Article

On a New Position Space Doubly Special Relativity Theory

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Abstract

The general consensus feeling amongst researchers is that it is generally difficult to obtain a position space Lorentz invariant Doubly Special Relativity (DSR) theory. In this reading, we propose such a theory. The Lorentz transformations are modified such that the resultant theory has not one, but two invariants – the speed of light $c = 2.99792458\times10^{8}\,\mathrm{ms}^{-1}$, and a minimum length ℓ_p. Actually, we achieve our desire by infusing Heisenberg's quantum mechanical uncertainty principle into the fabric of Minkowiski spacetime. It should be stated that this theory has been developed more as a mathematical exercise to obtain a physically reasonable as is possible a position space DSR theory that is *Lorentz invariant*. In the low energy regime, the theory gives the same predictions as Einstein's Special Theory *of* Relativity (STR).

Keywords: Doubly Special Relativity , Lorentz transformation, Special Theory of Relativity.

1 Introduction

In 2002, Professor Giovanni Amelino-Camelia of the University *of* Rome in Italy set-forth a fecund revision of Einstein's seemingly sacrosanct STR (Einstein 1905) by adding to it – *via* momentum space; an absolute universal minimum scale-length $[\ell_p = (G\hbar/c^3)^{\frac{1}{2}} = 5.11 \times 10^{-36}\,\mathrm{m}$, where $\hbar = 1.06 \times 10^{-35}\,\mathrm{Js}$, is Planck's normalised constant and $G = 6.667 \times 10^{-11}\,\mathrm{kg^{-1}m^3s^{-2}}$ is Newton's universal constant of gravitation]. The proposal by Professor Amelino-Camelia is popularly known as the Doubly Special Relativity (DSR) theory (Amelino-Camelia 2002, 2002*b*). So, to the already well established absolute universal constant – the sacrosanct speed of light $c = 2.99792458 \times 10^8\,\mathrm{ms}^{-1}$, Professor Giovanni Amelino-Camelia added a second, thus his theory contains not one, but two absolute universal constants (c, ℓ_p). Because the theory has two universal absolute constants, Professor Giovanni Amelino-Camelia dubbed it "*Doubly Special*" hence the name

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Doubly Special Relativity. Some call such theories Deformed Special Relativity while others call them Extended STR (*e.g.* Pavlopoulos 1967). However, the current popular name is Doubly Special Relativity.

The motivation for the formulation of DSR theories is well founded and is based on the subtle observations that: the Planck energy (or Planck scale, ℓ_p) is expected to play not only a fundamental role in any theory of quantum gravity but a pivotal role in setting the absolute universal scale-length at which quantum gravity effects cannot be neglected, leading to new physically observable phenomena. If Einstein's bare special relativity theory is to hold up exactly to this scale, then, inevitably – due to the Lorentz-FitzGerald contraction, different observers would observe quantum gravity effects at different scales. The simple fact is that the STR is believed to be hostile to the introduction of a universal length as a quintessential and sacrosanct constant of *Nature*, since for the STR, for two different observers in uniform relative motion, there can never be an absolute universal scale-length required by quantum gravity (see *e.g.* Pavlopoulos 1967).

From the foregoing, it follows that, quantum gravity would lead to a clear contradiction to the sacred Principle *of* Relativity that beholds that the Laws *of* Nature are the same for all inertial observers, hence, all inertial observers should agree on the existence of a physical phenomenon if of cause, they both are observing it. One can not – as would happen if quantum gravity occurs; claim a system has entered the quantum gravity regime while another observer observing the same physical system measures something to the contrary. In STR, they can disagree on the numerical values of their respective measurements; this disagreement in the numerical values of their measurements is (in the STR) resolved by the Lorentz transformations. If quantum gravity is to occur, contrary to all the dictates of binary logic, they will disagree on the existence of the physical phenomenon itself. This is where the need for a fundamental, absolute and universal scale-length comes in.

In the reading Nyambuya (2010), entitled "*Is Doubly Relativity Necessary*", it was argued using both physical and number theoretic arguments that the STR already implies the existence of an absolute minimum length. The resulting thesis from this reading (Nyambuya 2010) is that spacetime is no longer a continuum, but is composed of discrete nodes. The resultant quantized or discrete spacetime does not violate Lorentz invariance since Lorentz invariance does not require that spacetime be a continuum (Livine *&* Oriti 2004, Snyder 1947). The simple argument set-forth in Nyambuya (2010), goes as follows: from a physical standpoint, the fact that there exists a maximum absolute speed c implies there can be no object that can move from any two points in zero time interval. This directly points to the undeniable fact that there must always be a finite duration in the time interval when a material object is moved from one point to the other, otherwise there will exist not any such phenomena as an absolute maximum constant speed, c. If a zero time interval where permissible, the maximum allowed cosmic speed limit would be infinity – simple, there would exist no upper speed limit.

