

## Updated View about the Hierarchy of Planck Constants

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### Abstract

Few years has passed from the latest formulation for the hierarchy of Planck constant. The original hypothesis seven years ago was that the hierarchy is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of  $M^4$  and  $CP_2$ .

Few years ago came the realization that the hierarchy could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write  $\hbar_{eff} = n\hbar$  rather than  $\hbar = n\hbar_0$  as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. It was no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of  $M^4$  and  $CP_2$  but for some reason I kept this assumption.

In the recent formulation this assumption is made and the emphasis is on the interpretation of the multi-sheetedness (in the sense of Riemann surfaces) resulting as a multi-furcation for a preferred extremal taking place at the partonic 2-surfaces. This gives a connection with complexity theory (say in living systems), with transition to chaos, and with general ideas about fractality. Second quantization of the multi-furcation means accepting not only superpositions of branches as single particle states but also the analogs of many-particle states obtained by allowing several branches up to the maximum number. This revives the ideas of N-atom, N-molecule etc.. already given up as too adventurous.

The question whether gravitational Planck constant  $h_{gr}$  having gigantic values results as an effective Planck constant has remained open. A simple argument suggests that gravitational four-momentum could be identified as a projection of the inertial four-momentum to the space-time surface and that the square of the gravitational four-momentum obtained using the effective metric defined by the anti-commutators of the modified gamma matrices appearing in the modified Dirac equation naturally leads to the emergence of  $h_{gr}$ .

## 1 Introduction

During last years the work with TGD proper has transformed from the discovery of brave visions to the work of clock smith. The challenge is to fill in the details, to define various notions more precisely, and to eliminate the numerous inconsistencies.

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necessary to assume that the covering reduces to a Cartesian product of singular coverings of  $M^4$  and  $CP_2$  but for some reason I kept this assumption.

It seems that the time is ripe for checking whether some polishing of this formulation might be needed. In particular, the work with TGD inspired quantum biology suggests a close connection between the hierarchy of Planck constants and negentropic entanglement. Also the connection with anyons and charge fractionalization has remained somewhat fuzzy [5]. In particular, it seems that the formulation based on multi-furcations of space-time surfaces to  $N$  branches is not general enough: the  $N$  branches are very much analogous to single particle states and second quantization allowing all  $0 < n \leq N$ -particle states for given  $N$  rather than only  $N$ -particle states looks very natural: as a matter fact, this interpretation was the original one and led to the very speculative and fuzzy notion of  $N$ -atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of  $N$ -nuclei,  $N$ -atoms, and  $N$ -molecules.

## 2 Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [7].
2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order  $CP_2$  size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain:  $E = hf$  implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [5] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also  $\hbar_{gr}$  corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [1] who first introduced the notion of gravitational Planck constant as  $\hbar_{gr} = GMm/v_0$ ,

$v_0 < 1$  has interpretation as velocity light parameter in units  $c = 1$ . This would be true for  $GMm/v_0 \geq 1$ . The interpretation of  $\hbar_{gr}$  in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses  $M$  and  $m$ . The huge value of  $\hbar_{gr}$  means that the integer  $\hbar_{gr}/\hbar_0$  interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

It must be however emphasized that the interpretation of  $\hbar_{gr}$  could be different, and it will be found that one can develop an argument demonstrating how  $\hbar_{gr}$  with a correct order of magnitude emerges from the effective space-time metric defined by the anticommutators appearing in the modified Dirac equation.

4. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths  $\alpha = g^2/4\pi\hbar$ . If the effective value of  $\hbar$  replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle,  $\alpha$  is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter  $GMm/\hbar$  has gigantic value. Replacing  $\hbar$  with  $\hbar_{gr} = GMm/v_0$  the coupling strength becomes  $v_0 < 1$ .

### 3 Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of  $M^4$  and  $CP_2$  with numbers of sheets given by integers  $n_a$  and  $n_b$  and  $\hbar = n\hbar_0$ .  $n = n_a n_b$ .

