Article

Five Dimensional Plane Wave-like Solutions of Field Equations of Israel and Trollope in Peres Space-time

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Abstract

The plane wave-like solutions in Peres space-time of the field equations proposed by Israel and Trollope (1961) have been derived by Srivastava (1974) using the generalized Takeno's space-time in four dimension. We observed that the study regarding plane wave like solutions can further be extended to higher five dimension and therefore an attempt has been made in the present paper. Thus in this paper, we have obtained the five dimensional plane wave like solutions of field equations of Israel and Trollope's unified field theory in Peres space-time on the lines of Srivastava. It is found that the second set of field equations of Israel and Trollope's unified field theory does not have any plane wave like solution. It has been pointed out that the plane wave like solutions obtained here are the generalized to that of Srivastva(1974) because the work of Srivastva can be reproduced from ours by reducing the dimension.

Keywords : General relativity, Five-dimensional field equations of Israel and Trollope, Plane wave-like solution in Peres space-time.

1. Introduction

Takeno (1961) has solved field equations of general relativity as well as non-symmetric unified field theory proposed by Einstein and obtained plane wave solutions using his own space-time in four dimensions. Also he has investigated plane-like solutions using the space-time given by Peres (1959) in both the theories. Finally he has made the comparison of the plane wave solutions obtained in both the theories to identify which theory is appropriate for the study of physical phenomenon. Takeno's (1961) work on gravitational waves has been a source of inspiration to all the researchers in the field of relativity with a base for investigations on wave solutions of various field equations.

A new set of field equations, alternative to Einstein's unified field equations, as proposed by Israel and Trollope (1961) has been considered by Srivastava (1974) in the two types of generalized Takeno space-times, one proposed by Srivastava (1974) himself and other

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suggested by the Lal and Ali (1970) and obtained wave solutions of these field equations in four dimension. The plane wave-like solutions in Peres space-time of the field equations of Israel and Trollope (1961) have also been derived by Srivastava (1974). Lal and Pandey (1975) have generalized the Peres space-time and obtained plane wave like solutions of the field equations proposed by Finzi (1954). Recently Pradhan (1977) has introduced a new generalization of Peres space-time other than Lal and Pandey (1974) and investigated plane wave like solutions of field equations of Israel and Trollope (1961).

In a past few years there have been many attempts to construct a unified field theory based on the idea of multidimensional space-time. Most recent efforts have been diverted at studying theories in which the dimensions of the space-time are greater than (3+1) of the order which we observe. The idea that space-time should be extended from four to higher five dimension was introduced by Kaluza and Klein (1921,1926) to unify gravity and electromagnetism. Wesson (1983,1984) and Reddy D. R. K. (1999) have studied several aspects of five dimensional space-time in variable mass theory and bi-metric theory of relativity respectively. Inspired by work in string theory and other field theories, there has been a considerable interest in recent times to find solutions of the Einstein's field equations in dimensions greater than four.

In the present work we would like to find plane wave-like solutions in the frameworks of five-dimensional space-time. Many authors have extended Takeno's work in general as well as non-symmetric unified field theory proposed by Einstein to higher dimensions using the different space-times [Thengane (2000), Ambatkar (2002), Ladke (2004), Warade (2006) and Jumale (2006)]. With this motivation the study regarding plane wave-like solutions of the field equations of Israel and Trollope has been carried out to higher five dimensional Peres space-time on the lines of Srivastva (1 974) in the present paper.

Recently, Jumale (2006) has extended the work of Takeno (1961) in four dimensional Peres space-time to higher five dimension in the case of Einstein's non-symmetric unified field theory. This study regarding plane wave like solutions in ENSUFT can further be carried out to the case of Israel and Trollope's unified field theory and therefore an attempt has been made in this paper. To obtain the plane wave-like solutions in V_5, we consider the first set [I] of five dimensional field equations of Israel and Trollope's unified field theory as under

$$\theta^{ij}{}_{;}\mu^{*} = 0\,, \qquad (i, j = 1,2,3,4,5) \tag{1.1}$$

$$\Gamma_i^{*} = \Gamma^{*j}_{[ij]} = 0\,, \tag{1.2}$$

$$\theta^{[ij]}{}_{,j} = 0\,, \tag{1.3}$$

$$R_{(ij)} = \alpha M_{ij}(R_{[\,]})\,, \tag{1.4}$$

$$R = 0 \tag{1.5}$$

while the second set [II] of five dimensional field equations of their unified field theory consists of

$$\theta^{ij}{}_{;}\mu^{*} = 0\,, \tag{1.6}$$

$$\Gamma_i^{*} = \Gamma^{*j}_{[ij]} = 0\,, \tag{1.7}$$

$$\theta^{[ij]}{}_{,j} = 0\,, \tag{1.8}$$

$$bR(R_{(ij)} - \frac{1}{4}Rg_{ij}) = \alpha\, M_{ij}(R_{[\;]})\,, \tag{1.9}$$

where R be a non-vanishing constant.