From the foregoing, if there must be a finite duration and an absolute maximum constant speed, then – inevitably; there must exist the least possible time interval. Clearly, this time interval can not be infinitely small. By infinitely small, this is what we mean: for example, the least greatest number after zero is $0.\dot{0}1$ where the $\dot{0}$ represents an infinite number of zeros; the number $0.\dot{0}1$ is infinitely small. The least possible time interval can not take this value or any infinitely small numbered time interval e.g. $0.\dot{0}3707$, $0.\dot{0}7712$, $0.\dot{0}1507$ *etc.* The number of zeros between the comma and the first significant figure must be finite – *i.e.*, 0.[finite number of zeros]ABC... where, $\text{A} \in [1, 2, 3, 4, 5, 6, 7, 8, 9]$ is the first significant figure and $\text{B}, \text{C}, \dots \in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]$. More precisely, the least possible time interval must be expressible in scientific notion as a finite physical quantity. Take for example, how does one write the number $0.\dot{0}1$ in scientific notation to three significant figures? Accordingly, it must be $1.00 \times 10^{-\infty}$. Technically, the number $1.00 \times 10^{-\infty}$ is zero. So, technically, $0.\dot{0}1$ is not a finite time interval thus it can not be a physical time duration since physical time durations are required to be greater than zero if there is to be a finite upper cosmic speed limit. In summary, whatever its value, let us set the least possible time interval to be denoted or represented by the symbol t_p.

Now, the existence of a least time interval coupled with the existence of an absolute maximum speed implies there must exist a least possible space interval given by $\ell_p = ct_p$. This is a mathematical fact that the *number theorist* will affirm without any iota, shred or dot of doubt! From this, clearly, the very existence of an absolute maximum speed means there must exist an absolute least time interval and an absolute least space interval. Naturally, the question had to be raised, "*Is the DSR theory really necessary?*" This question was raised because the analysis presented in Nyambuya (2010) pointed to the fact that the STR *via* its prediction of an upper cosmic speed limit, does predict that an absolute minimum length must exist. The DSR theory as presented by Amelino-Camelia ($2002a, b$) strongly appeared to us to posit that because such a minimum length does not exist in the STR, it must be inserted by the lathe of hand. It is because of this, that we began to doubt the foundations of Amelino-Camelia ($2002a, b$)'s brilliant leap of imagination.

According to Nyambuya (2010), the STR predicts the existence of a minimum scale length. However, Einstein's STR does not make this fact explicit that such a scale exists; it is a fact deeply hidden in its beautiful fabric. With hindsight after much pondering and reflection on this matter, Amelino-Camelia ($2002a, b$) is right. The reading Nyambuya (2010) is ignorant of the important fact that this scale length predicted by the STR is a scale length which its observers can not agree as to what this value is. Clearly, Einstein's STR contradicts itself on this matter. There is only one thing to be done, that is, resolve this contradiction as has been attempted by the many variants of the DSR theories. At best, this problem must be solved not in momentum but position space.

The DSR theories, while they have a valid reason for their existence, they suffer from several conceptual difficulties that are yet to be resolved (see *e.g.* Aloisio et al. 2004,5). Amongst these problems and central to this reading is the conceptual difficulty that the DSR theories are a *priori* formulated in momentum space. Though there exists significant attempts on a position space DSR (Deriglazov *&* Rizzuti 2005, Deriglazov 2004, Gao *&*

Wu 2003, amongst others), there exists no consistent formulation of the model in position space. The position space DSR theories in existence today most often require a radial revision of the Lorentz transformation. It is the hope of this reading that this missing part of the DSR theory may (perhaps) begin to be furnished and that the search for a plausible solution may (perhaps) find its germ in the present work.

In our present endeavours, while we make appreciable and moderate provisions to pay attention to the physical world *i.e.*, by infusing Heisenberg's quantum mechanical uncertainty principle, we approach this problem more as a mathematical problem to attain at least two invariants at the position-level of Einstein's STR.

2 Momentum Space DSR

In addition to the sacrosanct invariant – the speed of light c; the introduction of an independent scale-length where first made by Pavlopoulos (1967,9). His estimate was $\ell_p \sim 10^{-13}\,\mathrm{m}$. Pavlopoulos was investigating the possible violation of Lorentz invariance at the atomic level, hence his independent scale length was chosen to coincide with the atomic-scale-length $10^{-13}\,\mathrm{m}$. As stated in the introductory section, the modern approach is motivated by the desire to find a quantum gravity theory. The first such approach is that by Amelino-Camelia ($2002a, b$). Most variants of DSR theories are descendants of this work by Amelino-Camelia ($2002a, b$). Therefore, our brief exposition of DSR theories centres on the work of Amelino-Camelia ($2002a, b$).

Other than the taxing and esoteric intellectual labour and pursuit to solve the contradiction brought fourth by the prospects of quantum gravity where an invariant energy scale (and consequently an invariant length scale) seems inevitable, DSR theories have some more realistic motivation to explain the observed time delays in the arrival times of Gamma-rays from Gamma Ray Bursts (GRBs). In the literature, GRBs were first reported by Klebesadel et al. (1973). GRBs are cosmic flashes of gamma rays associated with extremely energetic cosmic explosions coming from distant galaxies (Metzger 1997). GRBs are the most luminous electromagnetic events known to occur in the Universe. These bursts can last from 10 ms to several minutes; a typical burst lasts $20 - 40\,\mathrm{s}$. In relation to DSR theories, the problem with GRBs is as follows.

