Article

Does the Sum Rule Hold at the Big Bang?

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Abstract

In Dice 2010 Sumati Surya brought up a weaker Quantum sum rule as a biproduct of a quantum invariant measure space. Our question is stated as follows: Does it make sense to have disjointed sets to give us quantum conditions for a measure at the origin of the big bang? We argue that the answer is no, which has implications as to quantum measures and causal set structure. What is called equation (1) in the text requires a length, and interval, none of which holds at a point in space-time.singularity. What are the reasons? First, measurable spaces allow disjoint sets. Also, that smooth relations alone do not define separability or admit sets Planck's length, if it exists, is a natural way to get about the 'bad effects' of a cosmic singularity at the beginning of space-time evolution, but if a new development is to believed, namely by Stoica in the article, about removing the cosmic singularity as a break down point in relativity, there is nothing which forbids space-time from collapsing to a point. If that happens, the cautions as to no disjoint intervals at a point raise the questions as to the appropriateness of Surya's quantum measure. Since Stoica's re scaling of pressure and density involve the cube of the scale factor, a, the differentiability and smoothness issues of the Friedman and acceleration equation vanish, leading to problems with the Hausdorf limiting cases for disjoint open sets, which makes quantum vector meaures not feasibile, due to vanishing of disjoint sets, as we approach a point in space-time. The final conclusion is that the intial singularity has to be embedded into higher dimensions, as in String theory due to 4 dimensional problems with quatum measures which in themselves in four dimensions break down.

Keywords: quantum measures, spatial diffeomorphism, cylinder sets, Caratheodary-Hahn – Huvanek theorem, Big Bang singularity, causal sets.

I. Introduction

We consider in this paper whether singularity behavior in space-time can be affected by co-ordinate choices[1].

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[1] As noted by one of the Reviewers: "one can indeed look at the induced metric at light-cone boundary[. It] is just r_M^2dOmega^2. There is nothing pathological in its topology nor in the topology of future future lightcone when seens as parts of Minkowski space,[p]erfectly well defined but metrically 2-D since radial direction gives no contribution as a light-like direction. This example suggests that one should be extremely cautious in considerations related to singularity: so much depends on a choice of proper coordinates. In fact, holography suggests that one should consider light-cone boundary or initial singularity as sub-manifold of space-time."

A proper choice of coordinates is going to involve more than four dimensions and that what is chosen in four-dimensional space-time usually in the Roberson Walker metric will lead to a singularity problem. We claim that this affects the quantum mesure problem in four dimensions. The main point of the article is below where we outline how to fix the glaring problems in four-dimensional measures theory which we state as unphysical.

We note here that higher dimensions will, as in String theory, remove this problem. We wish to reconcile the four and higher dimensional examples of co ordinate behavior and reflect upon what the four dimensional representation does to quantum mesures, especially if there is a removal of the standard four dimensional representation of a mathematical singularity at the start of inflation. To do this, we will give an argument which will point in the direction of vanishing of disjoint shets in four dimensions leading to a break up of the quantum measure in four dimensions.

Our initial goal is to show that disjoint sets, are due to separability in a topological sense, and that at a point in space – time, that the very notion of separability breaks down completely [1].

Separability in a topological sense can be constructed as follows. A topological space X is said to be separable if X has a countable dense subset. In other words, there is a countable subset D of X such that closure (D) = X.

Equivalently, each nonempty open set in X intersects D. The fact is, that if there is a space – time point, that the countable subset D of X is such that the closure (D) = X. breaks down completely.

Afterwards, we should note that disjoint sets in a topological space, X, are due to working with X being a Hausdorff space.We then note the properties of Hausdorf spaces can be written follows:

1. If K is a compact subset of X and $y \in X$ is a point outside of K then y and K have disjoint neighborhoods, i.e. there exist an open neighborhood W_y of y and an open set $V_y \supset K$ for which $W_y \cap V_y = \phi$
2. Every compact subset of X is closed.
3. Any two disjoint compact subsets of X have disjoint open neighborhoods, i.e. if C and D are compact disjoint subsets of X, then there exist open sets $U \supset C$ and $V \supset D$ for which $U \cap V = \phi$

Note that when one has a point in space time, the there is not a comparable construction to closure (D) = X. or $U \cap V = \phi$.