Israel and Trollope have assumed $bR = 1$ therefore, the field equation (1.9) replaced by

$$R_{(ij)} - \frac{1}{4b}\, g_{ij} = \alpha\, M_{ij}(R_{[\;]})\,, \tag{1.10}$$

where α and b are constants and θ^{ij} is a contra variant tensor density defined by

$$\theta^{ij} = \sqrt{g}\,(g^{ij} + \alpha R^{[ij]}) \tag{1.11}$$

and $M_{ij}(R_{[\;]})$, is known as the Maxwellian of the tensor $R_{[ij]}$ which is defined by

$$M_{ij}(R_{[\;]}) = (1/4)g_{ij}(R_{[lm]}R^{[lm]}) - R_{[il]}R_{[km]}g^{lm}\,. \tag{1.12}$$

In both the two sets of field equations a semicolon (;) followed by asterisk $*$ denotes covariant differentiation with respect to connections Γ^{*i}_{jk} .The Ricci tensor R_{ij} is defined by

$$R_{ij} \equiv \Gamma^{l}_{il,j} - \Gamma^{l}_{ij,l} + \Gamma^{l}_{tj}\Gamma^{t}_{il} - \Gamma^{l}_{tl}\Gamma^{t}_{ij}\,. \tag{1.13}$$

The paper is organized as under: section 2 contains components of contra-variant tensor density. In the section 3, we have explained components of the tensors $g^{ij}, R^{[ij]}, g_{ij}, R_{[ij]}$ and $M_{ij}(R_{[\;]})$. Section 4 is dealt with solutions of the field equations (1.1), (1.2) and (1.3). Solutions of the field equations (1.4) and (1.5) are obtained in the fifth section. Section 6 is devoted to the second set of field equations of Israel and Trollope's unified field theory and in the last section of the present paper, we summarize and conclude the result.

2. Components of contra-variant tensor density θ^{ij} in V_5

Recently, Jumale (2006) has obtained the non-symmetric tensor g_{ij} as a plane wave like solutions of field equations of Einstein's non-symmetric unified field theory in the five dimensional Peres space-time :

$$ds^2 = -du^2 - dx^2 - dy^2 - dz^2 + dt^2 - 2f(u,x,y,Z)(dz-dt)^2 \tag{2.1}$$

such that
$$g_{ij} = \begin{bmatrix} -1 & 0 & 0 & \rho_1 & -\rho_1 \\ 0 & -1 & 0 & \rho_2 & -\rho_2 \\ 0 & 0 & -1 & \rho_3 & -\rho_3 \\ -\rho_1 & -\rho_2 & -\rho_3 & -(1+2f) & 2f \\ \rho_1 & \rho_2 & \rho_3 & 2f & (1-2f) \end{bmatrix}. \tag{2.2}$$

where f , ρ_1, ρ_2 and ρ_3 are functions of u, x, y and Z , $Z \equiv Z(z-t)$.
Here, we consider the non-symmetric covariant tensor S_{ij} in the same form as above

$$S_{ij} = \begin{bmatrix} -1 & 0 & 0 & \rho_1 & -\rho_1 \\ 0 & -1 & 0 & \rho_2 & -\rho_2 \\ 0 & 0 & -1 & \rho_3 & -\rho_3 \\ -\rho_1 & -\rho_2 & -\rho_3 & -(1+2f) & 2f \\ \rho_1 & \rho_2 & \rho_3 & 2f & (1-2f) \end{bmatrix}. \tag{2.3}$$

which implies that

$$\det.(S_{ij}) = g = mn > 0, \qquad m = -1 \text{ and } \quad n = -1. \tag{2.4}$$

It is to be noted that the symmetric part of S_{ij} corresponds to the metric tensor of the space time under consideration.