GRBs are electromagnetic waves, so, according to our most immaculate, pristine and foremost understanding of the Laws *of* Nature, they must all travel at the same speed of light which is $c = 2.99792458 \times 10^8\mathrm{ms}^{-1}$. So, if these bursts are coming from the same event, they must leave this event at the sametime. If this is true, it follows that they must arrive on Earth at the same time since they have travelled the same distance. However, this is not what is observed when our telescopes are pointed to the heavens – at least this is what the Fermi Large Area Telescope (Fermi-LAT) reveals to us. Gamma rays from these GRBs seem to arrive on Earth at different times (see *e.g.* Amelino-Camelia *&* Smolin 2009). There seems to be an energy dependence in the arrival times with with more energy gamma rays taking longer to arrive while less energetic gamma rays taking much less time to arrive. This obviously suggests an energy dependence on the speed on these electromagnetic waves, this is a clear violation of Lorentz invariance.

The DSR approach to solving this apparent dilemma is to modify the Einstein dispersion relation namely, $E^2 - \boldsymbol{p}^2c^2 = m_0^2c^4$, where $(\boldsymbol{p}, m_0, E)$ are the particle's momentum, rest mass and total energy respectively. Typically, DSR theorists propose the modification:

$$E^2 - \boldsymbol{p}^2c^2 = f(\boldsymbol{p}, E : \ell_p). \tag{2.1}$$

where the function $f(\boldsymbol{p}, E : \ell_p)$ is a variable and takes a form which is usually dependent on the phenomenon that one seeks to address. In the modification, ℓ_p enters the dispersion relation as a universal and absolute invariant scale-length. The momentum transformation relationship between any two initial systems will have to be modified such that ℓ_p (like the invariant light speed c), is the same for all inertial observers. Because one has to modify the Lorentz momentum transformations so as to include an invariant scale length, these theories are built in momentum (p_μ) and not position (x^μ) space, this is the reason for them to be called momentum space DSR theories. The equivalent in position space are known as position space DSR theories.

Now, with (2.1) given, we know that the group velocity of a wave-packet with energy E, and momentum $\boldsymbol{p}$, is given by:

$$\boldsymbol{v}_g = \frac{\partial E}{\partial \boldsymbol{p}}. \tag{2.2}$$

Differentiating the energy formula (2.1) with respect to $\boldsymbol{p}$, we obtain after some basic algebraic operations:

$$\boldsymbol{v}_g = \left(\frac{\frac{\boldsymbol{p}c}{E} + \frac{1}{2E}\frac{\partial f(\boldsymbol{p},E:\ell_p)}{c\partial \boldsymbol{p}}}{1 - \frac{1}{2E}\frac{\partial f(\boldsymbol{p},E:\ell_p)}{\partial E}} \right) c = \boldsymbol{\beta}_* c. \tag{2.3}$$

The above implies that $\boldsymbol{v}_g = \boldsymbol{v}_g(E, \boldsymbol{p} : \ell_p)$. If no signal can exceed c, then, the vector $\boldsymbol{\beta}_*$ is such that $|\boldsymbol{\beta}_*| \leq 1$. The case $|\boldsymbol{\beta}_*| = 1$ will occur when $f(\boldsymbol{p}, E : \ell_p) = 0$ and $E = |\boldsymbol{p}|c$. For reasons already stated, DSR theorists choose ℓ_p to coincide with the Planck length where quantum gravity effects are expected to take center stage. The Planck length is part of an esoteric set of about six fundamental units. This set of six fundamental units is listed in Table 1 below.

In-passing, we would like to point out that dispersion relations for which $f = f(\boldsymbol{p}, E)$ is a variable function are not unique to DSR theories but are also found in searches of curved spacetime Dirac equation such as *e.g.* Nyambuya (2008). The work Nyambuya (2008) derives three kinds of curved spacetime Dirac equations. Each of these three equations come along with its own unique dispersion relation. So, in a way, the curved spacetime Dirac theory of Nyambuya (2008) posits that electromagnetic waves must have three different speeds, with one of the speeds being the speed of light while the other two speeds should be less than the speed of light. These two speeds must be different

Table 1: **Planck Units**

Quantity	**Symbol**	**Formula**	**Value**
Length	ℓ_p	$\left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}}$	$5.11 \times 10^{-36}\,\mathrm{m}$
Time	t_p	$\left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}}$	$1.70 \times 10^{-44}\,\mathrm{s}$
Mass	m_p	$\frac{1}{2}\left(\frac{\hbar c}{G}\right)^{\frac{1}{2}}$	$3.44 \times 10^{-8}\,\mathrm{kg}$
Energy	E_*	$\frac{1}{2}\left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}}$	$1.93 \times 10^{18}\,\mathrm{GeV}$
Momentum	p_*	$\frac{1}{2}\left(\frac{\hbar c^3}{G}\right)^{\frac{1}{2}}$	$1.03\,\mathrm{kgms}^{-1}$
Temperature	T_p	$\left(\frac{\hbar c^5}{G k_B^2}\right)^{\frac{1}{2}}$	$4.49 \times 10^{31}\,\mathrm{K}$

from each other. If this theory (*i.e.* Nyambuya 2008) has any bearing or correspondence with reality, then, flushes of GRBs must arrive in three distinct groups. We will not go further into these rather polemical matters but simple leave them at this point. In the next section, we shall look at the quantum mechanical uncertainty principle, with in mind the hope of infusing this into Minkowski space, where upon, endeavours to include an invariant scale-length will be sought.