This lack of having at a point in space- time a topological set X with open subsets with these constcutions dooms having these properties. I.e if one does not have a Hausdorff space, one is going to find it impossible to form disjoint sets in a separable X if X is itself a point

When one does not have separable sub sets, at a single point, then the construction used by [1] for quantum measures breaks down. We review in Appendix A what happens due to Stoica's [2] treatment of the Friedman and acceleration equations and show it implies a smoothness condition which eliminates disjoint sets at a point, entirely. i.e. no pressure, density and scale factor.

It is noted that we are working with the formalism introduced by Surya [32] and submit that it breaks down at a singularity. The sum rule in particular in Eq. (1) will break down if there is no length, or specified interval. The reason for that break down is that there is nothing to measure, at a perfect point of space-time. Surya's paper [32] has at its end speculations as to how to avoid this issue, but the fact remains by elemetrary measure theory, as given by Hamos [4] that a measure requires intervals, and an interval does not exist at a perfect singularity. If one wants to have a measure zero object, that is fine, but a measure zero entity itself is not sufficent to justify a sum rule, as given in equation (1) which will be addressed later. Furthermore, the existence of another new paper by Stoica [5] removes the cosmic singularity at the start of the big bang as a mandatory break down point of general relativity.

While the existence of the pathological singularity can be treated by use of Planck's length, which can be used to construct disjoint sets, if Stoica is believable, this Planck's length is no longer essential,which brings up interesting questions so far avoided by main stream cosmologists. This paper merely brings up that issue, and asks what can be done to correct for it, at the point of the big bang. To do this, we revisit what happened in Surya's paper [3] in the DICE 2010 conference, and make a few suggestions of our own afterwards. Appendix A summarizes how Surya built up her qauntum measures and is mandatory reading for those wishing to understand how quantum measures are built up outside the point regime so specified by Surya which is clamied to break down in usual singularity regimes at the origin of the big bang[2].

Our contribution is to examine quantum measures assuming a non-string theory treatment of cosmology. And to arguem that the break down of a quantum measure in four dimensions necessitates use of higher dimensional embedding of the start of cosmologicla inflation.

II. Aftermath of Construction of the Spatial Diffeomorphism Leading to Quantum Measures

The main point of the formalism for Appendix B is of bi-additivity of D leading to the finite addivity of μ_V. The author asks readers to go to Appendix B to see the construction leading to the following equation, which in its creation uses disjoint sets, in an interval [4]

[2] The Reviewer noted that "[i]n Einstein's equations for RW cosmology of course rho, p, and scale factor a are scalar invariants which become infinite at the initial moment. This is a real physical singularity. In cosmic string dominated primordial cosmology however the mass per comoving volume vanishes at the singularity like a so that in this sense everything is non-singular. As mentioned Stoica shows that the equations can be redefined to get well-defined equations also at the singularity."

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right)=\sum_{i=1}^{n}\mu_V\left(\alpha_i\right) \qquad (1)$$

The use of finite additivity of μ_V is essential to the quantum measure prospect and in Appendix Binheriently involves use of disjoint sets. The reason for stating this shows up in the next section, C. We leave the issue of if a Planck's length is mandatory for initial cosmology to the conclusion with our own point of view. Should the existence of Planck's length be mandatory due to space-time evolution, then there is no question that (1) holds.

III. Arguments against Eq. (1) in the Vicinity/ Origin of the Big Bang Singularity

The main problem, as the author sees it, is insuring the existence of disjoint sets at a point of space-time. If one views a finite, infinitely small region of space-time, as given by Plank's interval as 1.616 times 10 ^-35 meters as contravening a space-time singularity, in relativity, then even in this incredibily small length, there can be disjoint sets, and then the math construction of Surya[3] goes through verbatim. Classical relativity theory though does not have a Planck interval, i.e. the singularity of space-time, so in effect in General relativity in its classical form will not have the construction so alluded to in (1) above. [5] written by Cristi Stoica gives a view of a beginning of space-time starting that does away completely with the space-time singularity, so mathematically, in a cosmos as constructed, if there is no singularity problem, there is then no restriction as to the collapse of space-time to an infinitely small point. In which then there would be no reason to appeal to a Planck's length graniness of space-time to enforce some rationality in the behavior of (quantum?) cosmology.

The precondition for a quantum measure μ_V for a quantum measurement is given by Eq. (1) [3] for n disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$. This Eq. (1) is a math precondition for μ_V being a vector measure over A. Eq (1) right at the point of the big bang cannot insure the existence of n disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$. Therefore at the loci of the big bang one would instead get, due to non-definable disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$, a situation definable as, at best.

