changes also and therefore codes information about the values of the measured observables.

2. The measurement interaction is assumed to be linear in the measured charges which must commute and therefore belong to the Cartan algebra. Cartan algebra plays a key role not only at quantum level but also at the level of space-time geometry since quantum critical conserved currents vanish for the Cartan algebra of isometries and the measurement interaction terms giving rise to conserved currents are possible only for Cartan algebras. Furthermore, modified Dirac equation makes sense only for the eigen states of Cartan algebra generators. The hierarchy of Planck constants realized in terms of the book like structure of the generalized imbedding space assigns to each CD (causal diamond) preferred Cartan algebra: in case of Poincare algebra there are two of them corresponding to linear and cylindrical M^4 coordinates. The origin of the hierarchy of Planck constants can be now understood from the basic quantum TGD and it relates directly with criticality [6, A1].

3. The values of Cartan charges are fed to 3-D Chern-Simons Dirac action via the measurement interaction term. Measurement interaction term corresponds to a term resulting from the $U(1)$ transformation ϕ of the CP_2 Kähler potential. Since this term is assigned only with the Chern-Simons Dirac action, it does not reduce to a mere gauge transformation with a trivial effect. This picture is consistent with the reduction of TGD to almost topological QFT [43] implied by electric-magnetic duality and the vanishing of the Coulomb interaction term in Kähler action [6, A1].

4. One can require that the propagating states are generalized eigenstates of the modified Dirac equation. The generalized eigenvalues are of form $D_{C-S}\Psi = \lambda^k \gamma_k \Psi$, where only the covariantly constant M^4 gamma matrices can appear. λ^k is completely analogous to four-momentum and the propagator is formally massless propagator so that ordinary twistor formalism should apply. The identification with actual four-momentum does not however make sense. This suggests that also massless gauge theories could make sense if the four-momenta do not correspond to the actual four-momenta.

1.2.3 CP breaking and matter-antimatter asymmetry

Chern-Simons Dirac action used to defined measurement interaction term breaks CP and T symmetries and therefore provides a first principle description for the breaking of these symmetries. CP breaking could also reflect to the discretization of the relative coordinate between the tips of the CD. One could label the positions of the lower tip of CD by M^4 and the relative positions of the upper tip by a discrete space consisting of discrete variants of hyperboloids with proper time coordinate coming as powers of 2. This CP and T breaking would be apparent and due to the fixing the rest system to the observer assigned with the "lower" boundary of CD serving as a role of medium forcing the CP breaking at the level sub-CDs. One can of course argue that the CP breaking induced by Chern-Simons action gives the special role for the "lower" boundary of CD. In fact, the breaking of Lorentz invariance at the level of CD (but not at the level of WCW) could even make possible a spontaneous breaking of CPT symmetry.

1.2.4 Weak form of electric-magnetic duality and Kähler function as Dirac determinant

The construction of WCW spinor structure in terms of induced spinor fields has been continual shifting between various options. Should one have 3-D or 4-D modified Dirac action at the fundamental level? Does the idea about TGD as almost topological QFT make sense or not? Is the identification of Kähler function as Dirac determinant really needed? Does Dirac determinant make even sense mathematically?

The weak form of electric-magnetic duality provides the a clearcut answer to most open questions of this kind. The reduction to almost topological QFT based on the weak form of electric-magnetic duality gives the explicit form of the WCW Kähler function Chern-Simons action, and one understand how the measurement interaction term affects it. This is of utmost importance for the construction of quantum TGD since WCW Kähler metric becomes directly calculable. The progress in some aspects however forces always to challenge the basic assumptions.

1. The basic idea has been that a correlation between 4-D geometry of the space-time sheet and quantum numbers would be achieved by the identification of the exponent of Kähler function as a Dirac determinant. The effect of the measurement interaction to the Kähler function is however induced by the same gauge transformation of the induced Kähler gauge potential appearing in Chern-Simons action as appears in Chern-Simons Dirac action. Therefore Dirac determinant is not needed to calculate the Kähler function and one can ask whether the identification of Kähler function as a Dirac determinant has any practical value. It has.

It turns out that the weak form of electric-magnetic duality leads to a beautiful generalization of the earlier solution asatz of the field equations (Kähler current is proportional to the instanton current which requires that both currents are Beltrami fields [52]) guaranteeing the reduction of the Kähler action to Chern-Simons term.

Another consequence is an explicit solution of Kähler Dirac equation and generalized eigenvalue equation for the Chern-Simons Dirac equation allowing to write an explicit formula for the Dirac determinant in terms of the geometric data about the orbit of the partonic 2-surface coded by the eigenvalues [9, A4]. Concerning the interpretation of Kähler function as a Dirac determinant the outcome is very encouraging since in the case of CP_2 vacuum extremals the resulting formula for Kähler function is consistent with the earlier conjecture leading to an expression of gravitational constant in terms of Kähler action for CP_2 type vacuum extremal and p-adic length scale. The detailed calculations are carried out in [9, A4].

2. One can still worry whether the measurement interaction is really needed. The propagator reduces formally to a massless Dirac propagator in which the analog of four-momentum is expressible in terms of quantum numbers propagating along the line of the generalized Feynman diagram. This would be a fantastic news for a believer in the twistor program since also massive case and virtual momenta could be treated. One could however argue that the road involving minimum amount of calculations is the safest one: why not to identify the four-momentum with the physical four-momentum and try to resolve the resulting problems? It turns out that this identification fails for several reasons [9, A4].

1.3 Identification of configuration space gamma matrices as super Hamiltonians

The basic super-algebra corresponds to the fermionic oscillator operators and can be regarded as a generalization $\mathcal{N}$ super algebras by replacing $\mathcal{N}$ with the number of solutions of the modified Dirac equation which can be infinite. This leads to QFT SUSY limit of TGD different in many respects crucially from standard SUSYs [13].

Configuration space gamma matrices identified as super generators of super-symplectic and are expressible in terms of these oscillator operators. Super-symplectic and super charges are assumed to be expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory.

2 Configuration space spinor structure: general definition

The basic problem in constructing configuration space spinor structure is clearly the construction of the explicit representation for the gamma matrices of the configuration space. One should be able to identify the space, where these gamma matrices act as well as the counterparts of the "free" gamma matrices, in terms of which the gamma matrices would be representable using generalized vielbein coefficients.

2.1 Defining relations for gamma matrices

The ordinary definition of the gamma matrix algebra is in terms of the anti-commutators

$$\{\gamma_A, \gamma_B\} = 2g_{AB} \ .$$

This definition served implicitly also as a basic definition of the gamma matrix algebra in TGD context until the difficulties related to the understanding of the configuration space d'Alembertian defined in terms of the square of the Dirac operator forced to reconsider the definition. If configuration space allows Kähler structure, the most general definition allows to replace the metric any covariantly constant Hermitian form. In particular, g_{AB} can be replaced with

$$\{\Gamma^{\dagger}_{\overline{A}}, \Gamma_B\} = iJ_{\overline{A}B} \ , \tag{2.1}$$

where J_{AB} denotes the matrix element of the Kähler form of the configuration space. The reason is that gamma matrices carry fermion number and are non-hermitian in all coordinate systems. This definition is numerically equivalent with the standard one in the complex coordinates but in arbitrary coordinates situation is different since in general coordinates iJ_{kl} is a non-trivial positive square root of g_{kl}. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly.

2.2 General vielbein representations

There are two ideas, which make the solution of the problem obvious.

1. Since the classical time development in bosonic degrees of freedom (induced gauge fields) is coded into the geometry of the configuration space it seems natural to expect that same applies in the case of the spinor structure. The time development of the induced spinor fields dictated by the TGD counterpart of the massless Dirac action should be coded into the definition of the configuration space spinor structure. This leads to the challenge of defining what classical spinor field means.

2. Since classical scalar field in the configuration space corresponds to second quantized boson fields of the imbedding space same correspondence should apply in the case of the fermions, too. The spinor fields of configuration space should correspond to second quantized fermion field of the imbedding space and the space of the configuration space spinors should be more or less identical with the Fock space of the second quantized fermion field of imbedding space or $X^4(X^3)$. Since classical spinor fields at space-time surface are obtained by restricting the spinor structure to the space-time surface, one might consider the possibility that life is really simple: the second quantized spinor field corresponds to the free spinor field of the imbedding space satisfying the counterpart of the massless Dirac equation and more or less standard anti-commutation relations. Unfortunately life is not *so* simple as the construction of configuration space spinor structure demonstrates: second quantization must be performed for induced spinor fields.

It is relatively simple to fill in the details once these basic ideas are accepted.

1. The only natural candidate for the second quantized spinor field is just the on X^4. Since this field is free field, one can indeed perform second quantization and construct fermionic oscillator operator algebra with unique anti-commutation relations. The space of the configuration space spinors can be identified as the associated with these oscillator operators. This space depends on 3-surface and strictly speaking one should speak of the Fock bundle having configuration space as its base space.