3 Quantum Fluctuations and the Uncertainty Principle

In conjunction with the famous uncertainty principle set-forth by the great German physicist, Weiner Heisenberg, in 1927, we are going to develop in the present, our ideas of spacetime quantum fluctuations. These ideas will be infused (injected) into the resultant position space DSR theory; these fluctuations will form the fundamental and foundational basis of the new theory.

With regard to fluctuations, it is perhaps safe to say that every physicist (be they theoretical, observational or experimental physicist) has, during one part of their training been to the laboratory where they have had to measure – using either a digital or analog voltmeter; the potential different across two points in an electrical circuit. As the physicist already knows very well, the voltage does not take a precisely defined value, but fluctuates in a given range. This behaviour is clearly seen in a digital voltmeter. In an analog voltmeter, the pointer dangles (back-and-forth) slightly about two extremes.

For example, the voltmeter reading may fluctuate between $9.00\,\mathrm{V}$ to $11.00\,\mathrm{V}$. The way to take down such a reading would be to record it as: $10.00 \pm 1.00\,\mathrm{V}$. Further, the way to interpret this reading is that the average (mean) value of the potential difference is $10.00\,\mathrm{V}$ with an uncertainty of magnitude $1.00\,\mathrm{V}$. This uncertainty is caused by electronic fluctuations. Obviously, this interpretation assumes that these fluctuations follow a normal Gaussian distribution. In general, we will have a reading "R" accompanied by a corresponding fluctuation "δR"; *i.e.* $R \pm \delta R$. At any given moment in time, the reading on the voltmeter will be anything in the range $R + \delta R$ to $R - \delta R$ *i.e.* $[R + \delta R, R - \delta R]$. One can not predict with 100% certainty what the voltmeter reading will be at any given moment. All they can do is to give a prediction with it attached a probability of occurrence as determined by Gaussian statistics (or any corresponding statistics that these fluctuations follow).

Redolently, we envision spacetime to be inherently and intrinsically inhabited by random-dynamic fluctuations. If (x, y, z, t) is a point on spacetime, this point has associated with it, the corresponding inherent and intrinsic fluctuations $(\delta x, \delta y, \delta z, \delta t)$. The kind of spacetime that we have in mind is one that is quantized as proposed in Nyambuya (2010) where each physical point in spacetime is separated from the next physical point by a distance $\ell_p = ct_p$. As with the case of the voltmeter, points in the quantised spacetime will be represented by:

$$x^\mu \pm \delta x^\mu, \tag{3.1}$$

where x^μ is the mean value of the point and δx^μ is the corresponding fluctuation of the point in question. This means that, at any given time, the position of any point in spacetime can never be known with 100% certainty.

Note that in the case of the voltmeter reading, "R" and "δR" are precisely fixed values. In the same manner "x^μ" and "δx^μ" are precisely fixed values. Further, the fluctuations "δx^μ" have no implicit nor explicit dependence on space nor time *i.e.*:

$$\frac{\partial(\delta x^\mu)}{\partial x^\mu} = \frac{d(\delta x^\mu)}{dx^\mu} \equiv 0, \tag{3.2}$$

thus a change in $x^\mu \pm \delta x^\mu$ *i.e.* $\Delta(x^\mu \pm \delta x^\mu)$ is such that:

$$\Delta(x^\mu \pm \delta x^\mu) = \Delta x^\mu \pm \delta x^\mu, \tag{3.3}$$

because $\Delta(\delta x^\mu) = \delta x^\mu$; this flows from the constraint that "δx^μ" has no implicit nor explicit dependence on space nor time. The spacetime fluctuations "δx^μ" are envisaged to be exactly the quantum fluctuations embodied in Heisenberg's 1927 uncertainty relation.

If δp and δE are momentum and energy random fluctuations of a particle and δx and δt are the corresponding fluctuations in the particle's position and time respectively, then according to Heisenberg's uncertainty principle, these fluctuations are related by the relations:

$$\delta p \delta x \geq \hbar \quad \text{and} \quad \delta E \delta t \geq \hbar. \tag{3.4}$$

Our hypothesis, which is drawn from the thesis set-forth in Nyambuya (2010) is that these space and time quantum fluctuations δx^j $(j = 1, 2, 3)$ and δt have a absolute and universal minimum value, *i.e.*:

$$\delta x^j \geq \ell_p \quad \text{and} \quad \delta t \geq t_p. \tag{3.5}$$

The values ℓ_p and t_p are measured by all observers to be the same, they are universal and absolute just as is the case with the speed of light c. The fact that these values are measured by all observers to be the same, this fact must be engraved or completely embodied in the Lorentz transformations at the position and not momentum level of the DSR theory. Therefore, the task at hand in creating a position space DSR theory is to modify the usual Lorentz transformations to reflect these "facts". Under certain conditions, the position space DSR theory, must, just like any other DSR theory, truncate to the usual STR.