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right)\neq\sum_{i=1}^{n}\mu_V\left(\alpha_i\right) \qquad (2)$$

Not being able to have a guarantee of having n disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$ because of singular conditions at the big bang will bring into question whether equation (1) can hold and the overall research endeavor of analyzing the existence of quantum measures μ_V. I.e., the triple $\left(\Omega, A, \mu_V\right)$ for quantum measures μ_V cannot be guaranteed to exist. Especially if there is no bar to a singularity existing as given by [5] And we look at whether there is sufficiently convergent behavior for μ_V, so that uniqueness of convergent sequenes is guaranteed by the Caratheodary-

Hahn –Huvanek theorem. If so, the following supremum expression for all FINITE partitions will lead to the equality expression for vector measures. This is what becomes very problematic if [5] is true about non pathological consequences of a BB singularity.

$$\left|\mu_V(\alpha)\right| = \sup_{\pi(\alpha)} \sum_{\rho} \left\|\mu_V(\alpha_\rho)\right\| \tag{3}$$

The singularity will not allow us to analyze disjoint partitions. What happens if instead of Eq. (3) [3] a situation for which there is longer finite partitions, ordered sets, but the replacement for Eq. (3) is now an inequality written as:

$$\left|\mu_V(\alpha)\right| \neq \sup_{\pi(\alpha)} \sum_{\rho} \left\|\mu_V(\alpha_\rho)\right\| \tag{4}$$

Or worse, a situation where there is no finite partially ordered set, i.e., no *cuasal* set? The inequality of Eq.(4) can occur if there is no finite disjoint sets to make a supremum over.

Eq. (1) depends upon having [3] an "*unconditional convergence of the vector measure over all partitions.*" Replace partitions with causal set structure, and one still has the same requirement of an *unconditional convergence of the vector set over all "causal set structure"* within a finite geometric regime of space-time. One does not get about the necessity of conversgence of sequences and sub sequences in a causal set structure. The convergence of sequences and sub sequences has the same rules as when causal set structure is replaced by partitions.Surya's construction [3] of taking a least upper bound (supremum) over finite partitions does not work if there are no finite partitions at a singularity.

VI. Conclusions

References [3] and [5] together suggest a way out of the impasse. First of all, the question we need to ask is, is the existence of a Planck length, as a minimum length mandatory as to space-time? If it is, the problem of the existence of disjoint intervals is solved. I.e. we need not worry, even if it is 10^-35 meters in length. If this minimum length exists, (1) holds everywhere.

If a mandatory minimum non-zero space-time interval is necessary then there is nothing which forbids the existence of (1) above. If such an interval does not exist, then (1) breaks down. Furthermore, the space of all infinitely differentiable functions is also separable, and a fundamental sequence is the sequence of all powers of x. This is shown by Taylor series and Weierstrass's theorem [1] . But having either Weirtrass theorem or Taylor series at a single point of space-time is a non starter, and also the dodge of using the simplification of a finite dimensional normed space breaks down. No longer at a point can, many of the computations be simplified by the existence of a finite basis, where every vector in the space is a linear combination of some subset of vectors in the basis. **One does not have a finite basis in a point of space time** [1].

It should be noted that Connes [6] outlines conditions for non commutative geometry in space-time for the development of exotic basis which in higher dimensions could restore separatable space, i.e. even Hausdorff behavior, as would be necessary for disjoint sets to exist. But such a development would be involving encasing the four dimensional singularity as embedded in a hierarchy of higher dimensional geometric spaces. With 3 dimensional space and time at a singular point, one does not have a Hausdorf metric sopace X, separatability and without having either of the above, then the construction for a quantum measure, as outlined and developed in the given Appendix A will not work out.

In essence, for making a consistent cosmology, our results argue in favor of a string theory style embedding of the start of inflation and what we have argued so far is indicating how typical four dimensional cosmologies have serious mathematical measure theoretic problems. These quantum measure theoretic probem are unphysical espeecially in light of the Stoica findings. [5]

Acknowledgements: I thank Johnathan Dickau for valuable discussions and assistance with this paper and one of the Reviewers for his astute observations quoted in the footnotes. I am deeply debted to my recently deceased father who discussed decades ago with me as to the worth of five-dimensional gemoetry, in cosmology. I further thank the Department of Physics, University of Chongqing, P.R. China, for affiliation, and Dr. Fangyu Li, who encouraged me to explore such issues. This work is supported in part by National Nature Science Foundation of China grant No. 11075224.