According to Israel and Trollope (1961), we have a relation between covariant tensor S_{ij} and contra-variant tensor density θ^{ij} such that

$$S_{i\alpha}\theta^{j\alpha} = \delta_i^j(\det\theta^{\nu\mu})^{1/2} = \delta_i^j(\det S_{ij})^{1/2} = \delta_i^j\sqrt{g}\,. \tag{2.5}$$

where $\det(\theta^{\nu\mu}) = \det(S_{ij})$. (2.6)

Putting values of S_{ij} from (2.3) in the equation (2.5), we get twenty five equations and after solving them we have the component of θ^{ij} as follows:

$$\theta^{ij} = \begin{bmatrix} -1 & 0 & 0 & \rho_1 & \rho_1 \\ 0 & -1 & 0 & \rho_2 & \rho_2 \\ 0 & 0 & -1 & \rho_3 & \rho_3 \\ -\rho_1 & -\rho_2 & -\rho_3 & \omega-1 & \omega \\ -\rho_1 & -\rho_2 & -\rho_3 & \omega & \omega+1 \end{bmatrix}, \tag{2.7}$$

where $\omega = \rho_1^2 + \rho_2^2 + \rho_3^2 + 2f$. (2.8)

3. Components of the tensors $g^{ij}, R^{[ij]}, g_{ij}, R_{[ij]}$ and $M_{ij}(R_{[\,]})$ in V_5

In this section we find the components of tensors $g^{ij}, R^{[ij]}, g_{ij}, R_{[ij]}$ and $M_{ij}(R_{[\,]})$.
The components of g^{ij} and non-vanishing components of $R^{[ij]}$ can be obtained by substituting the values of tensor density θ^{ij} into the equation (1.11) as under

$$g^{ij} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \omega'-1 & \omega' \\ 0 & 0 & 0 & \omega' & \omega'+1 \end{bmatrix} \tag{3.1}$$

where $\omega' = (\rho_1^2 + \rho_2^2 + \rho_3^2 + 2f)/\sqrt{g}$ (3.2)

and

$$R^{[14]} = -R^{[41]} = R^{[15]} = -R^{[51]} = \rho_1/\alpha\sqrt{g}\,,$$

$$R^{[24]} = -R^{[42]} = R^{[25]} = -R^{[52]} = \rho_2/\alpha\sqrt{g}\,,$$

$$R^{[34]} = -R^{[43]} = R^{[35]} = -R^{[53]} = \rho_3 / \alpha\sqrt{g} . \tag{3.3}$$

From (3.1) the components of g_{ij} are calculated as

$$g_{ij} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -(\omega'+1) & \omega' \\ 0 & 0 & 0 & \omega' & -(\omega'-1) \end{bmatrix}. \tag{3.4}$$

From (3.3) and (3.4), we can easily obtained the expressions of non-vanishing components of $R_{[ij]}$ such that

$$R_{[14]} = -R_{[41]} = R_{[15]} = -R_{[51]} = \rho_1 / \alpha ,$$

$$R_{[24]} = -R_{[42]} = R_{[25]} = -R_{[52]} = \rho_2 / \alpha ,$$

$$R_{[34]} = -R_{[43]} = R_{[35]} = -R_{[53]} = \rho_3 / \alpha . \tag{3.5}$$

It has been observed that $R_{[ij]}$ $R^{[ij]}$ vanishes identically. Therefore, expression for Maxwellian $M_{ij}(R_{[\,]})$ reduces to

$$M_{ij}(R_{[\,]}) = -R_{[il]}R_{[km]}g^{lm} = -\frac{1}{2}(R_{[il]}R_{[jm]} + R_{[li]}R_{[mj]}) . \tag{3.6}$$

From (3.6) the non-vanishing components of Maxwellian are calculated as

$$M_{44} = M_{55} = -M_{45} = -M_{54} = (\rho_1^2 + \rho_2^2 + \rho_3^2) / \alpha^2 . \tag{3.7}$$

These values of $g^{ij}, R^{[ij]}, g_{ij}, R_{[ij]}$ and $M_{ij}(R_{[\,]})$ are useful to find out the solutions of field equations of Israel and Trollope unified field theory.

4. Solutions of the field equations (1.1), (1.2) and (1.3) in V_5

In this section we solve the field equations (1.1), (1.2) and (1.3) of Israel and Trollope in higher five dimensional Peres space-time (2.1).