The constraints (3.5) coupled with (3.4) imply a lower limit on the momentum-energy fluctuations, that is:

$$\delta p^j \geq p_* \quad \text{and} \quad \delta E \geq E_*, \tag{3.6}$$

where p_* and E_* are the minimum possible momentum and energy fluctuations respectively. Like the space and time fluctuations, the values of p_* and E_* are measured by all observers to be the same, they are universal and absolute just as is the case with the speed of light c.

4 Position Space DSR Theory

A theory will be compatible with the DSR principles if there is complete equivalence of inertial observers (*i.e.* if it is compatible with the *Relativity Principle*) and the laws of transformation between inertial observers are characterized by two scales, a maximum speed scale and a minimum length scale (Amelino-Camelia 2010). The theory to be set-up in this section espouses these requirements with the addition that it is fully compliant with Lorentz symmetry (invariance).

If we have two observers, the primed and the un-primed, such that the spacetime intervals that these measure for a particular event are $(\Delta x', \Delta y', \Delta z', \Delta t')$ and $(\Delta x, \Delta y, \Delta z, \Delta t)$ respectively, and the un-primed observer is moving along the positive $x-axis$ with a speed v, then, according to Einstein's STR, the measurements of these two observers are related by the Lorentz transformations:

$$\begin{array}{rll} \Delta x' = & \Gamma(\Delta x + v\Delta t) & (\mathbf{a}) \\ \Delta y' = & \Delta y & (\mathbf{b}) \\ \Delta z' = & \Delta z & (\mathbf{c}) \\ ic\Delta t' = & i\Gamma(c\Delta t + v\Delta x/c) & (\mathbf{d}) \end{array}. \tag{4.1}$$

Our proposal is to include a scalar function ϕ which embody the information of the quantum fluctuations into the Lorentz transformations. A *priori*, we are of the strong belief that a fundamental DSR theory must be formulated on a fundamental fabric or garment such as spacetime. Momentum Space is a secondary space derived from the fabric or garment of spacetime itself.

4.1 Derivation of the New Position Space DSR Transformations

In 1905 when Einstein derived the transformations (4.1), he assumed that these transformations must be linear. He proposed that these linear transformations must have the form:

$$\begin{array}{rll} \Delta x' = & A\Delta x - Bv\Delta t & \ldots\ (\mathbf{a}) \\ \Delta y' = & \Delta y & \ldots\ (\mathbf{b}) \\ \Delta z' = & \Delta z & \ldots\ (\mathbf{c}) \\ ic\Delta t' = & iCc\Delta t - iDv\Delta x/c & \ldots\ (\mathbf{d}) \end{array}. \tag{4.2}$$

where A, B, C and D where assumed to be constants. Given these linear transformations, the line element emerging from them is:

$$\begin{array}{c} ds'^2 = -c^2\Delta t'^2 + \Delta x'^2 + \Delta y'^2 + \Delta z'^2 = \\ -\left(C^2 - \frac{B^2v^2}{c^2}\right)c^2\Delta t^2 + \left(A^2 + \frac{D^2v^2}{c^2}\right)\Delta x^2 + \Delta y^2 + \Delta z^2 - (AB - CD)\, v\Delta t\Delta x = ds^2. \end{array} \tag{4.3}$$

This line element is such that the coefficients of $(c^2\Delta t'^2, \Delta x'^2, \Delta y'^2, \Delta z'^2)$ and $(c^2\Delta t^2, \Delta x^2, \Delta y^2, \Delta z^2)$ must be equal; that of $v\Delta t\Delta x$ must be equal to zero. The means:

$$\begin{array}{ll} A^2 - \frac{D^2v^2}{c^2} = 1, & \ldots(\mathbf{a}) \\ C^2 - \frac{B^2v^2}{c^2} = 1, & \ldots(\mathbf{b}) \\ AB - CD = 0. & \ldots(\mathbf{c}) \end{array} \tag{4.4}$$

As a minimum requirement, the symmetry of the Lorentz transformations requires that $A = C = \Gamma_\phi$. From [4.4 (c)], this requirement $A = C$, implies $B = D$. Now, our contribution is that, we shall set $B = D = \phi\Gamma_\phi$; this setting does not destroy the symmetry of the Lorentz transformations for as long as ϕ is a scaler, *i.e.* it is the same for both observers. From all this, it follows that:

$$\Gamma_\phi = \frac{1}{\sqrt{1 - \phi^2\boldsymbol{v}^2/c^2}}. \tag{4.5}$$

Thus, for the just proposed spacetime – we have; just as in Einstein's STR, the spacetime signature $[-1, +1, +1, +1]$ of the primed and the un-primed observer's spacetime manifolds being identical. What this means is that, all observations and Laws *of* Nature discovered by one inertial observer, are equivalent to those of any other inertial observer. Simple stated, the just proposed DSR theory upholds *Lorentz invariance.*

Lorentz invariance is the idea that the result of any physical experiment should stay the same whether the experimental apparatus is "*motionless*" or "*travelling*" at some great constant speed relative to some "*fixed*" reference system. Lorentz invariance is a fundamental pillar and cornerstone of Einstein's STR. Be that it may, most DSR theories violate this principle.