2. The gamma matrices of the configuration space (or rather fermionic Kac Moody generators) are representable as super positions of the fermionic oscillator algebra generators:

$$\begin{aligned} \Gamma_A^+ &= E_A^n a_n^\dagger \\ \Gamma_A^- &= \bar{E}_A^n a_n \\ iJ_{A\bar{B}} &= \sum_n E_A^n \bar{E}_B^n \end{aligned} \tag{2.2}$$

   where E_A^n are the vielbein coefficients. Induced spinor fields can possess zero modes and there is no oscillator operators associated with these modes. Since oscillator operators are spin 1/2 objects, configuration space gamma matrices are analogous to spin 3/2 spinor fields (in a very general sense). Therefore the generalized vielbein and configuration space metric is analogous to the pair of spin 3/2 and spin 2 fields encountered in super gravitation! Notice that the contractions $j^{Ak}\Gamma_k$ of the complexified gamma matrices with the isometry generators are genuine spin 1/2 objects labeled by the quantum numbers labeling isometry generators. In particular, in CP_2 degrees of freedom these fermions are color octets.

3. A further great idea inspired by the symplectic [25] and Kähler [24] structures of the configuration space is that configuration gamma matrices are actually generators of super-symplectic symmetries. This simplifies enormously the construction allows to deduce explicit formulas for the gamma matrices.

2.3 Configuration space Clifford algebra as a hyper-finite factor of type II_1

The naive expectation is that the trace of the unit matrix associated with the Clifford algebra [26] spanned by configuration space sigma matrices is infinite and thus defines an excellent candidate for a source of divergences in perturbation theory. This potential source of infinities remained un-noticed until it became clear that there is a connection with von Neumann algebras [38]. In fact, for a separable Hilbert space defines a standard representation for so called [39]. This guarantees that the trace of the unit matrix equals to unity and there is no danger about divergences coming from infinite traces.

2.3.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [39].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

2.3.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac based on the notion of delta function, plus the emergence of Feynman graphs, the possibility to formulate the notion of delta function rigorously in terms of distributions, and the emergence of path integral approach meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [44, 45] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [40, 41] relate closely to type II_1 factors. In topological quantum computation [49] based on braid groups [42] modular S-matrices they play an especially important role.

2.3.3 Clifford algebra of configuration space as von Neumann algebra

The Clifford algebra of the configuration space provides a school example of a hyper-finite factor of type II_1, which means that fermionic sector does not produce divergence problems. Super-symmetry means that also "orbital" degrees of freedom corresponding to the deformations of 3-surface define similar factor. The general theory of hyper-finite factors of type II_1 is very rich and leads to rather detailed understanding of the general structure of S-matrix in TGD framework. For instance, there is a unitary evolution operator intrinsic to the von Neumann algebra defining in a natural manner single particle time evolution. Also a connection with 3-dimensional topological quantum field theories and knot theory, conformal field theories, braid groups, quantum groups, and quantum counterparts of quaternionic and octonionic division algebras emerges naturally (for classical numbers fields see [27, 28, 29]). These aspects are discussed in detail in [14].

3 Does modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as the exponent of Kähler function of world of classical worlds (WCW) expressible and that Kähler function reduces to Kähler action for a preferred extremal of Kähler action.

3.1 What are the basic equations of quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

1. Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents are conserved. There is infinite hierarchy of these currents and they define fermionic counterparts for zero modes. Space-time sheets can be also regarded as hyper-quaternionic surfaces. What these statements precisely mean has become clear only during this year. A rigorous proof for the equivalence of these two identifications is still lacking.

2. The purely quantal equations are associated with the representations of various super-conformal algebras and with the modified Dirac equation. The requirement that there are deformations of the space-time surface -actually infinite number of them- giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations. The precise form of the modified Dirac equation is not however completely fixed without further input. Quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

3. The notion of weak electric-magnetic duality generalizing the notion of electric-magnetic duality [9, A4] leads to a detailed understanding of how TGD reduces to almost topological quantum field theory [9, A4]. If Kähler current defines Beltrami flow [52] it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to

a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

4. The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD \times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

3.2 Quantum criticality and modified Dirac action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The question leading to a considerable progress in the problem was simple: Under what conditions the modified Dirac action allows to assign conserved fermionic currents with the deformations of the space-time surface? The answer was equally simple: These currents exists only if these deformations correspond to vanishing second variations of Kähler action - which is what criticality is. The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

3.2.1 Quantum criticality and fermionic representation of conserved charges associated with second variations of Kähler action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been slow. Modified Dirac action provides excellent candidates for quantum counterparts of Noether charges. Unfortunately, the isometry charges vanish for Cartan algebras. The only manner to obtain non-trivial isometry charges is to add a direct coupling to the charges in Cartan algebra as will be found later. This addition involves Chern-Simons Dirac action so that the original intuition guided by almost TQFT idea was not wrong after all.

1. Conservation of the fermionic current requires the vanishing of the second variation of Kähler action

1. The modified Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the modified Dirac action under this deformation vanishes. The vanishing of the first variation for the modified Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the modified Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J_k^\alpha \Psi \ , \\ J_k^\alpha &= \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h_\beta^l}\delta h_\beta^k + \frac{\partial^2 L_K}{\partial h_\alpha^k \partial h^l}\delta h^l \ . \end{aligned} \tag{3.1}$$

Here h_β^k denote partial derivative of the imbedding space coordinate with respect to space-time coordinates. This term must vanish:

$$D_\alpha J_k^\alpha = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J_k^α does not define conserved classical charge in the general case.

2. It is essential that the modified Dirac equation holds true so that the modified Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the modified Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{3.2}$$

Here $1/D$ is the inverse of the modified Dirac operator defining the counterpart of the fermionic propagator.

3. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{3.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the modified Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance [32, 34]. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing modified gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha\Psi + \overline{\Psi}\hat{\Gamma}^\alpha\delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha\Psi \ . \tag{3.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra [35].

4. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right-handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the modified Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

5. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

2. About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding charges are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates. Therefore one cannot represent isometry charges as fermionic bilinears. Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities but this is probably not enough. This can be seen as a problem.

(a) Four-momentum and color Cartan algebra emerge naturally in the representations of super-conformal algebras. In the case of color algebra the charges in the complement of the Cartan algebra can be constructed in standard manner as extension of those for the Cartan algebra using free field representation of Kac-Moody algebras. In string theories four-momentum appears linearly in bosonic Kac-Moody generators and in Sugawara construction [36] of super Virasoro generators as bilinears of bosonic Kac-Moody generators and fermionic super Kac-Moody generators [35]. Also now quantized transversal parts for M^4 coordinates could define a second quantized field having interpretation as an operator acting on spinor fields of WCW. The angle coordinates conjugate to color isospin and hyper charge take the role of M^4 coordinates in case of CP_2.

(b) Somehow one should be able to feed the information about the super-conformal representation of the isometry charges to the modified Dirac action by adding to it a term coupling fermionic current to the Cartan charges in general coordinate invariant and isometry invariant manner. As will be shown later, this is possible. The interpretation is as measurement interaction guaranteeing also the stringy character of the fermionic propagators. The values of the couplings involved are fixed by the condition of quantum criticality assumed in the sense that Kähler function of WCW suffers only a $U(1)$ gauge transformation $K \to K + f + \overline{f}$, where f is a holomorphic function of WCW coordinates depending also on zero modes.

(c) The simplest addition involves the modified gamma matrices defined by a Chern-Simon term at the light-like wormhole throats and is sum of Chern-Simons Dirac action and corresponding coupling term linear in Cartan charges assignable to the partonic 2-surfaces at the ends of the throats. Hence the modified Dirac equation in the interior of the space-time sheet is not affected and nothing changes as far as quantum criticality in interior is considered.

2. The action defined by four-volume gives a first glimpse about what one can expect. In this case modified gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

3. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a on-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs^k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

4. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

5. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

3. Critical super algebra and zero modes

The relationship of the critical super-algebra to configuration space geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the configuration space metric and thus correspond to zero modes. This conforms with the fact that configuration space metric vanishes identically for canonically imbedded M^4. Zero modes do not seem to correspond to gauge degrees of freedom so that the super-conformal algebra associated with the zero modes has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give configuration space metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that the configuration space line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

4. Connection with quantum criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. For some mysterious reason I failed to realize that quantum criticality realized as the vanishing of the second variation makes possible a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality. Both the super-symmetry of D_K and conservation Dirac Noether currents for modified Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$. Recall that the formulation of Equivalence Principle in string picture demonstrated that the reduction of stringy dynamics to that for free strings requires that second variation with respect to M^4 coordinates vanish. This condition would guarantee the conservation of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers but not those for gravitational color quantum numbers. Encouragingly, the action of CP_2 type vacuum extremals having random light-like curve as M^4 projection have vanishing second variation with respect to M^4 coordinates (this follows from the vanishing of Kähler energy momentum tensor, second fundamental form, and Kähler gauge current). In this case however the momentum is vanishing.