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded  $M^4$  in  $M^4 \times CP_2$  have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of  $CP_2$  coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents  $\partial L_K/\partial(\partial_\alpha h^k)$  defining the modified gamma matrices [10] and gradients  $\partial_\alpha h^k$  is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of  $CD$  carrying the elementary particle quantum numbers this implies that the two normal derivatives of  $h^k$  are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to  $N$  branches  $b_i$  of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches  $b_i$  and  $b_j$  of multi-furcation.  $N$ -particle state would correspond to  $N$ -sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization  $N = n_a n_b$  occurs but now

$n_a$  and  $n_b$  would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than  $M^4$  and  $CP_2$  as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only  $N$ -sheeted covering corresponding to a situation in which all  $N$  branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single  $n$ -sub-furcations of  $N$ -furcation is selected. The most general state of this kind involves superposition of various  $n$ -sub-furcations.

## 4 Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.
2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.
3. In the case of massless particles the scaling of wavelength in the effective scaling of  $\hbar$  can be understood if dark  $n$ -photons consist of  $n$  photons with energy  $E/n$  and wavelength  $n\lambda$ .
4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

On the other hand, p-adic thermodynamics predicts that massive elementary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the  $n$ -electron has same mass as electron, the mass for dark single electron state would be scaled down by  $1/n$ . This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

This suggests that for fermions the basic scaling rule does not hold true for Compton length  $\lambda_c = \hbar/m$ . Could it however hold for de-Broglie lengths  $\lambda = \hbar/p$  defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an  $1/N$ -fold reduction of density that takes place in the delocalization of the single particle states to the  $N$  branches of the cover, implies that the volume per particle increases by a factor  $N$  and single particle wave function is delocalized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would

increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling  $\hbar \rightarrow k\hbar$  in the formula  $E_n = (n + 1/2)\hbar eB/m$  implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have  $k$ -particle state formed from cyclotron states in  $N$ -fold branched cover of space-time surface. Each branch would carry magnetic field  $B$  and ion or electron. This would give a total cyclotron energy equal to  $kE_n$ . These cyclotron states would be excited by  $k$ -photons with total energy  $E = khf$  and for large enough value of  $k$  the energies involved would be above thermal threshold. In the case of  $Ca^{++}$  one has  $f = 15$  Hz in the field  $B_{end} = .2$  Gauss. This means that the value of  $\hbar$  is at least the ratio of thermal energy at room temperature to  $E = hf$ . The thermal frequency is of order  $10^{12}$  Hz so that one would have  $k \simeq 10^{11}$ . The number branches would be therefore rather high.
2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of  $k$  photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of  $N$ -furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be  $n = 2$ -particle states associated with  $N$ -furcation.

There are experimental findings suggesting that photosynthesis involves delocalized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement automatically.
2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark  $n$ -photons exciting all  $n$  electrons simultaneously.  $n$ -photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to  $n$ -photons in  $N$ -furcation in biosphere.
3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore  $n = 1$  dark photons delocalized to the branches of the  $N$ -furcation. They would induce delocalized single electron excitation in WCW rather than 3-space.

## 5 Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by  $n$ . This corresponds effectively to the scaling  $\alpha_K \rightarrow \alpha_K/n$  induced by the scaling  $\hbar_0 \rightarrow n\hbar_0$ .

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now

the same happens at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in  $E^3$  are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of  $N$  sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge  $q/N$  for teh analogs of plane waves.

Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability  $p = 1/N$  from which one can deduce that charge is  $q/N$ .

2. This is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionalization and fractionalization of spin.
3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionalization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through  $2\pi$  at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and  $N + 1$ :th branch corresponds to the original one. This suggests that angular momentum fractionalization should take place for  $M^4$  angle coordinate  $\phi$  because for it  $2\pi$  rotation could lead to a different sheet of the effective covering.

The orbital angular momentum eigenstates would correspond to waves  $\exp(i\phi m/N)$ ,  $m = 0, 2, \dots, N-1$  and the maximum orbital angular momentum would correspond the sum  $\sum_{m=0}^{N-1} m/N = (N-1)/2$ . The sum of spin and orbital angular momentum be therefore fractional.