Now, our next contribution is that this newly introduced function ϕ must define or embody the required threshold of an absolute and universal minimum length or maximum energy scale. If $\delta l = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$ represents the resultant quantum mechanical fluctuations in position, and if our requirement is that whenever $\delta l = \ell_p$ all observers must agree on their measurements, we find that this function ϕ must be defined as:

$$\phi = \sqrt{1 - \left(\frac{\ell_p}{\delta l}\right)^2}. \tag{4.6}$$

Therefore, the new Lorentz transformations become:

$$\begin{array}{rll} \Delta x' = & \Gamma_\phi \left(\Delta x - \phi v \Delta t\right) & (\mathbf{a}) \\ \Delta y' = & \Delta y & (\mathbf{b}) \\ \Delta z' = & \Delta z & (\mathbf{c}) \\ ic\Delta t' = & i\Gamma_\phi \left(c\Delta t - \phi v \Delta x / c\right) & (\mathbf{d}) \end{array}. \tag{4.7}$$

The function ϕ can be defined in terms of energy fluctuations. If these energy fluctuations are such as $\delta E \longmapsto E_p$, $\phi \longmapsto 0$: just as in the case where this function is defined in terms length fluctuations, then one finds that, we will have:

$$\phi = \sqrt{1 - \left(\frac{\delta E}{E_p}\right)^2}. \tag{4.8}$$

The above means there exists a relationship between δE and δl, *i.e.*:

$$\delta E \delta l = \hbar. \tag{4.9}$$

For δl and δt, we expect that $\delta l = c\delta t$. If this holds, then:

$$\delta E \delta t = \hbar. \tag{4.10}$$

The quantum mechanical uncertainty relationship of Heisenberg, *i.e.* $\delta p \delta l \geq \hbar$ and $\delta E \delta t \geq \hbar$, do not give a clearly defined relationship between δE and δt, but merely give an inequality. If the above (4.10) holds true, then, δE and δt takes-up the equality-sign in the inequality $\delta E \delta t \geq \hbar$, thus giving a clearly defined relationship between these two quantities, *i.e.*, for a given δE-fluctuation, there is a clearly defined and predictable δt-fluctuation. From this, if $\delta p = \delta E/c$, it follows that:

$$\delta p \delta l = \hbar. \tag{4.11}$$

We shall take (4.10) and (4.11) as defining the relationship between δp, δl, δE, and δt. These fluctuations are in conformity with Heisenberg's uncertainly requirements.

4.2 Absolute and Universal State

When the spacetime fluctuations reach their absolute limit *i.e.* $\phi \equiv 0$, which occurs when $\delta l = c\delta t = \ell_p$ and $\delta E = c\delta p = E_p$, spacetime for this reference system becomes absolute and universal, *i.e.*:

$$\begin{array}{rl} \Delta x' = & \Delta x \\ \Delta y' = & \Delta y \\ \Delta z' = & \Delta z \\ ic\Delta t' = & ic\Delta t \end{array} . \tag{4.12}$$

All observers will agree universally and absolutely on the state of such a system. That is, numerically, they will measure the same space and time intervals hence they will measure every observable associated with this system to have exactly the same numerical values since all other observables are but derivatives of space and time intervals. This system has reached its absolute state.

Conversely, this observer in the absolute reference system has the privilege that they will make measurements of events measured by any other observer in the universe in manner such that they will always agree numerically and absolutely on the measurements made by this observer about measurements in this observer's system. For example if the reader where to make a length (or time) measurement in their own inertial reference system, say 10 cm (or 10 s, for example on their clock), the observer in the absolute reference system will also measure exactly 10 cm (or 10 s) for the same event in their absolute reference system. Other inertial observers that are not in this absolute state will not agree on numerically on that the observe in the other inertial observer's reference system.

4.3 Position Space DSR Postulates

The ideas just laid down above comprise the position space DSR theory that we wish to put forward. In summary, this DSR theory can be summed up in the form of four postulates, and these are:

1. All the Laws *of* Nature are the same in all inertial reference systems.

2. The speed of light c in vacuum, is an absolute universal constant that all inertial observers will measure and agree exactly on its numerical value.

3. The space and time quantum fluctuations δl and δt are such that:

   $$\delta l \geq \ell_p \text{ and } \delta t \geq t_p,$$

   where – like the speed of light, c; ℓ_p and t_p are absolute universal constants which are the same for all inertial observers.

4. The quantum mechanical uncertainty of Heisenberg, *i.e.* $\delta p \delta x \geq \hbar$ and $\delta E \delta t \geq \hbar$; coupled to the spacetime fluctuations, lead to limits on the momentum energy quantum fluctuations such that:

$$\delta p \geq p_p \text{ and } \delta E \geq E_p,$$

where again – like the speed of light, c; E_p and p_p are absolute universal constants which are the same for all inertial observers.