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the modified Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with Super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [15] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. A breakthrough in understanding of the criticality was the discovery that the realization that the hierarchy of singular coverings of $CD \times CP_2$ needed to realize the hierarchy of Planck constants could correspond directly to a similar hierarchy of coverings forced by the factor that classical canonical momentum densities correspond to several values of the time derivatives of the imbedding space coordinates led to a considerable progress if the understanding of the relationship between criticality and hierarchy of Planck constants [6, A1]. Therefore the problem which led to the geometrization program of quantum TGD, also allowed to reduce the hierarchy of Planck constants introduced on basis of experimental evidence to the basic quantum TGD. One can say that the 3-surfaces at the ends of CD *resp.* wormhole throats are critical in the sense that they are unstable against splitting to n_b *resp.* n_a surfaces so that one obtains space-time surfaces which can be regarded as surfaces in $n_a \times n_b$ fold covering of $CD \times CP_2$. This allows to understand why Planck constant is effectively replaced with $n_a n_b \hbar_0$ and explains charge fractionization.

3.2.2 Preferred extremal property as classical correlate for quantum criticality, holography, and quantum classical correspondence

The Noether currents assignable to the modified Dirac equation are conserved only if the first variation of the modified Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ wï£¡th the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of configuration space represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the configuration space metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [51] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

3.3 Handful of problems with a common resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete. Let us begin by listing the problems.

1. The condition that modified Dirac action allows conserved charges leads to the condition that the symmetries in question give rise to vanishing second variations of Kähler action. The interpretation is as quantum criticality and there are good arguments suggesting that the critical symmetries define an infinite-dimensional super-conformal algebra forming an inclusion hierarchy related to a sequence of symmetry breakings closely related to a hierarchy of inclusions of hyper-finite factors of types II_1 and III_1. This means an enormous generalization of the symmetry breaking patterns of gauge theories.

   There is however a problem. For the translations of M^4 and color hyper charge and isospin (more generally, any Cartan algebra of $P \times SU(3)$) the resulting fermionic charges vanish. The trial for the crossword in absence of nothing better would be the following argument. By the abelianity of these charges the vanishing of quantal representation of four-momentum and color Cartan charges is not a problem and that classical representation of these charges or their super-conformal representation is enough.

2. Modified Dirac equation is satisfied in the interior of space-time surface always. This means that one does not obtain off-mass shell propagation at all in 4-D sense. Effective 2-dimensionality suggests that off mass shell propagation takes place along wormhole throats. The reduction to almost topological QFT with Kähler function reducing to Chern-Simonst type action implied by the weak form of electric-magnetic duality and a proper gauge choice for the induced Kähler gauge potential implies effective 3-dimensionality at classical level. This inspires the question whether Chern-Simons type action resulting from an instanton term could define

the modified gamma matrices appearing in the 3-D modified Dirac action associated with wormhole throats and the ends of the space-time sheet at the boundaries of CD.

The assumption that modified Dirac equation is satisfied also at the ends and wormhole throats would realize effective 2-dimensionality as conditions on the boundary values of the 4-D Dirac equation but would would not allow off mass shell propagation. Therefore one could argue that effective 2-dimensionality in this sense holds true only for incoming and outgoing particles.

The reduction of Kähler action to Chern-Simons term together with effective 2-dimensionality suggests that Kähler function corresponds to an extremum of this action with a constraint term due to the weak form of electric-magnetic duality. Without this term the extrema of Chern-Simons action have 2-D CP_2 projection not consistent with the weak form of electric-magnetic duality. The extrema are not maxima of Kähler function: they are obtained by varying with respect to tangent space data of the partonic 2-surfaces. Lagrange multiplier term induces also to the modified gamma matrices a contribution which is of the same general form as for any general coordinate invariant action.

3. Quantum classical correspondence requires that the geometry of the space-time sheet should correlate with the quantum numbers characterizing positive (negative) energy part of the quantum state. One could argue that by multiplying WCW spinor field by a suitable phase factor depending on the charges of the state, the correspondence follows from stationary phase approximation. This crossword looks unconvincing. A more precise connection between quantum and classical is required.

4. In quantum measurement theory classical macroscopic variables identified as degrees of freedom assignable to the interior of the space-time sheet correlate with quantum numbers. Stern Gerlach experiment is an excellent example of the situation. The generalization of the imbedding space concept by replacing it with a book like structure implies that imbedding space geometry at given page and for given causal diamond (CD) carries information about the choice of the quantization axes (preferred plane M^2 of M^4 *resp.* geodesic sphere of CP_2 associated with singular covering/factor space of CD *resp.* CP_2). This is a big step but not enough. Modified Dirac action as such does not seem to provide any hint about how to achieve this correspondence. One could even wonder whether dissipative processes or at least the breaking of T and CP characterizing the outcome of quantum jump sequence should have space-time correlate. How to achieve this?

Each of these problems makes one suspect that something is lacking from the modified Dirac action: there should exist an elegant manner to feed information about quantum numbers of the state to the modified Dirac action in turn determining vacuum functional as an exponent Kähler function identified as Kähler action for the preferred extremal assumed to be dictated by by quantum criticality and equivalently by hyper-quaternionicity.

This observation leads to what might be the correct question. Could a general coordinate invariant and Poincare invariant modification of the modified Dirac action consistent with the vacuum degeneracy of Kähler action allow to achieve this information flow somehow? In the following one manner to achieve this modification is discussed. It must be however emphasized that I have considered many alternatives and the one discussed below finds its justification only from the fact that it is the simplest one found hitherto.

3.3.1 The identification of the measurement interaction term

The idea is simple: add to the modified Dirac action a term which is analogous to the Dirac action in $M^4 \times CP_2$. One can consider two options according to whether the term is assigned with interior or with a 3-D light-like 3-surface and last years have been continual argumentation about which option is the correct one.

1. The additional term would be essentially the analog of the ordinary Dirac action at the imbedding space level.

$$\begin{aligned} S_{int} &= \sum_A Q_A \int \overline{\Psi} g^{AB} j_{B\alpha} \hat{\Gamma}^\alpha \Psi \sqrt{g} d^4x \ , \\ g_{AB} &= j^k_A h_{kl} j^l_B \ , \ g^{AB} g_{BC} = \delta^A_C \ , \\ j_{B\alpha} &= j^k_B h_{kl} \partial_\alpha h^l \ . \end{aligned} \tag{3.5}$$

The sum is over isometry charges Q_A interpreted as quantal charges and j^{Ak} denotes the Killing vector field of the isometry. g^{AB} is the inverse of the tensor g_{AB} defined by the local inner products of Killing vectors fields in M^4 and CP_2. The space-time projections of the Killing vector fields $j_{B\alpha}$ have interpretation as classical color gauge potentials in the case of $SU(3)$. In M^4 degrees of freedom and for Cartan algebra of $SU(3)$ $j_{B\alpha}$ reduce to the gradients of linear M^4 coordinates in case of translations. Modified gamma matrices could be assigned to Kähler action or its instanton term or with Chern-Simons action.

2. The added term containing quantal charges must make sense in the modified Dirac equation. This requires that the physical state is an eigenstate of momentum and color charges. This allows only color hyper-charge and color isospin so that there is no hope of obtaining exactly the stringy formula for the propagator. The modified Dirac operator is given by

$$\begin{aligned} D &= D + D_{int} = \hat{\Gamma}^\alpha D_\alpha + \hat{\Gamma}^\alpha \sum_A Q_A g^{AB} j_{B\alpha} \\ &= \hat{\Gamma}^\alpha (D_\alpha + \partial_\alpha \phi) \ , \ \partial_\alpha \phi = \sum_A Q_A g^{AB} j_{B\alpha} \ . \end{aligned} \tag{3.6}$$

The conserved fermionic isometry currents are

$$J^{A\alpha} = \sum_B Q_B \overline{\Psi} g^{BC} j_C^k h_{kl} j_A^l \hat{\Gamma}^\alpha \Psi = Q_A \overline{\Psi} \hat{\Gamma}^\alpha \Psi \ . \tag{3.7}$$

Here the sum is restricted to a Cartan sub-algebra of Poincare group and color group.

3. An important restriction is that by four-dimensionality of M^4 and CP_2 the rank of g_{AB} is 4 so that g^{AB} exists only when one considers only four conserved charges. In the case of M^4 this is achieved by a restriction to translation generators $Q_A = p_A$. g_{AB} reduces to Minkowski metric and Killing vector fields are constants. The Cartan sub-algebra could be however replaced by any four commuting charges in the case of Poincare algebra (second one corresponds to time translation plus translation, boost and rotation in given direction). In the case of $SU(3)$ one must restrict the consideration either to $U(2)$ sub-algebra or its complement. $CP_2 = SU(3)/SU(2)$ decomposition would suggest the complement as the correct choice. One can indeed build the generators of $U(2)$ as commutators of the charges in the complement. On the other hand, Cartan algebra is enough in free field construction of Kac-Moody algebras.

4. What is remarkable that for the Cartan algebra of $M^4 \times SU(3)$ the measurement interaction term is equivalent with the addition of gauge part $\partial_\alpha \phi$ of the induced Kähler gauge potential A_α. This property might hold true for any measurement interaction term. This also suggests that the change in Kähler function is only the transformation $A_\alpha \rightarrow A_\alpha + \partial_\alpha \phi$, $\partial_\alpha \phi = \sum_A Q_A g^{AB} j_{B\alpha}$.