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Appendix A

Stoica [2] does a re-scaling of the pressure and density along t he following lines, namely the initial Friedman equation is changed i.e. it starts with

$$\rho = \frac{3}{\kappa} \cdot \frac{\dot{a}^2 + k}{a^2} \quad (1a)$$

Furthermore we also have the accelleration equation given by

$$\rho + 3p = -\frac{6}{\kappa} \cdot \frac{\ddot{a}}{a^2} \quad (2a)$$

Using the re-scaling of [2] using Σ as part of a 'typical space'

$$\det g = -a^6 \det_3 g\Sigma \Leftrightarrow \sqrt{-g} = a^3 \sqrt{g\Sigma} \quad (3a)$$

We then re-scale the density and also the pressure as follows:

$$\begin{aligned} \tilde{\rho} &= \rho a^3 \sqrt{g\Sigma} \\ \tilde{p} &= p a^3 \sqrt{g\Sigma} \end{aligned} \quad (4a)$$

This will lead to

$$\begin{aligned} \tilde{\rho} &= \frac{a}{\kappa}\left(\dot{a}^2 + k\right)\sqrt{g\Sigma} \\ \tilde{\rho} + 3\tilde{p} &= -\frac{6}{\kappa} \cdot a^2 \ddot{a} \sqrt{g\Sigma} \end{aligned} \quad (5a)$$

The upshot is, as explained in [2] that then;

$$\begin{aligned} a(0) = 0 &\Leftrightarrow \tilde{\rho}(0) = \rho a(0)^3 \sqrt{g\Sigma} = 0 \\ a(0) = 0 &\Leftrightarrow \tilde{p}(0) = p a(0)^3 \sqrt{g\Sigma} = 0 \end{aligned} \quad (6a)$$

So then the acceleration equation and Friedman equation vanish at $a(0) = 0$

Appendix B

We introduce the formalism by appealing to the concept of spatial diffeomorphism [4] as a necessary condition for linking the physics of what happens at a singularity to outside of the singularity of inflation generated space time geometry. Trivially, a diffeomorphism involves an infinitely differentiable, one-to-one mapping of the model to itself. In contrast, there is a breakdown of differentiability at the start of the big bang, based on non-loop-quantum-gravity theories.

We submit that the difficulties in terms of consistency of Eq. (1) of this document. In terms of initial causal structural breakdown -- which we claim leads to Eq. (1) being re written as an inequality -- one has to come up with a different way to embed quantum measures within a superstructure, as noted in the conclusions of this paper. Spatial diffiomorphisms as stated in [4] do not work unless there is a lattice structure, effectively doing away with a singularity. If the lattice structure is not used, differentiability breaks down and one does not have one-to- one mapping of the physics of the big bang singularity onto the rest of the inflationary process. We submit that this breakdown would then make Eq. (b1) not definable.As to the measure set structure, the readers are referred to [4] to get the foundations of the measure theory structure understood. The rest of this text is an adoptation of what was done in [3], with the author's re interpretation of what the significance is of quantum measures as stated in [3], in the vicinity of a singularity.

The author's main point is that there is a break down of measurable structure, starting with definitions given in [3] and [4] where the concept of disjoint sets becomes meaningless in a point of space. In the causal set approach, the probabilities are held to be Markovian [3], label-independent and adhere to Bell's inequality. The author of [3] refers to a sequential growth called a classical transition percolation model. Then reference [3] extends the classical transition percolation model to complex models involving quantum measures in the definition of a (quantum) complex percolation model. Reference [3] defines the amplitude of transition as follows. For a quantum measure space defined as triple as given by (Ω, A, μ_V), with μ_V a yet to be defined vector measure, A is an event algebra or set of propositions about the system, and Ω is the sample space of histories or space-time configurations.

Let $p \ \varepsilon \ C$ be amplitude of transition, instead of a probability; and set $\psi(C^n)$ as the amplitude for a transition from an empty set to n element of a causal set C^n, and with $Cyl(C^n)$ cylinder set as a subset of Ω containing labeled past finite causal sets whose first n elements form the causal subset C^n. Note that the cylinder sets form event algebra A with measure given by form the sub-causal set C^n. Here, ψ is a complex measure on A, so then ψ is a vector measure [3]. This is the primary point of breakdown that occurs in the case of a space time singularity. Away from the singularity we will be working with the physics of

$$D\left(Cyl\left(C^n\right), Cyl\left(C'^n\right)\right) = \psi^*\left(C^n\right)\psi\left(C'^n\right) \qquad \text{(b1)}$$