The first postulate tells us that insofar as the formulation of Physical Laws is concerned, all systems of reference in uniform translatory motion are equivalent; a law discovered in one such system, holds in any such system which is in uniform translatory motion. The upholding of this law by the present DSR theory is important as it makes it Lorentz invariant.

4.4 Plausible Thermodynamic Link to Spacetime Fluctuations

In this section, we make the *all-daring* proposal, postulate or hypothesis, that makes a thermodynamic link to physical measurements. First we ask, what are these quantum fluctuations? What is their measure in the real world? We know that thermal energy is a manifestation of random dynamic fluctuations. Our proposal is that the energy fluctuations δE should manifest in the real world as thermal energy. That is, as spacetime undergoes fluctuations, it should affect any particle in its vicinity. From the position-momentum uncertainty relation ($\delta p \delta l = \hbar$), if we take that the thermal energy of a particle is given by $c\delta p$, that is, $c\delta p = \frac{3}{2}k_B T$; from $\delta p \delta l = \hbar$, we will have $\delta l = 2\hbar c/3k_B T$. Plugging this into (4.6), we will have:

$$\phi = \sqrt{1 - \left(\frac{T}{T_p}\right)^2}. \qquad (4.13)$$

This suggestion must taken as nothing but what it is – a suggestion; nothing more and nothing less. Taken seriously, it would mean that the thermodynamic state of a system does influence the measurements. Note that, since $\varphi < 0$, this means $T > 0$, that is, the state $T = 0$ must be unattainable. If it is unattainable, there must exist a minimum possible thermodynamic temperature scale T_{mim}. What determines this minimum temperature is the largest possible fluctuation δl_{max}. If the Universe is finite (as we believe or would like to believe to be true), then, its size must give δl_{max}, that is $\delta l_{max} = R_u = ct = c/H$, where H is the Hubble constant (or the Hubble parameter: it depends on how one interprets this quantity). Therefore:

$$T_{\mathrm{min}} = \frac{2}{3}\left(\frac{\hbar}{k_B}\right) H = 1.21 \times 10^{-29}\,\mathrm{K}. \qquad (4.14)$$

The point is – there must be a minimum non-zero thermodynamic temperature. As the Universe evolves, this minimum temperature tends to zero. It will never exactly be equal to zero, but forever approach this zero of temperature. If the Universe is infinite, then $\delta l_{max} = \infty$, leading to $T_{\min} = 0$, in which case, the zero of temperature will be very much attainable. However, if the restriction $\varphi < 0$ is to be upheld at all costs, then, the Universe will have to be finite in spatial extent. There will have to exist a time dependent minimum temperature $T_{\min}(t)$.

4.5 STR's Light Speed Singularity

One important and significant improvement to the STR that has been made in the present DSR theory is that there no-longer exists a singularity for the case $|\boldsymbol{v}| = c$. For example, the relativistic mass of a particle is – in the prsent DSR theory, given by:

$$m = \frac{m_0}{\sqrt{1 - \phi^2 \boldsymbol{v}^2/c^2}}, \text{ and for the case } |\boldsymbol{v}| = c, \text{ we will have : } m = \frac{m_0}{\sqrt{1 - \phi^2}}. \quad (4.15)$$

Since $(0 \leq \phi < 1)$, m can not take an undefined value as is the case in the STR *i.e.* $m = m_0/0$ for $|\boldsymbol{v}| = c$. Therefore, this bug found in the STR can be "considered fixed" or non-existent in the present DSR theory.

4.6 Velocity Addition Formula

The resultant velocity addition formula emerging from the transformations (4.7) is:

$$\boldsymbol{v}'_p = \frac{\boldsymbol{v}_p + \phi \boldsymbol{v}}{1 + \phi \frac{\boldsymbol{v} \cdot \boldsymbol{v}_p}{c^2}}, \quad (4.16)$$

where $\boldsymbol{v}_p$ is the velocity of a particle along the $x-axis$ in the un-primed system, $\boldsymbol{v}'_p$ is the corresponding velocity of particle as measured in the primed system and $\boldsymbol{v}$ is the relative velocity of the two systems. From this velocity addition formula, it is seen that no matter the value of ϕ (*i.e.*, $0 \leq \phi < 1$), the speed of light c, is an invariant, it is the same for all inertial observers. Further, if $\phi = 0$, then, the speed of a particle in this reference system is the same for all inertial observers. Thus, any system of reference that has attained the quantum state $\phi = 0$ is in an absolute state of being as all observers will measure the same numerical values for the any observable emerging from such as system. Furthermore, from (4.16), it is clear that the thermodynamic quantum state of a system as defined by ϕ, does affect the velocity of particles in this system as seen by external observers.