5. Recall that the ϕ for $U(1)$ gauge transformations respecting the vanishing of the Coulomb interaction term of Kähler action [6, A1] the current $j_K^\alpha \phi$ is conserved, which implies that the change of the Kähler action is trivial. These properties characterize the gauge transformations respecting the gauge in which Coulombic interaction term of the Kähler action vanishes so that Kähler action reduces to 3-dimensional generalized Chern-Simons term if the weak form of electric-magnetic duality holds true guaranteeing among other things that the induced Kähler field is not too singular at the wormhole throats [6, A1]. The scalar function assignable to the measurement interaction terms does not have this property and this is what is expected since it must change the value of the Kähler function and therefore affect the preferred extremal.

Concerning the precise form of the modified Dirac action the basic clue comes from the observation that the measurement interaction term corresponds to the addition of a gauge part to the induced CP_2 Kähler gauge potential A_α. The basic question is what part of the action one assigns the measurement interaction term.

1. One could define the measurement interaction term using either the four-dimensional instanton term or its reduction to Chern-Simons terms. The part of Dirac action defined by the instanton term in the interior does not reduce to a 3-D form unless the Dirac equation defined by the instanton term is satisfied : this cannot be true. Hence Chern-Simons term is the only possibility.

   The classical field equations associated with the Chern-Simons term cannot be assumed since they would imply that the CP_2 projection of the wormhole throat and space-like 3-surface are 2-dimensional. This might hold true for space-like 3-surfaces at the ends of CD and incoming and outgoing particles but not for off mass shell particles. This is however not a problem since $D_\alpha \hat{\Gamma}^\alpha_{C-S}$ for the modified gamma matrices for Chern-Simons action does not contain second derivatives. This is due to the topological character of this term. For Kähler action second derivatives are present and this forces extremal property of Kähler action in the modified Dirac Kähler action so that classical physics results as a consistency condition.

2. If one assigns measurement interaction term to both D_K and D_{C-S} the measurement interaction corresponds to a mere gauge transformation for AS_α and is trivial. Therefore it seems that one must choose between D_K or D_{C-S}. At least formally the measurement interaction term associated with D_K is gauge equivalent with its negative D_{C-S}. The addition of the measurement interaction to D_K changes the basis for the 4-D induced spinors by the phase $exp(-iQK\phi)$ and therefore also the basis for the generalized eigenstates of D_{C-S} and this brings in effectively the measurement interaction term affecting the Dirac determinant.

3. The definition of Dirac determinant should be in terms of Chern-Simons action induced by the instanton term and identified as a product of the generalized eigenvalues of this operator. The modified Dirac equation for Ψ is consistent with that for its conjugate if the coefficient of the instanton term is real and one uses the Dirac action $\overline{\Psi}(D^{\rightarrow} - D^{\leftarrow})\Psi$ giving modified Dirac equation as

$$D_{C-S}\Psi + \frac{1}{2}(D_\alpha \hat{\Gamma}^\alpha_{C-S})\Psi = 0 \ . \tag{3.8}$$

   As noticed, the divergence of gamma matrices does not contain second derivatives in the case of Chern-Simons action. In the case of Kähler action they occur unless field equations equivalent with the vanishing of the divergence term are satisfied.

   Also the fermionic current is conserved in this case, which conforms with the idea that fermions flow along the light-like 3-surfaces. If one uses the action $\overline{\Psi}D^{\rightarrow}\Psi$, $\overline{\Psi}$ does not satisfy the Dirac equation following from the variational principle and fermion current is not conserved. Also if the Chern-Simons term is imaginary - as a naive idea about dissipation would suggest- the Dirac equation fails to be consistent with the conjugation.

4. Off mass shell states appear in the lines of the generalized Feynman diagrams and for these D_{C-S} cannot annihilate the spinor field. The generalized eigen modes lf D_{C-S} should be such that one obtains the counterpart of Dirac propagator which is purely algebraic and does not therefore depend on the coordinates of the throat. This is satisfied if the generalized eigenvalues are expressible in terms of covariantly constant combinations of gamma matrices and here only M^4 gamma matrices are possible. Therefore the eigenvalue equation reqards as

$$D\Psi = \lambda^k\gamma_k\Psi \ , \ D = D_{C-S} + D_\alpha\hat{\Gamma}^\alpha_{C-S} \ , \ D_{C-S} = \hat{\Gamma}^\alpha_{C-S}D_\alpha \ . \tag{3.9}$$

Here the covariant derivatives D_α contain the measurement interaction term as an apparent gauge term. Covariant constancy allows to take the square of this equation and one has

$$(D^2 + \left[D, \lambda^k\gamma_k\right])\Psi + = \lambda^k\lambda_k\Psi \ . \tag{3.10}$$

The commutator term is analogous to magnetic moment interaction. The generalized eigenvalues correspond to $\lambda = \sqrt{\lambda^k\lambda_k}$ and Dirac determinant is defined as a product of the eigenvalues. λ is completely analogous to mass. For incoming lines this mass would vanish so that all incoming particles irrespective their actual quantum numbers would be massless in this sense and the propagator is indeed that for a massless particle. Note that the eigen modes define the boundary values for the solutions of $D_K\Psi = 0$ so that the values of λ indeed define the counterpart of the momentum space.

This transmutation of massive particles to effectively massless ones might make possible the application of the twistor formalism as such in TGD framework [12]. $N = 4$ SUSY is one of the very few gauge theory which might be UV finite but it is definitely unphysical due to the masslessness of the basic quanta. Could the resolution of the interpretational problems be that the four-momenta appearing in this theory do not directly correspond to the observed four-momenta?

3.3.2 Objections

The alert reader has probably raised several critical questions. Doesn't the need to solve λ_k as functions of incoming quantum numbers plus the need to construct the measurement interactions makes the practical application of the theory hopelessly difficult? Could the resulting pseudo-momentum λ_k correspond to the actual four-momentum? Could one drop the measurement interaction term altogether and assume that the quantum classical correspondence is through the identification of the eigenvalues as the four-momenta of the on mass shell particles propagating at the wormhole throats? Could one indeed assume that the momenta have a continuous spectrum and thus do not depend on the boundary conditions at all? Usually the thinking is just the opposite and in the general case would lead to to singular eigen modes.

1. Only the information about four-momentum would be fed into the space-time geometry. TGD however allows much more general measurement interaction terms and it would be very strange if the space-time geometry would not correlate also with the other quantum numbers. Mass formulas would of course contain information also about other quantum numbers so that this claim is not quite justified.

2. Number theoretic considerations and also the construction of octonionic variant of Dirac equation [19, A7] force the conclusion that the spectrum of pseudo four-momentum is restricted to a preferred plane M^2 of M^4 and this excludes the interpretation of λ^k as a genuine four-momentum. It also improves the hopes that the sum over pseudo-momenta does not imply divergences.

3. Dirac determinant would depend on the mass spectrum only and could not be identified as exponent of Kähler function. Note that the original guideline was the dream about stringy propagators. This is achieved for $\lambda_A\lambda^A = n$ in suitable units. This spectrum would of course also imply that Dirac determinant defined in terms of ζ function regularization is independent of the space-time surface and could not be identified with the exponent of Kähler function. One must of course take the identification of exponent of Kähler function as Dirac determinant as an additional conjecture which is not necessary for the calculation of Kähler function if the weak form of electric-magnetic duality is accepted.

4. All particles would behave as massless particles and this would not be consistent with the proposed Feynman diagrammatics inspired by zero energy ontology. Since wormhole throats carry on mass shell particles with positive or negative energy so that the net momentum can be also space-like propagators diverge for massless particles. One might overcome this problem by assuming small thermal mass (from p-adic thermodynamics [5]) and this is indeed assumed to reduce the number of generalized Feynman diagrams contributing to a given reaction to finite number.

Second objection of the skeptic reader relates to the delicacies of $U(1)$ gauge invariance. The modified Dirac action seems to break gauge symmetries and this breaking of gauge symmetry is absolutely essential for the dependence of the Dirac determinant on the quantum numbers. It however seems that this breaking of gauge invariance is only apparent.

1. One must distinguish between genuine $U(1)$ gauge transformations carried out for the induced Kähler gauge potential A_α and apparent gauge transformations of the Kähler gauge potential A_k of $S^2 \times CP_2$ induced by symplectic transformations deforming the space-time surface and affect also induced metric. This delicacy of $U(1)$ gauge symmetry explains also the apparent breaking of $U(1)$ gauge symmetry of Chern-Simons Dirac action due to the presence of explicit terms A_k and A_α.

2. CP_2 Kähler gauge potential is obtained in complex coordinates from Kähler function as $(K_{\xi^i}, K_{\overline{\xi^i}}) = (\partial_{\xi^i} K, -\partial_{\overline{\xi^i}} K)$. Gauge transformations correspond to the additions $K \rightarrow K + f + \overline{f}$, where f is a holomorphic function. Kähler gauge potential has a unique gauge in which the Kähler function of CP_2 is $U(2)$ invariant and contains no holomorphic part. Hence A_k is defined in a preferred gauge and is a gauge invariant quantity in this sense. Same applies to S^2 part of the Kähler potential if present.