This is done for a cylinder set [3], where γ is a given path, and γ^t as a truncated path, with $cyl\left(\gamma^t\right)$ a subset of Ω and $\mu\left(cyl\left(\gamma^t\right)\right)=P\left(\gamma^t\right)$, with $P\left(\gamma^t\right)$ the probability of a truncated path, with a given initial $\left(x_i,t_i\right)$ to final $\left(x_f,t_f\right)$ spatial and times. Note that the μ measure would be for $\mu: A \to R^+$ obeying the weaker Quantum sum rule [7]

$$\mu(\alpha\cup\beta\cup\gamma)=\mu(\alpha\cup\beta)+\mu(\alpha\cup\gamma)+\mu(\beta\cup\gamma)-\mu(\alpha)-\mu(\beta)-\mu(\gamma) \qquad \text{(b2)}$$

This probability would be a quantum probability which would *not* obey the classical rule of Kolmogrov [3]

$$P\left(\gamma_1\cup\gamma_2\right)=P\left(\gamma_1\right)+P\left(\gamma_2\right) \qquad \text{(b3)}$$

The actual probability used would have to take into account quantum interference. That is due to Eq. (1b) and Kolmogrov probability no longer applying, leading to [1]

$$cyl\left(\gamma^t\right)\equiv\left\{\gamma\in\Omega\,|\,\gamma(t')=\gamma^t(t')\ for\quad all\quad 0\le t'\le t\right\} \qquad \text{(b4)}$$

Here, $D: \mathrm{A}\times\mathrm{A}\to C$ is a decoherence functional [1], which is (i) Hermitian, (ii) finitely biadditive, and (iii) strongly additive [8], i.e., the eignvalues of D constructed as a matrix over the histories $\left\{\alpha_i\right\}$ are non-negative.A quantum mesurement is then defined via

$$\mu(\alpha)=D(\alpha,\alpha)\ge 0 \qquad \text{(b5)}$$

A quantum vector measurment is defined via

$$\mu_V(\alpha):=[\chi_\alpha]\in H \qquad \text{(b6)}$$

Where

$$\chi_\alpha(\beta)=\begin{cases}1\\0\end{cases}\quad,\qquad \chi_\alpha(\beta)=\ 1\text{ if }\quad \beta=\alpha,\ \chi_\alpha(\beta)=0\text{ if }\quad\beta\ne\alpha \qquad \text{(b7)}$$

Also V is the vector space over A with an inner product given by

$$\langle u,v\rangle_V\equiv\sum_{\alpha\in A}\sum_{\beta\in A}u^*(\alpha)v(\beta)\cdot D(\alpha,\beta) \qquad \text{(b8)}$$

with a Hilbert space H constructed by taking a sequence of Cauchy sequences $\left\{u_i\right\}$ sharing an equivalence relationship

$$\left\{u_i\right\}\sim\left\{v_i\right\}\text{ if }\ \lim_{i\to\infty}\left\|u_i-v_i\right\|_V=0 \qquad \text{(b9)}$$

So then as given in [1], the following happens,

$$\left[\left\{u_i\right\}\right]+\left[\left\{v_i\right\}\right]\equiv\left[\left\{u_i+v_i\right\}\right] \quad \text{(b10)}$$

$$\left[\left\{\lambda u_i\right\}\right]\equiv\lambda\left[\left\{u_i\right\}\right] \quad \text{(b11)}$$

$$\left\langle\left[\left\{u_i\right\}\right],\left[\left\{v_i\right\}\right]\right\rangle\equiv\lim_{i\to\infty}\left\langle u_i,v_i\right\rangle_V \quad \text{(b12)}$$

This is for all $\left[\left\{u_i\right\}\right],\left[\left\{v_i\right\}\right]\in H$ and $\lambda\in C$ so then the quantum measure is defined for $\mu_V : A\rightarrow H$ so the inner product on H is

$$\left\langle\mu_V\left(\alpha\right),\mu_V\left(\beta\right)\right\rangle=D\left(\alpha,\beta\right) \quad \text{(b13)}$$

b

The claim associated with Eq. (b1) above is that since ψ is a complex measure of A , Eq. (b1) corresponds to an unconditional convergence of the vector measure over all partitions. Secondly according to the Caratheodary-Hahn theorem there is unconditional convergence for classical stochastic growth, but this is not necessarily always true for a quantum growth process.

The main point of the formalism for Eq. (b13) is of bi-additivity of D leading to the finite addivity of μ_V

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right)=\sum_{i=1}^{n}\mu_V\left(\alpha_i\right) \quad \text{(b14)}$$