The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by  $2\pi$  does nothing for the 3-surface. Hence fractionalization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionalization however leads to problems with fractionalization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

## 6 Negentropic entanglement between branches of multi-furcations

The application of negentropic entanglement and effective hierarchy of Planck constants to photosynthesis and metabolism [3] suggests that these two notions might be closely related. Negentropic entanglement is possible for rational (and even algebraic) entanglement probabilities. If one allows number theoretic variant of Shannon entropy based on the p-adic norm for the probability appearing as argument of logarithm [4], it is quite possible to have negative entanglement entropy and the interpretation is as genuine information carried by entanglement. The superposition of state pairs  $a_i \otimes b_i$  in entangled state would represent instances of a rule. In the case of Schrödinger cat the rule states that it is better to not open the bottle: understanding the rule consciously however requires that cat is somewhat dead! Entanglement provides information about the relationship between two systems. Shannon entropy represents lack of information about single particle state.

Negentropic entanglement would replace metabolic energy as the basic quantity making life possible. Metabolic energy could generate negentropic entanglement by exciting biomolecules to negentropically entangled states. ATP providing the energy for generating the metabolic entanglement could also itself carry negentropic entanglement, and transfer it to the target by the emission of large  $\hbar$  photons.

How the large  $\hbar$  photons could carry negentropic entanglement?

1. In zero energy ontology large  $\hbar$  photons could carry the negentropic entanglement as entanglement between positive and negative energy parts of the photon state.
2. The negentropic entanglement of large  $\hbar$  photon could be also associated with its positive or energy part or both. Large  $\hbar_{eff} = n\hbar$  photon with  $n$ -fold energy  $E = n \times hf$  is  $n$ -sheeted structure consisting of  $n$ -photons with energy  $E = hf$  delocalized in the discrete space formed by the  $N$  space-time sheets. The  $n$  single photon states can entangle and since the branches effectively form a discrete space, rational and algebraic entanglement is very natural. There are many options for how this could happen. For instance, for  $N$ -fold branching the superposition of all  $N!/(N-n)!n!$  states obtained by selecting  $n$  branches are possible and the resulting state is entangled state. If this interpretation is correct, the vacuum degeneracy and multi-furcations implied by it would be the quintessence of life.
3. The identification of negentropic entanglement as entanglement between branches of a multi-furcation is not the only possible option. The proposal is that non-localized single particle excitations of cyclotron condensate at magnetic flux tubes give rise to negentropic entanglement relevant to living matter. Dark photons could transfer the negentropic entanglement possibly assignable to electron pairs of ATP molecule.
4. The negentropic entanglement associated with cyclotron condensate could be associated with the branches of the large  $\hbar$  variant of the condensate. In this case single particle excitation would not be sum of single particle excitations at various positions of 3-space but at various sheet of covering representing points of WCW. If each of the  $n$  branches carries  $1/n$ th part of electron one would have an anyonic state in WCW.
5. One can also make a really crazy question. Could it be that ATP and various bio-molecules form  $n$ -particle states at the  $n$ -sheet of  $N$ -furcations and that the bio-chemistry involves simultaneous reactions of large numbers of biomolecules at these sheets? If so, the chemical reactions would take place as large number of copies.

Note that in this picture the breaking of time reversal symmetry [2] in the presence of metabolic energy feed would be accompanied by evolution involving repeated multi-furcations leading to increased complexity. TGD based view about the arrow of time implies that for a given  $CD$  this evolution has definite direction of time. At the level of ensemble it implies second law but at the level of individual system means increasing complexity.

## 7 Dark variants of nuclear and atomic physics

During years I have in rather speculative spirit considered the possibility of dark variants of nuclear and atomic - and perhaps even molecular physics. Also the notion of dark cyclotron state is central in the quantum model of living matter. One such notion is the idea that dark nucleons could realize vertebrate genetic code [8].

Before the real understanding what charge fractionalization means it was possible to imagine several variants of say dark atoms depending on whether both nuclei and electrons are dark or whether only electrons are dark and genuinely fractionally charged. The recent picture however fixes these notions completely. Basic building bricks are just ordinary nuclei and atoms and they form  $n$ -particle states associated with  $n$ -branches of  $N$ -furcation with  $n = 1, \dots, N$ . The fractionalization for a single particle state delocalized completely to the discrete space of  $N$  branches as the analog of plane wave means that single branch carries charge  $1/N$ .

The new element is the possibility of  $n$ -particle states populating  $n$  branches of the  $N$ -furcation: note that there is superposition over the states corresponding to different selections of these  $n$  branches.  $N - k$

and  $k$ -nuclei/atoms are in sense conjugates of each other and they can fuse to form  $N$ -nuclei/ $N$ -atoms which in fermionic case are analogous to Fermi sea with all states filled.