3. A_α should be also gauge invariant under gauge transformation respecting the vanishing of Coulombic interaction energy. The allowed gauge transformations $A_\alpha \rightarrow A_\alpha + \partial_\alpha \phi$ must satisfy $D_\alpha(j_K^\alpha \phi) = 0$. If the scalar function ϕ reduces to constant at the wormhole throats and at the ends of the space-time surface D_{C-S} is gauge invariant. The gauge transformations for which ϕ does not satisfy this condition are identified as representations of critical deformations of space-time surface so that the change of A_α would code for this kind of deformation and indeed affect the modified Dirac operator and Kähler function (the change would be due to the change of zero modes).

3.3.3 Some details about the modified Dirac equation defined by Chern-Simons action

First some general comments about D_{C-S} are in order.

1. Quite generally, there is vacuum avoidance in the sense that Ψ must vanish in the regions where the modified gamma matrices vanish. A physical analogy for the system consider is a charged particle in an external magnetic field. The effective metric defined by the anti-commutators of the modified gamma matrices so that standard intuitions might not help much. What one would naively expect would be analogs of bound states in magnetic field localized into regions inside which the magnetic field is non-vanishing.

2. If only CP_2 Kähler form appears in the Kähler action, the modified Dirac action defined by the Chern-Simons term is non-vanishing only when the dimension of the CP_2 projection of the 3-surface is $D(CP_2) \geq 2$ and the induced Kähler field is non-vanishing. This conforms with the properties of Kähler action. The solutions of the modified Dirac equation with a vanishing eigenvalue λ would naturally correspond to incoming and outgoing particles.

3. $D(CP_2) \leq 2$ is apparently inconsistent with the weak form of electric-magnetic duality requiring $D(CP_2) = 3$. The conclusion is wrong: the variations of Chern-Simons action are subject to the constraint that electric-magnetic duality holds true expressible in terms of Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\gamma}) \sqrt{g_4} d^3x \ . \tag{3.11}$$

This gives a constraint force to the field equations and also a dependence on the induced 4-metric so that one has only almost topological QFT. This term also guarantees the M^4 part of WCW Kähler metric is non-trivial. The condition that the ends of space-time sheet and wormhole throats are extrema of Chern-Simons action subject to the electric-magnetic duality constraint is strongly suggested by the effective 2-dimensionality.

4. Electric-magnetic duality constraint gives an additional term to the Dirac action determined by the Lagrange multiplier term. This term gives an additional contribution to the modified gamma matrices having the same general form as coming from Kähler action and Chern-Simons action. In the following this term will not be considered. For the extremals it only affects the modified gamma matrices and leaves the general form of solutions unchanged.

In absence of the constraint from the weak form of electric-magnetic duality the explicit expression of D_{C-S} is given by

$$\begin{aligned} D &= \hat{\Gamma}^\mu D_\mu + \frac{1}{2} D_\mu \hat{\Gamma}^\mu \ , \\ \hat{\Gamma}^\mu &= \frac{\partial L_{C-S}}{\partial_\mu h^k} \Gamma_k = \epsilon^{\mu\alpha\beta} \left[2J_{kl} \partial_\alpha h^l A_\beta + J_{\alpha\beta} A_k \right] \Gamma^k D_\mu \ , \\ D_\mu \hat{\Gamma}^\mu &= B_K^\alpha (J_{k\alpha} + \partial_\alpha A_k) \ , \\ B_K^\alpha &= \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ , \ J_{k\alpha} = J_{kl} \partial_\alpha s^l \ , \ \hat{\epsilon}^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma} \sqrt{g_3} \ . \end{aligned} \tag{3.12}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma} =$ does not depend on the induced metric.

The extremals of Chern-Simons action without constraint term satisfy

$$B_K^\alpha (J_{kl} + \partial_l A_k) \partial_\alpha h^l = 0 \ , \ B_K^\alpha = \epsilon^{\alpha\beta\gamma} J_{\beta\gamma} \ . \tag{3.13}$$

For a non-vanishing Kähler magnetic field B^α these equations hold true when CP_2 projection is 2-dimensional. This implies a vanishing of Chern-Simons action in absence of the constraint term realizing electric-magnetic duality, which is therefore absolutely essential in order for having a non-vanishing WCW metric.

Consider now the situation in more detail.

1. Suppose that one can assign a global coordinate to the flow lines of the Kähler magnetic field. In this case one might hope that ordinary intuitions about motion in constant magnetic field might be helpful. The repetition of the discussion of [6, A1] leads to the condition $B \wedge dB = 0$ implying that a Beltrami flow for which current flows along the field lines and Lorentz forces vanishes is in question. This need not be the generic case.

2. With this assumption the modified Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{3.14}$$

3. The general solutions of the modified Dirac equation is covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \tag{3.15}$$

   The solution to this condition can be written immediately in terms of a non-integrable phase factor $Pexp(i\int A_r dr)$, where integration is along curve with constant transversal coordinates. If $\hat{\Gamma^v}$ is light-like vector field also $\hat{\Gamma^v}\Psi_0$ defines a solution of D_{C-S}. This solution corresponds to a zero mode for D_{C-S} and does not contribute to the Dirac determinant. Note that the dependence of these solutions on transversal coordinates of X_l^3 is arbitrary.

4. The formal solution associated with a general eigenvalue can be constructed by integrating the eigenvalue equation separately along all coordinate curves. This makes sense if r indeed assigned to light-like curves indeed defines a global coordinate. What is strange that there is no correlation between the behaviors with respect longitudinal coordinate and transversal coordinates. System would be like a collection of totally uncorrelated point like particles reflecting the flow of the current along flux lines. It is difficult to say anything about the spectrum of the generalized eigenvalues in this case: it might be that the boundary conditions at the ends of the flow lines fix the allowed values of λ. Clearly, the Beltrami flow property is what makes this case very special.

3.3.4 A connection with quantum measurement theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

1. The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

2. One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

3. What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

   (a) The hint as how this description could be achieved comes from a long standing un-answered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the modified Dirac action? At least this question might make sense.

   (b) It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^{\alpha} &= \overline{\Psi} O \hat{\Gamma}^{\alpha} \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB}J\} \ . \end{aligned} \tag{3.16}$$

Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

(c) Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a + bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

(d) Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

(e) There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

(f) The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0} = kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

4. Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

5. Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

6. The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

7. One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the modified Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z) + \overline{f(Z)}$, where Z denotes complex coordinates of WCW, the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

3.3.5 New view about gravitational mass and matter antimatter asymmetry

The physical interpretation of the additional term in the modified Dirac action might force quite a radical revision of the ideas about matter and antimatter.

1. The term $p_A \partial_\alpha m^A$ contracted with the fermion current is analogous to a gauge potential coupling to fermion number. Since the additional terms in the modified Dirac operator induce stringy propagation, a natural interpretation of the coupling to the induced spinor fields is in terms of gravitation. One might perhaps say that the measurement of four momentum induces gravitational interaction. Besides momentum components also color charges take the role of gravitational charges. As a matter fact, any observable takes this role via coupling to the projections of Killing vector fields of Cartan algebra. The analogy of color interactions with gravitational interactions is indeed one of the oldest ideas in TGD.

2. The coupling to four-momentum is through fermion number (both quark number and lepton number). For states with a vanishing fermion number isometry charges therefore vanish. In this framework matter antimatter asymmetry would be due to the fact that matter (antimatter) corresponds to positive (negative) energy parts of zero energy states for massive systems so that the contributions to the net gravitational four-momentum are of same sign. Could antimatter be unobservable to us because it resides at negative energy space-time sheets? As a matter fact, I proposed already years ago that gravitational mass is essentially the magnitude of the inertial mass but gave up this idea.

3. Bosons do not couple at all to gravitation if they are purely local bound states of fermion and anti-fermion at the same space-time sheet (say represented by generators of super Kac-Moody algebra). Therefore the only possible identification of gauge bosons is as wormhole contacts. If the fermion and anti-fermion at the opposite throats of the contact correspond to positive and negative energy states the net gravitational energy receives a positive contribution from both sheets. If both correspond to positive (negative) energy the contributions to the net four-momentum have opposite signs. It is not yet clear which identification is the correct one.

3.4 Generalized eigenvalues of D_{C-S} and General Coordinate Invariance

The fixing of light-like 3-surface to be the wormhole throat at which the signature of induced metric changes from Minkowskian to Euclidian corresponds to a convenient fixing of gauge. General Coordinate Invariance however requires that any light-like surface Y_l^3 parallel to X_l^3 in the slicing is equally good choice. In particular, it should give rise to same Kähler metric but not necessarily the same exponent of Kähler function identified as the product of the generalized eigenvalues of $D_{C,S}$ at Y_l^3.