Additionally, if the ideas propagated herein prove to have any correspondence with physical and natural reality, then, it is safe to say that the present DSR theory predicts that once a system attains a temperature $T = T_p = 4.49 \times 10^{31}$ K, this system is an absolute and universal state of existence because for such a system, events therein give identical measurements by every other inertial observers, *i.e.* their time and space intervals are absolute and universal to everyone in the Universe. However, if $(0 < \phi < 1 :\Longrightarrow \delta x^{\mu} > \ell_p)$, then, the state as just described is simple unattainable, it is an impossible state to be achieved by any physical system. From the foregoing discussion, the need to have engraved in the Laws *of* Nature the invariance of ℓ_p becomes crystal clear, it can not be over-emphasised nor over-stated.

5 Discussion and Conclusion

5.1 General Discussion

We have presented a new position space DSR theory. This we have done in the light of the prevalent view that DSR theories are considered to be highly speculative theories existing with no evidence from experience. Because of their shear lake of contact with experience, there are considered by the majority of the High-Energy Physics community as not so promising. Be that it may, true is also the fact that the progress of science deepens not only on the exploration of promising ideas. Sometimes, we have to explore ideas that spring logically from the present knowledge. DSR theories exist on a firm logical footing of the requirement to marry the very small (quantum physics) and the very large (general relativity) into a quantum theory of gravity.

Our desire here has been to fill in the gap of the implied existence of a fundamental unit of length (and time) as urged in Nyambuya (2010). This fundamental unit of length has been identified with the Planck-length as is the case with all DSR theories in existence today. The work by Amelino-Camelia (2002a, b) set the stage for Planck-length motivated DSR theories. These searches (DSR theories) seek a quantum gravity theory from a bottom top approach as compared to the top-down approaches of say string theories (and related theories).

On that pedestal, if the ideas set-forth in Nyambuya (2010) are to be believed or acceptable as plausible, then, space must be granular and time is not continuous. These suggestions are directly implied by the upper cosmic speed, *i.e.* the speed of light in vacuum. What we have done here is to suggest that these points must fluctuation, thus, we envision a granular spacetime with discrete points that are under constant random quantum mechanical fluctuations.

Additionally – out of shear curiosity; if the fluctuations $(\delta x^{\mu} \geq \ell_p)$, represent events in the observable (measurable) part of the Universe, it is natural to ask if there exists events such that $(\delta x^{\mu} < \ell_p)$? Obviously, if such events exists, they must exist in the non-observable (non-measurable) part of the Universe. There is no *priori* nor *posteriori* reason to think that events such that $(\delta x^{\mu} < \ell_p)$ must not exist. Thus, there is no *priori* nor *posteriori* reason to believe that everything that exists must be observable (measurable). This brings us to the question of darkmatter and darkenergy. We have no desire to touch these topics of darkmatter and darkenergy but merely want to raise this to the attention of the interested reader that may take this idea further.

A noteworthy point, is that, one considerable, significant and perhaps important improvement to Einstein's STR that has been made in the present DSR theory by the introduction of the scaler ϕ, is the removal of the singularity occurring at $|\boldsymbol{v}| = c$. Whatever interpretation that can, may or might be given to this function, one thing is clear, if it is restricted to values: $(0 \leq \phi < 1)$ this singularity vanishes. It is surprising that this singularity in the STR has been welcome as pointing to the fact that the light speed barrier is un-attainable while on the same footing and pedal, the general relativity singularity occurring when an object attains the Schwarzchild radius has been rejected with researchers advocating for a modification of general relativity around that Schwarzchild radius regime. Both singularities must either be reject or welcome. We should not welcome one and reject the other.

Another way of looking at this speed singularity is that the introduction of the δl dependence into the Lorentz transformation formulae – is effectively replacing c with a δl dependent speed *i.e.*, $c' = c/\phi(\delta l)$. Written in this way, one can equality claim that one must identify c as a singular speed, rather than adding it into the theory by the *lathe of hand* as is normally done. That is to say, one postulates a universal and absolute cosmic speed limit which is the same for all observers, where this speed limit is dependent on the spacetime fluctuations δl. This is an equally valid way of picturing the present modification. The speed of light in empty space becomes dependent on the spacetime fluctuations in such a manner that the upper cosmic speed limit of material particles can never attain the speed $c = 2.99792458 \times 10^8\,\mathrm{ms}^{-1}$, which itself becomes the singular speed.

5.2 Conclusion

Assuming the correctness (acceptability) of what has been presented herein, in a rather succinct manner, we hereby make the following conclusions:

1. *Without having to drastically modify the Lorentz transformations – as demonstrated herein; it is possible to achieve a position space DSR theory where an invariant scale length which is the same for all observers everywhen and everywhere in the Universe.*

2. *The proposed position space DSR allows for ponderable material particles whose speeds are less than the speed of light to be accelerated to the speed of light without encountering a singularity when $|\boldsymbol{v}| = c$ is attained, as is the case with Einstein's STR. This means that, the Einstein singularity encountered at $|\boldsymbol{v}| = c$ in the STR, has here been eliminated.*

3. *It has been demonstrated that as the spacetime fluctuations approach the Planck-scale, spacetimes tends to become absolute, that is, they are the same for all observers everywhere in the Universe.*

4. *This work must be taken more as an intellectual exercise that is yet to find contact with experience (i.e. observational and experimental science).*

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