Bio-molecules seem to obey symbolic dynamics which does not depend much on the chemical properties: this has motivated various linguistic metaphors applied in bio-chemistry to describe the interactions between DNA and related molecules. This motivated the wild speculation was that  $N$ -atoms and even  $N$ -molecules could make possible the emergence of symbolic representations with  $n \leq N$  serving as a name of atom/molecule and that  $k$ - and  $N - k$  atom/molecule would be analogous to opposite sexes in that there would be strong tendency for them to fuse together to form  $N$ -atom/-molecule. For instance, in bio-catalysis  $k$ - and  $N - k$ -atoms/molecules would be paired. The recent picture about  $n$  and  $N - k$  atoms seems to be consistent with these speculations which I had already given up as too crazy. It is difficult to avoid even the speculation that bio-chemistry could replace chemical reactions with their  $n$ -multiples. Synchronized quantum jumps would allow to avoid the distasteful effects of state function reductions on quantum coherence. The second manner to say the same thing is that the effective value of Planck constant is large.

## 8 What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression  $\hbar_{gr} = GMm/v_0$ , where  $v_0 < 1$  has interpretation as velocity parameter in the units  $c = 1$ . Can one interpret also  $\hbar_{gr}$  as effective value of Planck constant so that its values would correspond to multifurcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for  $\hbar_{gr}$ ? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of  $\hbar_{gr}$  naturally?

1. Gravitational four-momentum can be defined as a projection of the  $M^4$ -four-momentum to space-time surface. It's length can be naturally defined by the effective metric  $g_{eff}^{\alpha\beta}$  defined by the anticommutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the modified Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of  $CD$  and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.
2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the  $M^4$  metric or rather - to its  $M^2$  projection:  $g_{eff}^{kl} = K^2 m^{kl}$ .

One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses  $M$  and  $m$  as

$$g_{eff}^{\alpha\beta} p_\alpha p_\beta = g_{eff}^{\alpha\beta} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl} p_k p_l = n^2 \frac{\hbar^2}{L^2} . \quad (8.1)$$

Here  $L$  would correspond to the length of the flux tube mediating gravitational interaction and  $p_k$  would be the momentum flowing in that flux tube.  $g_{eff}^{kl} = K^2 m^{kl}$  would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} .$$

$\hbar_{gr}$  could be identified in this simplified situation as  $\hbar_{gr} = \hbar/K$ .



3. Nottale's proposal requires  $K = GMm/v_0$  for the space-time sheets mediating gravitational interacting between massive objects with masses  $M$  and  $m$ . This gives the estimate

$$p_{gr} = \frac{GMm}{v_0} \frac{1}{L} . \quad (8.2)$$

For  $v_0 = 1$  this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude.  $v_0$  is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of  $v_0$  to  $v_0 \simeq 2^{-11}$  in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value  $GMm/v_0$ . Einstein's equations  $T = \kappa G + \Lambda g$  give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of  $h_{gr}$  approaches infinity. At the flux tubes mediating gravitational interaction one expects  $T$  to be proportional to the factor  $GMm$  simply because they mediate the gravitational interaction.
5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta} L_\alpha L_\beta = g_{eff}^{kl} L_k L_l = l(l+1)\hbar^2 . \quad (8.3)$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1) \frac{\hbar^2}{K^2} . \quad (8.4)$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

One might counter argue that if gravitational 4- momentum square is proportional to inertial 4- momentum squared, then Equivalence Principle implies that  $h_{gr}$  can have only single value. In ZEO however all wormhole throats - also virtual - are massless and the argument fails. The varying  $h_{gr}$  can be assigned only with longitudinal and transversal momentum squared separately but not to the ratio of gravitational and inertial 4-momenta squared which both vanish.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between  $m_{eff}^{kl} = Km^{kl}$  could make sense as a quantum average. Also the fact, that the constant  $v_0$  varies, could be understood from the dynamical character of  $m_{eff}^{kl}$ .

## 9 Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and the hierarchy is only effective. Anyonic physics and effective charge fractionalization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [10]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level, to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [9] so that conformal dynamics represents conformal evolution) [6].

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