General Coordinate Invariance requires that the components of Kähler metric of configuration space defined in terms of Kähler function as

$$G_{k\bar{l}} = \partial_k \partial_{\bar{l}} K = \sum_i \partial_k \partial_{\bar{l}} \lambda_i$$

remain invariant under this flow. Here complex coordinate are of course associated with the configuration space. This is the case if the flow corresponds to the addition of sum of holomorphic function $f(z)$ and its conjugate $\overline{f}(z))$ which is anti-holomorphic function to K. This boils down to the scaling of eigenvalues λ_i by

$$\lambda_i \rightarrow exp(f_i(z) + \overline{f_i(z)})\lambda_i \ . \tag{3.17}$$

If the eigenvalues are interpreted as vacuum conformal weights, general coordinate transformations correspond to a spectral flow scaling the eigenvalues in this manner. This in turn would induce spectral flow of ground state conformal weights if the squares of λ_i correspond to ground state conformal weights.

4 Representations for the configuration space gamma matrices in terms of super-symplectic charges at light cone boundary

During years I have considered several variants for the representation of WCW gamma matrices and each of these proposals has had some weakness.

1. One question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians. Magnetic flux Hamiltonians do not refer to the space-time dynamics implying genuine 2-dimensionality, which is a catastrophe. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed effective 2-dimensionality is achieved. The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians.

2. In the original proposal for WCW gamma matrices the covariantly constant right handed spinors played a key role. This led to interpretational problems with quarks. Are they needed at all or do leptons and quarks define somehow equivalent representations? I discovered only recently a brutally simple but deadly objection against this approach: the resulting WCW gamma matrices do not generate all WCW spinors from Fock vacuum. Therefore all modes of the induced spinor fields must be used.

The latter objection forced to realize that nothing is changed if one replaces the covariantly constant right handed neutrino with the collection of quark spinor modes q_n *resp.* leptonic spinor modes L_n multiplied by the contractions $J_{A+} = j^{Ak}\Gamma_k$ *resp.* its conjugate $J_{A-} = j^{A\bar{k}}\Gamma_{\bar{k}}$. It is essential that only of these contractions is used for a given H-chirality.

1. If the anti-commutator of the spinor fields is or form $J = J_{\alpha\beta}\epsilon^{\alpha\beta}\delta^2(x,y)$ at X^2 for magnetic flux Hamiltonians and appropriate generalization of this fro the sum of magnetic and electric flux Hamiltonians, the "half-Poisson bracket" $\partial_k H_A J^{kl}\partial_{\bar{l}} H_B$ from the quark spinor field and its conjugate as anti-commutator from the leptonic spinor field can combine to the full Poisson bracket if the remaining factors are identical.

2. This happens if the quark modes and lepton-like modes are in 1-1 correspondence and the contractions of the eigenmodes resulting in the contraction satisfy $\overline{q}_m\gamma^0 q_n = \overline{L}_m\gamma^0 L_n = \Phi_{mn}$. The resulting Hamiltonians define an X^2-local algebra: that this extension is needed became obvious already earlier. A stronger condition is that the spinors can be expressed in terms of scalar function bases $\{\Phi_m\}$ so that one would have $q_{m,i} = \{\Phi_m\}q_i$ and $L_{m,i} = \{\Phi_m\}L_i$ so that one would assign to the super-currents the local Hamiltonians $\Phi_m H_A$.

3. One could of course still argue that it is questionable to use sum of quark and lepton gamma matrices since this the resulting objects to not have a well defined fermion number and cannot be used to generate physical states from vacuum. How seriously this argument should be taken is not clear to me at this moment. One could of course consider also a scenario in which one divides leptonic (or quark) modes to two classes analogous to quark and lepton modes and uses J_{A_+} *resp.* J_{A_-} for these two classes.

In any case, the recent view is that all modes of the induced spinor fields must be used, that lepton-quark degeneracy is absolutely essential for the construction of WCW geometry, and that the original super-symmetrization of the flux Hamiltonians combined with weak electric-magnetic duality is the correct approach. There are also fermionic Noether charges and their super counterparts implied by the criticality but these can be assigned with zero modes.

This section represents both the earlier version of the construction of configuration gamma matrices and the construction introducing explicitly the notion of finite measurement resolution. The motivation for the latter option is that if the number the generalized eigen modes of modified Dirac operator is finite, strictly local anti-commutation relations fail unless one restricts the set of points included to that corresponding to number theoretic braid. In the following integral expressions for configuration space Hamiltonians and their super-counterparts are derived first. After that the motivations for replacing integrals with sums are discussed and the expressions for Hamiltonians and super Hamiltonians are derived.

4.1 Magnetic flux representation of the super-symplectic algebra

In order to derive representation of the configuration space gamma matrices and super charges it is good to restate the basic facts about the magnetic flux representation of the configuration space gamma matrices using the original approach based on 2-dimensional integrals.

4.2 Quantization of the modified Dirac action and configuration space geometry

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. The naive approach would be based on standard canonical quantization of induced spinor fields by posing anti-commutation relations between Ψ and canonical momentum density $\partial L/\partial(\partial_t\Psi)$.

4.2.1 Generalized magnetic and electric fluxes

Isometry invariants are just a special case of fluxes defining natural coordinate variables for the configuration space. Canonical transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local symplectic transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{s,n,k}$ defining the Lorentz covariant function basis H_A, $A \equiv (a,s,n,k)$ at the light cone boundary: $H_A = H_a \times f(s,n,k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind magnetic or electric flux via the following formulas:

$$Q_{m/e}(H_A|X^2) = \int_{X^2} H_A J_{m/e} \ . \tag{4.1}$$

Here the magnetic (electric) flux J_m (J_e) denotes the flux associated with induced Kähler field and its dual which is well-defined since X^2 is part of 4-D space-time surface.

The flux Hamiltonians

$$Q_i(H_A|X^2) = Q_i(H_A|X^2) \ , \ A \equiv (a,s,n,k) \tag{4.2}$$

provide a representation of WCW Hamiltonians as far as the "kinetic" part of Kähler form is considered.

4.2.2 Anti-commutation relations between oscillator operators associated with same partonic 2-surface

The construction of WCW gamma matrices leads to the anti-commutation relations given by

$$\begin{aligned} \{\overline{\Psi}(x)\gamma^0, \Psi(x)\} &= [J_e + J_m)\delta^2_{x,y} \ , \\ J_e &= \int J^{03}\sqrt{g_4} \ . \end{aligned} \tag{4.3}$$

Kähler magnetic flux $J_m = \epsilon^{\alpha\beta}J_{\alpha\beta}\sqrt{g_2}$ has no dependence on the induced metric.

If the weak- form of the electric-magnetic duality holds true, Kähler electric flux relates to it via the formula

$$J^{03}\sqrt{g_4} = KJ_{12} \ ,$$

where K is symplectic invariant and identifiable in terms of Kähler coupling strength from classical charge quantization condition for Kähler electric flux. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar = 4\pi\alpha_K$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant. The arguments leading to the identification $\epsilon \pm 1$ at the opposite boundaries of CD are discussed in [6, A1]. An alternative identification is as $\epsilon = 0$ but predicts that WCW is trivial in M^4 degrees of freedom if Kähler function reduces to Chern-Simons terms.

The general form of the anti-commutation relations is therefore

$$\{\overline{\Psi}(x)\gamma^0, \Psi(x)\} = (1+K)J\delta^2_{x,y} \ . \tag{4.4}$$

What is nice that at the limit of vacuum extremals the right hand side vanishes when both J and J^1 vanish so that spinor fields become non-dynamical. One can criticize the non-vanishing of the anti-commutator for vacuum extremals of Kähler action.

For the latter option the fermionic counterparts of local flux Hamiltonians can be written in the form

$$\begin{aligned} H_{A,\pm,n} &= \epsilon_q(A,\mp,n)H_{A,\pm,q,n} + \epsilon_L(A,\pm)H_{A,\mp,L,n} \quad , \\ H_{A,+,q,n} &= \oint \overline{\Psi}J^A_+ q_n d^2x \ , \\ H_{A,-,q,n} &= \oint \overline{q}_n J^A_- \Psi d^2x \ , \\ H_{A,-,L,n} &= \oint \overline{\Psi}J^A_+ L_n d^2x \ , \\ H_{A,+,L,n} &= \oint \overline{L}_n J^A_- \Psi d^2x \ , \\ J^A_+ &= j^{Ak}\Gamma_k \ , \ J^A_- = j^{A\overline{k}}\Gamma_{\overline{k}} \ . \end{aligned} \tag{4.5}$$

The commutative parameters $\epsilon_q(A,\pm,n)$ *resp.* $\epsilon_L(A,\pm,n)$ are assumed to carry quark *resp.* lepton number opposite to that of $H_{A,\mp,q,n}$ *resp.* $H_{A,\mp,L,n}$ and satisfy $\epsilon_i(A,+,n)\epsilon_i(A,-,n) = 1$. One encounters a hierarchy discrete algebras satisfying this condition in the construction of a symplectic analog of conformal quantum field theory required by the construction of quantum TGD [3]. Associativity condition fixes uniquely the commutative multiplication of these units and analogs of plane waves with discrete momentum are in question.

