Article

Kantowaski-Sachs Dark Energy Model in *f(R,T)* Gravity

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Abstract

The dark energy model with EoS parameter are derived for Kantowaski-Sachs space-time filled with perfect fluid source in the frame work of $f(R,T)$ gravity (Harko et al., arXiv:1104.2669v2 [gr-qc], 2011). To obtain a determinate solution special law of variation for Hubble's parameter proposed by Berman [(NuvoCimentoB, 74,183(1983)] is used. We have also assumed that the scalar expansion is proportional to shear and the EoS parameter is proportional to skewness parameter. In fact, the possibility of reconstruction of the Kantowaski-Sachs cosmology with an appropriate choice of a function $f(T)$ has been proved in $f(R,T)$ gravity. It is observed that the EoS parameter, skewness parameters in the model turn out to be functions of cosmic time. Some physical and kinematical properties of the model are also discussed.

Keywords: Dark energy, Constant deceleration parameter, *f(R,T)* gravity, Kantowaski-Sachs space-time.

1. Introduction

Cosmological data from a wide range of sources have indicated that our Universe is undergoing an accelerating expansion [1–3]. Basically, two kinds of alternative explanations have been proposed for this unexpected observational phenomenon. One is the dark energy with a sufficient negative pressure, which induces a late-time accelerating cosmic expansion. Currently, there are many candidates of dark energy, such as the cosmological constant, quintessence, phantom, quintom, and so on. The other is the modified gravity, which originates from the idea that the general relativity is incorrect in the cosmic scale and therefore needs to be modified. Noteworthy amongst them are $f(R)$ theory of gravity formulated by Nojiri and Odintsov [4] and $f(R,T)$ theory of gravity proposed by Harko et al. [5].

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The idea of introducing additional terms of the Ricci scalar to the Einstein-Hilbert action did not begin years ago with the $f(R)$ gravity paper by Carroll et. al.[6]. Recently, Harko et al. [5] generalized $f(R)$ gravity by introducing an arbitrary function of the Ricci scalar R and the trace of the energy-momentum tensor T. The dependence of T may be introduced by exotic imperfect fluids or quantum effects (conformal anomaly). As a result of coupling between matter and geometry motion of test particles is non geodesic and an extra acceleration is always present. In $f(R,T)$ gravity, cosmic acceleration may result not only due to geometrical contribution to the total cosmic energy density but it also depends on matter contents. This theory can be applied to explore several issues of current interest and may lead to some major differences. Houndjo [7] developed the cosmological reconstruction of $f(R,T)$ gravity for $f(R,T) = f_1(T) + f_2(T)$ and discussed transition of matter dominated phase to an acceleration phase and in [8] the authors consider cosmological scenarios based on $f(R,T)$ theory and the function $f(R,T)$ is numerically reconstructed from holographic dark energy. Lobo [9], Capozziello and Faraoni [10], Nojiri and Odintsov [11, 12]. Felice and Tsujikawa [13] , Multamaki and Vilja[14,15], Chiba et al.[16]and Shamir [17] are some of the authors who have investigated several aspects of modified $f(R)$ gravity models which show the unification of early time inflation and late time acceleration.

The accelerating expansion of the universe may be explained in context of the dark energy. Due to negative pressure, the simplest way for modeling the dark energy is the Einstein's cosmological constant. Inspite of several attempts to identify the candidates for dark energy still cosmic acceleration is a challenge for modern cosmology. However dark energy has conventionally been characterized by the EoS parameter mentioned by $\omega(t) = \frac{p}{\rho}$ which is not necessarily constant where p is the fluid pressure and ρ is energy density (Carroll and Hoffman [18]). Ray et al. [19], Akarsu and Kilinc [20-21], Yadav et al. [22], Yadav and Yadav [23], Pradhan and Amirhashchi [24], Pradhan et al. [25] are some of the authors who have obtained dark energy models with variable EoS parameter. Pradhan et al. [26], Yadav [27] have discussed some Bianchi type dark energy models in general relativity. Recently, Katore et.al.[28] have studied a cosmological model in the presence of perfect fluid and dark energy.

In this paper we discuss Kantowaski–Sachs dark energy cosmological model, with variable EoS parameter, in $f(R,T)$ gravity choosing an appropriate form of $f(T)$ proposed by Harko et al [5].