The field equation (1.1) is equivalent to

$$S^{*}_{ij;\mu} = 0\,, \tag{4.1}$$

which is nothing but the first equation of Einstein's non-symmetric unified field theory and has already been solved by Jumale (2006) in [1]. The equation (4.1) enables us to calculate the components of Γ^{*i}_{jk}. Therefore, we can consider the components of Γ^{*i}_{jk} are the same as that of Γ^{i}_{jk} given in [1] such that

$$\Gamma^k_{11} = \Gamma^k_{22} = \Gamma^k_{33} = 0\,,\ \Gamma^k_{13} = -\Gamma^k_{31} = \{0,0,0,-(\partial_3\rho_1 - \partial_1\rho_3)/2,-(\partial_3\rho_1 - \partial_1\rho_3)/2\}\,,$$

$$\Gamma^k_{14} = -\Gamma^k_{15} = \{-\partial_1\rho_1,-(\partial_2\rho_1 + \partial_1\rho_2)/2,-(\partial_3\rho_1 + \partial_1\rho_3)/2,\partial_1 f - \alpha_1 - \bar{\rho}_1,\partial_1 f - \alpha_1 - \bar{\rho}_1\}\,,$$

$$\Gamma^k_{41} = -\Gamma^k_{51} = \{\partial_1\rho_1,(\partial_2\rho_1 + \partial_1\rho_2)/2,(\partial_3\rho_1 + \partial_1\rho_3)/2,\partial_1 f - \alpha_1 + \bar{\rho}_1,\partial_1 f - \alpha_1 + \bar{\rho}_1\}\,,$$

$$\Gamma^k_{34} = -\Gamma^k_{35} = \{-(\partial_3\rho_1 + \partial_1\rho_3)/2,-(\partial_3\rho_2 + \partial_2\rho_3)/2,-\partial_3\rho_3,\partial_3 f - \alpha_3 - \bar{\rho}_3,\partial_3 f - \alpha_3 - \bar{\rho}_3\}\,,$$

$$\Gamma^k_{43} = -\Gamma^k_{53} = \{(\partial_3\rho_1 + \partial_1\rho_3)/2,(\partial_3\rho_2 + \partial_2\rho_3)/2,\partial_3\rho_3,\partial_3 f - \alpha_3 + \bar{\rho}_3,\partial_3 f - \alpha_3 + \bar{\rho}_3\}\,,$$

$$\Gamma^k_{44} = -\Gamma^k_{45} = -\Gamma^k_{54} = \Gamma^k_{55} = \{-\partial_1 f + 2\alpha_1,-\partial_2 f + 2\alpha_2,-\partial_3 f + 2\alpha_3,\bar{f},\bar{f}\}\,,$$

$$\Gamma^k_{12} = -\Gamma^k_{21} = \{0,0,0-(\partial_2\rho_1 - \partial_1\rho_2)/2,-(\partial_2\rho_1 - \partial_1\rho_2)/2\}\,,$$

$$\Gamma^k_{23} = -\Gamma^k_{32} = \{0,0,0-(\partial_3\rho_2 - \partial_2\rho_3)/2,-(\partial_3\rho_2 - \partial_2\rho_3)/2\}\,,$$

$$\Gamma^k_{24} = -\Gamma^k_{25} = \{-(\partial_2\rho_1 + \partial_1\rho_2)/2,-\partial_2\rho_2,-(\partial_3\rho_2 + \partial_2\rho_3)/2,\partial_2 f - \alpha_2 - \bar{\rho}_2,\partial_2 f - \alpha_2 - \bar{\rho}_2\}\,,$$

$$\Gamma^k_{42} = -\Gamma^k_{52} = \{(\partial_2\rho_1 + \partial_1\rho_2)/2,\partial_2\rho_2,(\partial_3\rho_2 + \partial_2\rho_3)/2,\partial_2 f - \alpha_2 + \bar{\rho}_2,\partial_2 f - \alpha_2 + \bar{\rho}_2\} \tag{4.2}$$

where $\alpha_1 = -[2\rho_1\partial_1\rho_1 + \rho_2(\partial_2\rho_1 + \partial_1\rho_2) + \rho_3(\partial_3\rho_1 + \partial_1\rho_3)]/2\,,$

$$\alpha_2 = -[2\rho_2\partial_2\rho_2 + \rho_1(\partial_2\rho_1 + \partial_1\rho_2) + \rho_3(\partial_3\rho_2 + \partial_2\rho_3)]/2\,,$$

$$\alpha_3 = -[2\rho_3\partial_3\rho_3 + \rho_1(\partial_3\rho_1 + \partial_1\rho_3) + \rho_2(\partial_3\rho_2 + \partial_2\rho_3)]/2\,. \tag{4.3}$$

Putting the components of Γ^{*i}_{jk} in field equation (1.2) of Israel and Trollope unified field theory, we observed that it is satisfied under the condition

$$\partial_1\rho_1 + \partial_2\rho_2 + \partial_3\rho_3 = 0\,. \tag{4.4}$$

The field equation (1.3) of Israel and Trollope's unified field theory is also identically satisfied by substituting the values of $\theta^{[ij]}$ from (2.7).