Suppose that there is a one-one correspondence between quark modes and leptonic modes is satisfied and the label n decomposes as $n = (m,i)$, where n labels a scalar function basis and i labels spinor components. This would give

$$\begin{aligned} q_n = q_{m,i} &= \Phi_m q_i \ , \\ L_n = L_{m,i} &= \Phi_m L_i \ , \\ \overline{q}_i\gamma^0 q_j &= \overline{L}_i\gamma^0 L_j = g_{ij} \ . \end{aligned} \tag{4.6}$$

Suppose that the inner products g_{ij} are constant. The simplest possibility is $g_{ij} = \delta_{ij}$ Under these assumptions the anti-commutators of the super-symmetric flux Hamiltonians give flux Hamiltonians.

$$\{H_{A,+,n}, H_{A,-,n}\} = g_{ij}\oint \overline{\Phi}_m\Phi_n H_A J d^2x \ . \tag{4.7}$$

The product of scalar functions can be expressed as

$$\overline{\Phi}_m\Phi_n = c_{mn}^{\ \ k}\Phi_k \ . \tag{4.8}$$

Note that the notion of symplectic QFT [11] led to a scalar function algebra of similar kind consisting of phase factors and there excellent reasons to consider the possibility that there is a deep connection with this approach.

One expects that the symplectic algebra is restricted to a direct sum of symplectic algebras localized to the regions where the induced Kähler form is non-vanishing implying that the algebras associated with different region form to a direct sum. Also the contributions to configuration space metric are direct sums. The symplectic algebras associated with different region can be truncated to finite-dimensional spaces of symplectic algebras associated with the regions in question. As far as coordinatization of the reduced configuration space is considered, these symplectic sub-spaces are enough. These truncated algebras naturally correspond to the hyper-finite factor property of the Clifford algebra of configuration space.

4.2.3 Generalization of WCW Hamiltonians and anti-commutation relations between flux Hamiltonians belonging to different ends of CD

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [7, A2, 8]

$$Q(H_A) = \int H_A J d^2x \ . \tag{4.9}$$

   works for the kinetic terms only since J is not expectred to be the same at the ends of the line.

   The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. Perhaps the only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

   Recall that also the construction of number theoretic braids and symplectic QFT [11] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

   This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = (1+K)\int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2s_\pm = (1+K)\int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1,s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2x_\pm \ . \tag{4.10}$$

   Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

   The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J^+_{kl} + J^-_{kl} \ , \\ J^{kl}_\pm &= \partial_\alpha s^k \partial_\beta s^l J^{\alpha\beta}_\pm \ . \end{aligned} \tag{4.11}$$

   The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the modified Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing J with $X\partial(s^1,s^2)/\partial(x^1_\pm,x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+,s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

4.3 Expressions for configuration space super-symplectic generators in finite measurement resolution

The expressions of configuration space Hamiltonians and their super counterparts just discussed were based on 2-dimensional integrals. This is problematic for several reasons.

1. In p-adic context integrals do not makes sense so that this representation fails in p-adic context (for pe-adic numbers see[30]). Sums would be more appropriate if one wants number theoretic universality at the level of basic formulas.

2. The use of sums would also conform with the notion of finite measurement resolution having discretization in terms of intersections of X^2 with number theoretic braids as a space-time correlate.

3. Number theoretic duality suggests a unique realization of the discretization in the sense that only the points of partonic 2-surface X^2 whose $\delta M^4_\pm$ projections commute in hyper-octonionic sense and thus belong to the intersections of the projection $P_{M^4}(X^2)$ with radial light-like geodesics $M_\pm$ representing intersections of $M^2\subset M^4\subset M^8$ with $\delta M^4_\pm\times CP_2$ contribute to the configuration space Hamiltonians and super Hamiltonians and therefore to the configuration space metric.

Clearly, finite measurement resolution seems to be an unavoidable aspect of the geometrization of the configuration space as one can expect on basis of the fact that configuration space Clifford algebra provides representation for hyper-finite factors of type II_1 whose inclusions provide a representation for the finite measurement resolution. This means that the infinite-dimensional configuration space can be represented as a finite-dimensional space in arbitrary precise approximation so that also also configuration Clifford algebra and configuration space spinor fields becomes finite-dimensional.

The modification of anti-commutation relations to this case is

$$\{\overline{\Psi}(x_m)\gamma^0,\Psi(x_n)\} = (1+K)J\delta_{x_m,x_n} . \tag{4.12}$$

Note that the constancy of γ^0 implies a complete symmetry between the two points. The number of points must be the maximal one consistent with the Kronecker delta type anti-commutation relations so that information is not lost.

The question arises about the choice of the points x_m. This choice should general coordinate invariant. The number theoretic vision leads to the notion of number theoretic braid defined as the set of points common to real and p-adic variant of X^2. The points of the number theoretic braid are excellent candidates for points x_n. The p-adic variant exists only if X^2 is defined by rational functions with coefficients which are possibly algebraic and thus make sense both in real and p-adic sense. These points belong to the algebraic extension of rational numbers appearing in the representation of X^2 as an algebraic surface but one can consider quite generally the possibility that the points of the number theoretic braid are rational or in a finite algebraic extension of rationals. What is important that if one restricts the consideration to rational points this criterion makes sense even if X^2 is not algebraic. In the generic case one can expect that the number of these points is finite.

4.4 Configuration space geometry and hierarchy of inclusions of hyper-finite factors of II_1

The configuration space metric defined as anti-commutators of the configuration space gamma matrices is extremely degenerate since it effectively corresponds to a quadratic form in N-dimensional space, where N_m is the total number of the eigenmodes of D_K. Since two Hamiltonians whose values and corresponding Killing vector fields co-incide at the points of B are equivalent for given ray $M_\pm$, it is natural to pose a cutoff in the number of Hamiltonians used for the representation of reduced configuration space in given region inside which induced Kähler form is non-vanishing. The natural manner to pose this cutoff is by ordering the representations with respect to dimension and eigenvalue of Casimir operator for the irreducible representations of $SO(3)\times SO(4)$ in case of M^8 and for the representations of $SO(3)\times SU(3)$ in case of H.

This boils down to a hierarchy of approximate representations of the configuration space as Kähler manifold with spinor structure with a truncation of the Clifford algebra to a finite dimensional Clifford algebra. This is in spirit with the proposed interpretation of the inclusion sequence of hyper-finite factors of type II_1 and with the very notion of hyper-finiteness. A surprisingly concrete connection of the configuration space geometry with generalized eigenvalue spectrum of $D_K(X^3)$ and basic quantum physics results. For instance, from the general expression of Kähler metric in terms of Kähler function

$$G_{k\bar{l}} = \partial_k\partial_{\bar{l}}K = \frac{\partial_k\partial_{\bar{l}}exp(K)}{exp(K)} - \frac{\partial_k exp(K)}{exp(K)}\frac{\partial_{\bar{l}}exp(K)}{exp(K)} , \tag{4.13}$$

and from the expression of $exp(K) = \prod_i \lambda_i$ as the product of of finite number of eigenvalues of $D_K(X^3)$, the expression

$$G_{k\bar{l}} = \sum_i \frac{\partial_k \partial_{\bar{l}} \lambda_i}{\lambda_i} - \frac{\partial_k \lambda_i}{\lambda_i} \frac{\partial_{\bar{l}} \lambda_i}{\lambda_i} \tag{4.14}$$

for the configuration space metric follows. Here complex coordinates refer to the complex coordinates of configuration space.

A good candidate for these complex coordinates are the complex coordinates of $S^2 \times S$, $S = CP_2$ or E^4, for the points of B so that a close connection with the geometry of imbedding space is obtained. Once these coordinates have been specified G can be contracted with the Killing vector fields of configuration space isometries defining the coordinates for the truncated configuration space. By studying the behavior of eigenvalue spectrum under small deformations of X_l^3 by symplectic transformations of $\delta CD \times S$ the components of G can be estimated.

References

Online books about TGD

[1] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry.*
`http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html`.

[2] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time.*
`http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html`.

[3] M. Pitkänen (2006), *Quantum TGD.*
`http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html`.

[4] M. Pitkänen (2006), *TGD as a Generalized Number Theory.*
`http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html`.

[5] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy.*
`http://tgd.wippiespace.com/public_html/paddark/paddark.html`.

References to the chapters of books and articles

[6] The chapter *Identification of the Configuration Space Kähler Function* of [1].
`http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#kahler`.

[7] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles* of [1].
`http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#compl1`.

[8] The chapter *Configuration Space Spinor Structure* of [1].
`http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#cspin`.

[9] The chapter *Does the Modified Dirac Equation Define the Fundamental Action Principle?* of [1].
`http://tgd.wippiespace.com/public_html/tgdgeom/tgdgeom.html#Dirac`.

[10] The chapter *Construction of Quantum Theory: Symmetries* of [3].
`http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#quthe`.

[11] The chapter *Construction of Quantum Theory: S-matrix* of [3].
`http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#towards`.

[12] The chapter *Twistors, N=4 Super-Conformal Symmetry, and Quantum TGD* of [3].
`http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#twistor`.