2. Metric and Field Equations

We consider spatially homogeneous and anisotropic Kantowaski – Sachs metric given by

$$ds^2 = dt^2 - A^2 dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{1}$$

where A and B are functions of t only.

The Energy momentum tensor for anisotropic dark energy is given by

$$\begin{aligned} T_j^i &= diag[\rho, -p_x, -p_y, -p_z] \\ &= diag[1, -w_x, -w_y, -w_z], \end{aligned} \tag{2}$$

where ρ is the energy density of the fluid and p_x, p_y, p_z are the pressure along x,y,z axis respectively.

The Energy momentum tensor can be parameterized as

$$T_j^i = diag[1, -w, -(w+\delta), -(w+\delta)], \rho \tag{3}$$

For the sake of simplicity we choose $w_x = w$ and the skewness parameter δ are the deviations from *w* on *y* and *z* axis respectively. Now verying the action

$$S = \frac{1}{16\pi} \int f(R,T) \sqrt{-g}\, d^4 x + \int L_m \sqrt{-g}\, d^4 x, \tag{4}$$

of the gravitational field with respect to the metric tensor components g_{ij} we obtain the field equation of $f(R,T)$ gravity model as (Harko et. al. [5])

$$f_R(R,T) R_{ij} - \frac{1}{2} f(R,T) R_{ij} + (g_{ij\rightarrow} - \nabla_i \nabla_j) f_R(R,T) = 8\pi T_{ij} - f_R(R,T) T_{ij} - f_T(R,T)\theta_{ij}, \tag{5}$$

where $T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g})}{\partial g^{ij}}L_m, \theta_{ij} = -2T_{ij} - pg_{ij},$ (6)

$f(R,T)$ is an arbitrary function if Ricci Scalar R and of the trace T of the stress energy tensor of matter T_{ij} and L_m is the matter Langrangian density and in the present study we have assumed that the stress energy tensor of matter as

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \quad (7)$$

Now assuming that the function $f(R,T)$ given by

$$f(R,T) = R + 2f(T), \quad (8)$$

where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter and using the equation (6) and (7), the field equation (5) takes the form

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}, \quad (9)$$

where the overhead prime indicates differentiation with respect to argument.

We also choose

$$f(T) = \mu T, \quad (10)$$

where μ is constant.

Now assuming commoving coordinate system, the field equations (9) for the metric (1) with the help of equations (2), (3) and (10) can be written as

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} = \rho[(8\pi + 2\mu)w - (1 - 3w - 2\delta)] - 2\mu p, \quad (11)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} = \rho[(8\pi + 2\mu)(w + \delta) - (1 - 3w - 2\delta)] - 2\mu p, \quad (12)$$

$$2\frac{A_4}{A}\frac{B_4}{B}+\frac{B_4^2}{B^2}+\frac{1}{B^2}=-\rho\left[(8\pi+2\mu)+(1-3w-2\delta)\right]-2\mu p, \tag{13}$$

where *4* denotes differentiation with respect to time *t*.

3. Solution of the field equations

The average scale factor $R(t)$ and the spatial volume V are defined as

$$R(t)=\sqrt[3]{AB^2}, \qquad V=R^3=AB^2. \tag{14}$$

The generalized mean Hubble parameter H is given by

$$H=\frac{1}{2}\left(H_1+H_2+H_3\right), \tag{15}$$

where $H_1=\frac{A_4}{A}$, $H_2=H_3=\frac{B_4}{B}$, is the directional Hubble parameter in the direction of x, y, z axis respectively.

Using equations (14) and (15), we obtain

$$H=\frac{1}{2}\left(H_1+H_2+H_3\right)=\frac{R_4}{R}, \tag{16}$$

The Expansion Scalar θ and the Shear Scalar σ are given by

$$\theta=\frac{A_4}{A}+2\frac{B_4}{B}, \tag{17}$$

$$\sigma^2=\frac{1}{3}\left(\frac{A_4}{A}-\frac{B_4}{B}\right)^2, \tag{18}$$

The mean anisotropic parameter Am is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{19}$$

Where $\Delta H_i = H_i - H$,

The field equations (11)-(13) are three independent equations in six Unknown.