5. Solutions of the field equations (1.4) and (1.5) in V_5

Using the components of Γ^{*i}_{jk} from (4.2) the non-vanishing symmetric components of $R^*{}_{ij}$ are obtained as follows:

$$R^*{}_{(44)} = -R^*{}_{(45)} = R^*{}_{(55)} = Q \tag{5.1}$$

where $Q = -\Delta f + 2(\partial_1\alpha_1 + \partial_2\alpha_2 + \partial_3\alpha_3) + \{(\partial_1\rho_1)^2 + (\partial_2\rho_2)^2 + (\partial_3\rho_3)^2 + (\partial_2\rho_1 + \partial_1\rho_2)^2/2$

$$+ (\partial_3\rho_1 + \partial_1\rho_3)^2/2 + (\partial_3\rho_2 + \partial_2\rho_3)^2/2\} - (\partial_1\bar{\rho}_1 + \partial_2\bar{\rho}_2 + \partial_3\bar{\rho}_3)\,. \tag{5.2}$$

Substituting the components of M_{ij} and $R_{(ij)} = R^*{}_{(ij)}$ from (3.7) and (5.1) respectively in the field equation (1.4), we obtain the following equation

$$Q = [(\rho_1^2 + \rho_2^2 + \rho_3^2)/\alpha] \tag{5.3}$$

which is satisfied under the condition

$$Q - [(\rho_1^2 + \rho_2^2 + \rho_3^2)/\alpha] = 0\,. \tag{5.4}$$

The equation (1.5) of Israel and Trollope's unified field theory is identically satisfied.

We observed that g_{ij} *given by* (3.4) *are the wave solutions of the set* [I] *of field equations of unified field theory of Israel and Trollope, in the five dimensional Peres space-time, under the conditions* (4.4) *and* (5.4).

The following section is dealt with the study of plane wave-like solutions of the second set of field equations of Israel and Trollope's unified field theory in five dimensional Peres space-time.

6. Solution of the set (II) of field equations in V_5

With the choice $bR=1$, in the second set of field equation, we observed that the components of θ^{ij}, g^{ij}, g_{ij}, $R^{[ij]}$, $R_{[ij]}$ are the same as in the case of field equations of set [I]. These components have already been found in the previous section. The field equations (1.6), (1.7) and (1.8) are the same as first three field equations in the set [I] and have already been solved in the earlier section.

Substituting the values of $R^{*}_{(ij)}$, g_{ij} and M_{ij} from (5.1), (3.4) and (3.7) respectively in the field equation (1.10) for $[ij=11,12,13,22,23,33]$, we find that $1=0$ which is absurd.

Therefore we have, in the second set of field equations of Israel and Trollope's unified field theory, five dimensional plane wave-like solutions in the space-time of Peres (2.1) *do not exist.*

Concluding Remark

In this paper we have carried out the five dimensional work of Jumale (2006) regarding plane wave-like solutions of the field equations of Einstein's non-symmetric unified field theory to the case of Israel and Trollope's unified field theory and obtained plane wave-like solutions in higher five dimensional Peres space-time on the lines of Srivastava (1974).

We observed that the field equation (1.1) of Israel and Trollope is equivalent to the first equation of Einstein's non-symmetric unified field theory and has already been solved by Jumale (2006). The field equation (1.2) of Israel and Trollope's unified field theory is satisfied identically for the values given in (4.2). The field equation (1.3) of Israel and Trollope's unified field theory is also identically satisfied by substituting the values of $\theta^{[ij]}$ from (2.7) and the equation (1.5) of Israel and Trollope is identically satisfied.

The work presented here is the generalization of the plane wave like solutions obtained by Srivastava (1974) in four dimensional Peres space-time. It has been observed that all the results obtained here in higher five dimension are in the format of Srivastava (1974) in four dimensional Peres space-time. Four dimensional work of Srivastava (1974) can be reproduced from the work carried out here by reducing the dimension'.

It is to be noted that there is scope to study the plane wave like solutions in the generalized Peres space-time proposed by Lal and Pandey (1974) as well as by A. Pradhan (1977) in higher dimensions. We think that these higher five dimensional plane wave-like solutions should bring some additional information and therefore, they need to be further investigated.

It is to be noted that in the second set of field equations of Israel and Trollope's unified field theory, five dimensional plane wave-like solutions in the space-time of Peres (2.1) do not exist.

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