[13] The chapter *Does the QFT Limit of TGD Have Space-Time Super-Symmetry?* of [3].
`http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#susy`.

[14] The chapter *Was von Neumann Right After All* of [3].
`http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#vNeumann`.

[15] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [3].
`http://tgd.wippiespace.com/public_html/tgdquant/tgdquant.html#Planck`.

[16] The chapter *Basic Extremals of Kähler Action* of [2].
`http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#class`.

[17] The chapter *TGD and Cosmology* of [2].
`http://tgd.wippiespace.com/public_html/tgdclass/tgdclass.html#cosmo`.

[18] The chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [4].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visiona.

[19] The chapter *TGD as a Generalized Number Theory: Infinite Primes* of [4].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#visionc.

[20] The chapter *Riemann Hypothesis and Physics* of [4].
http://tgd.wippiespace.com/public_html/tgdnumber/tgdnumber.html#riema.

[21] The chapter *The Recent Status of Le ptohadron Hypothesis* of [5].
http://tgd.wippiespace.com/public_html/paddark/paddark.html#leptc.

Links to articles in this issue

[A1] M. Pitkänen (2010), *Physics as Infinite-dimensional Geometry I: Identification of the Configuration Space Kähler Function.* Prespacetime Journal July Vol. 1 Issue 4 Page 540-561.

[A2] M. Pitkänen (2010), *Physics as Infinite-dimensional Geometry II: Configuration Space Kähler Geometry from Symmetry Principles.* Prespacetime Journal July Vol. 1 Issue 4 Page 562-580.

[A4] M. Pitkänen (2010), *Physics as Infinite-dimensional Geometry IV: Weak Form of Electric-Magnetic Duality and Its Implications.* Prespacetime Journal July Vol. 1 Issue 4 Page 562-580.

[A7] M. Pitkänen (2010), *Physics as Generalized Number Theory III: Infinite Primes.* Prespacetime Journal July Vol. 1 Issue 4 Page 664-691.

Mathematics

Various geometries

[22] *Symmetric space.* http://en.wikipedia.org/wiki/Symmetric_space.

[23] *Harmonic analysis.* http://en.wikipedia.org/wiki/Harmonic_analysis.

[24] *Kähler manifold.* http://en.wikipedia.org/wiki/KÃďhler_manifold.

[25] *Symplectic manifold.* http://en.wikipedia.org/wiki/Symplectic_manifold.
Symplectic geometry. http://en.wikipedia.org/wiki/Symplectic_geometry.
A. C. da Silva (2004), *Symplectic geometry.* http://www.math.princeton.edu/~acannas/symplectic.pdf.

[26] *Clifford algebra.* http://en.wikipedia.org/wiki/Clifford_algebra.

Number theory

[27] *Fields.* http://en.wikipedia.org/wiki/Field_(mathematics).
D. A. Marcus (1977), *Number Fields.* Springer Verlag. http://www.springer.com/mathematics/numbers/book/978-0-387-90279-1.

[28] *Quaternions.* http://en.wikipedia.org/wiki/Quaternion.

[29] *Octonions.* http://en.wikipedia.org/wiki/Octonions.
J. C. Baez (2001), *The Octonions*, Bull. Amer. Math. Soc. 39 (2002), 145-205.
http://math.ucr.edu/home/baez/Octonions/octonions.html.

[30] http://en.wikipedia.org/wiki/P-adic_number.
F. Q. Gouvêa (1997), *p-adic Numbers: An Introduction*, Springer.
L. Brekke and P. G. O Freund (1993), *p-Adic Numbers and Physics*, Phys. Rep. , vol 233, No 1.

[31] *The Riemann zeta function interpreted as partition function.* http://www.secamlocal.ex.ac.uk/people/staff/mrwatkin//zeta/physics2.htm.

Conformal symmetries

[32] *Scale invariance vs. conformal invariance.*http://en.wikipedia.org/wiki/Conformal_field_theory#Scale_invariance_vs._conformal_invariance.

[33] *Chern-Simons theory.* http://en.wikipedia.org/wiki/ChernâĂŞSimons_theory.

[34] V. G. Knizhnik (1986), *Superconformal algebras in two dimensions.* Teoret. Mat. Fiz., Vol. 66, Number 1, pp. 102-108. *Super Virasoro algebra.* http://en.wikipedia.org/wiki/Super_Virasoro_algebra.

[35] *Kac-Moody algebra.* http://en.wikipedia.org/wiki/KacâĂŞMoody_algebra.
P. Windey (1986), *Super-Kac-Moody algebras and supersymmetric 2d-free fermions.* Comm. in Math. Phys. Vol. 105, No 4.
S. Kumar (2002), *Kac-Moody Groups, their Flag Varieties and Representation Theory.* Progress in Math. Vol 204. A Birkhauser Boston book. http://www.springer.com/birkhauser/mathematics/book/978-0-8176-4227-3.

[36] H. Sugawara (1968), *A field theory of currents*, Phys. Rev., 176, 2019-2025.

[37] D. S. Freed (1985): *The Geometry of Loop Groups* (Thesis). Berkeley: University of California.

Von Neumann algebras and quantum groups

[38] J. Dixmier (1981), *Von Neumann Algebras*, Amsterdam: North-Holland Publishing Company. [First published in French in 1957: Les Algebres d'Operateurs dans l'Espace Hilbertien, Paris: Gauthier-Villars.].

[39] V. F. R. Jones (1983), *Braid groups, Hecke algebras and type II_1 factors*, Geometric methods in operator algebras, Proc. of the US-Japan Seminar, Kyoto, July 1983.

[40] C. Kassel (1995), *Quantum Groups*, Springer Verlag.

[41] C. Gomez, M. Ruiz-Altaba, G. Sierra (1996), *Quantum Groups and Two-Dimensional Physics*, Cambridge University Press.

[42] C. N. Yang, M. L. Ge (1989), *Braid Group, Knot Theory, and Statistical Mechanics*, World Scientific.
V. F. R. Jones, *Hecke algebra representations of braid groups and link polynomial*, Ann. Math., 126(1987), 335-388.

Quantum field theories and string model

[43] *Topological quantum field theory.* http://en.wikipedia.org/wiki/Topological_quantum_field_theory.
E. Witten (1988), *Topological quantum field theory.* Communications in Mathematical Physics 117 (3): 353-386,http://projecteuclid.org/euclid.cmp/1104161738.

[44] E. Witten 1989), *Quantum field theory and the Jones polynomial*, Comm. Math. Phys. 121 , 351-399.

[45] S. Sawin (1995), *Links, Quantum Groups, and TQFT's*, q-alg/9506002.

[46] *Conformal field theory.* http://en.wikipedia.org/wiki/Conformal_field_theory.
A. A. Belavin, A. M. Polyakov, A. B. Zamolodchikov (1984), *Infinite conformal symmetry in two-dimensional quantum field theory*, Nucl. Phys. B241 (1984) 333-380.

[47] *Chern-Simons theory.* http://en.wikipedia.org/wiki/ChernâĂŞSimons_theory.

[48] Schwartz, J., H. (ed) (1985): *Super strings. The first 15 years of Superstring Theory.* World Scientific.

Physics

[49] M. Freedman, H. Larsen, and Z. Wang (2002), *A modular functor which is universal for quantum computation*, Found. Comput. Math. 1, no 2, 183-204. Comm. Math. Phys. 227, no 3, 605-622. quant-ph/0001108.
M. H. Freedman (2001), *Quantum Computation and the localization of Modular Functors*, Found. Comput. Math. 1, no 2, 183-204.
M. H. Freedman (1998), *P/NP, and the quantum field computer*, Proc. Natl. Acad. Sci. USA 95, no. 1, 98-101.
A. Kitaev (1997), Annals of Physics, vol 303, p.2. See also *Fault tolerant quantum computation by anyons*, quant-ph/9707021.
L. H. Kauffmann and S, J. Lomonaco Jr. (2004), *Braiding operations are universal quantum gates*, arxiv.org/quant-ph/0401090.
Paul Parsons (2004) , *Dancing the Quantum Dream*, New Scientist 24. January.
http://www.newscientist.com/hottopics.

[50] *Quantum criticality.* http://en.wikipedia.org/wiki/Quantum_criticality.

[51] *Self-Organized Criticality.* http://en.wikipedia.org/wiki/Self-organized_criticality.

[52] A. Lakthakia (1994), *Beltrami Fields in Chiral Media*, Series in Contemporary Chemical Physics - Vol. 2, World Scientific, Singapore.
D. Reed (1995), in *Advanced Electromagnetism: Theories, Foundations, Applications*, edited by T. Barrett (Chap. 7), World Scientific, Singapore.
O. I Bogoyavlenskij (2003), *Exact unsteady solutions to the Navier-Stokes equations and viscous MHD equations.* Phys. Lett. A, 281-286.
J. Etnyre and R. Ghrist (2001), *An index for closed orbits in Beltrami field.* ArXiv:math.DS/01010.
G. E. Marsh (1995), *Helicity and Electromagnetic Field Topology* in *Advanced Electromagnetism*, Eds. T. W. Barrett and D. M. Grimes, Word Scientific.