Hence to find deterministic solution three more conditions are necessary, we consider the following conditions,

(i) we apply the variation of Hubble Parameters proposed by Berrman that yields constant deceleration parameter as

$$q = -\frac{RR_{44}}{R_4^2}, \tag{20}$$

where the scale factor R is given by (14)

(ii) we assume that the Expansion Scalar θ is proportional to the Shear Scalar σ which gives us

$$A = B^m, \tag{21}$$

where $m > 1$ is a constant.

(iii) The E0S parameter w is proportional to skewness parameter δ such that

$$w + \delta = 0, \tag{22}$$

The solution of equation (20) is given by

$$R(t) = (at+b)^{\frac{1}{1+q}}, \tag{23}$$

where $a \neq 0$ and b are constant of integration and $1+q > 0$ for accelerating expansion of the universe.

Now using equations (20), (21), and (23) the expansion for the metric coefficient in the field equations are

$$B = (at+b)^{\frac{3}{(1+q)(m+2)}}, \tag{24}$$

$$A = (at+b)^{\frac{3m}{(1+q)(m+2)}}, \tag{25}$$

With the suitable choice of coordinates and constants, the metric (1) with the help of (24) and (21) can be

$$ds^2 = dT^2 - T^{\frac{6m}{(1+q)(m+2)}} dR^2 - T^{\frac{6}{(1+q)(m+2)}} (d\theta^2 + \sin^2\theta d\phi^2), \tag{26}$$

4. Some Physical properties of the model

Equation (26) represents Kantowaski–Sachs Dark Energy Model in $f(R,T)$ Gravity with the following physical and Kinematical parameters of the model which are important for discussing the physical of the cosmological model.

The Spatial volume in the model is

$$V = T^{\frac{3}{1+q}}. \tag{27}$$

The generalized Hubble Parameter is

$$H = \frac{a}{(1+q)T}. \tag{28}$$

The Scalar expansion in the model is

$$\theta = \frac{3a}{(1+q)T}. \tag{29}$$

The Shear Scalar in the model is

$$\sigma^2 = \frac{7}{18}\frac{a^2}{(1+q)^2 T^2}. \tag{30}$$

Mean anisotropy parameter is

$$A_m = \frac{1}{3}\frac{X^2}{(m+2)^2 a^2}, \text{ where } X^2 = X_1^2 + 2X_2^2. \tag{31}$$

The anisotropic parameter measures a constant value. This indicates that the universe expands isotropically.

The energy density in the model is

$$\rho = -p = \frac{1}{(8\pi + 2\mu)}\left\{\frac{k_1}{(1+q)^2(m+2)^2 T^2} - T^{\frac{-6}{(1+q)(m+2)}}\right\}, \tag{32}$$

where $k_1 = (6-3q)m^2a^2 + (-3q-12)ma^2 + (6q-12)a^2$.

The EoS and Skewness parameter in the model are

$$w = -\delta = -1 - \frac{1}{\rho(8\pi + 2\mu)}\left\{\frac{k_2}{(1+q)^2(m+2)^2 T^2}\right\}, \tag{33}$$

where $k_2 = (24+6q)ma^2 + (12q-6)a^2$.

It may be observed that the cosmological model in $f(R,T)$ gravity is free from initial singularity i.e. at $T = 0$, the special volume in these model increases as T increases confirming accelerated expansion of the universe.

It can also be observed that $H, \theta, \sigma, \delta, w, \rho$ are functions of T and vanishes for large T while they diverge for $T = 0$.

Also, $\frac{\sigma^2}{\theta^2} \neq 0$, and hence the model does not approach isotropy for large values of T, However the model become isotropic for $m = 1$.

5. Conclusions

It is well known that anisotropic dark energy models with variable E o S parameter in modified theories of gravity play a vital role in the discussion of the accelerated expansion of the universe which is the crux of the problem in the present scenario. In this paper we have investigated homogeneous and anisotropic Kantowaski-Sachs dark energy model in $f(R,T)$ gravity with variable E o S parameter in the presence of perfect fluid source. It is observed that EoS parameter, skewness parameters in the model are all functions of time. It can also be seen that the model is accelerating, expanding, non-rotating and has no initial singularity. This model confirms the high redshift supernova experiment.

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