Article

On Lorentz-invariant Theory of Gravitation Part 2: The Nature of Pre-spacetime & Its Geometrization

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Abstract

In the modern theory of gravity - general relativity - the cause of the gravitational field is the curvature of space-time, which is not a material object. The question of, how non-material space-time can cause a material gravitational field is still here unanswered. In the framework of proposed electromagnetic theory of gravitation, which is based on the nonlinear theory of elementary particles (NTEP), it is assumed that there is a material carrier, whose characteristics allow us to introduce the concept of space-time, uniting general relativity and quantum field theory. This medium we conventionally call "pre-spacetime." The purpose of this paper is to determine the nature of pre-spacetime and methods for its geometrization.

Keywords: gravitation theory, Lorentz-invariant gravitation theory, non-linear quantum theory, space-time background, physical vacuum.

1. Statement of problems

1. Einstein's theory is a theory of the macrocosm. The basis of this theory, which is conventionally called general relativity, is the notion of space-time. On one hand, the gravitational field is here a consequence of the curvature of space-time, which generates a pseudo-Riemannian space-time. However, the gravitational field is declared as cause of the curvature of space-time and the appearance of a pseudo-Riemannian space-time. Obviously, the simultaneous existence of these two statements does allow neither to determine the cause of gravitation, nor to give a definition of the space-time.

In general relativity, as a representative of pseudo-Riemannian space-time is defined the mathematical object, which called metric tensor. However, the elements of the metric tensor are relative dimensionless quantities, i.e., non-material objects and do not correspond to any material objects of nature. On the other hand, the gravitational field is a material field. In general relativity, the question how can non-material space-time produce a material object, is also not explained.

2. Our approach to the theoretical description of gravity is based on the quantum theory of elementary particles, expressed as a nonlinear form of an axiomatic theory of elementary particles (NTEP). This approach is based on the fact that elementary particles are the primary elements of matter, which generate a gravitational field and are involved in the gravitational interaction. Obviously, the existence of the gravitational field of macroscopic objects (planets, stars, etc.) is the consequence of this fact. In other words, the macroscopic theory of gravity must be a consequence of gravitational theory of the microworld, but not vice versa.

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The elementary particle theory describes the behavior of particle-fields in space-time. This behaviour is characterized by the invariance with respect to Lorentz transformations. Such space-time is called pseudo-Euclidean space-time. But this space-time is not itself a cause of material field production. It performs the usual role of some parameters, describing the existence of fields. Since the elementary particles are the primary material objects of the Universe, we must assume that namely the pseudo-Euclidean space-time is the primary space-time of the Universe. But then there is a need to link it with the pseudo-Riemannian space-time of gravitation theory.

3. In our approach (see articles, outlining NTEP in the “Prespacetime journal”), we assume that the gravitational field is a consequence of the "curvature" (i.e., nonlinearity) of the electromagnetic field. Indeed, as is shown in the framework of NTEP, the movement of EM fields in a closed curved path is the origin of appearance of massive particles. In this case, the "bending" of the field, and not a massless mathematical object, as space-time, produces the matter. At the same time it generates the material gravitational field. It follows that the gravitational field is some residual non-linear electromagnetic field. Therefore the following question arises: what the space-time curvature of general relativity doing here?

4. Indeed, despite these oddities, the GTR is tested well enough, at least for weak gravitational fields. Therefore, the principle of correspondence between the old and new theories requires the solution the question of how the description of gravity in general relativity by means of "curvature" of space-time is connected with the description the origin of gravitation through the "bending" of the fields in NTEP. In other words, we must answer the question, of what is there in nature - physical, material, that is mathematically described as space and time, and how does it relate to the conversion of matter fields.

This (yet unknown) physical body we will call pre-spacetime. This article is devoted to the elucidation of its nature and connection with the geometric space-time. To avoid controversy, we will specify the initial definitions of the concepts adopted in modern physical theory.

1.1. Some definitions

1. (PhED, 1965): "**Space and time** are common forms of coordination of material objects and their states.

Space is a set of relations, expressing the coordination of coexisting objects: their location relative to each other and relative magnitude (distance and orientation).

Time is a set of relations that express the coordination of successive states (events): their sequence and duration. "

The abovementioned coordination is given by the reference frame system and linked to them coordinate systems.

2. (TD, 1989): "**The reference frame system** is a real or conditionally solid body, which is connected with the coordinate system, equipped with a clock and used to determine the position in space of the physical objects (particles, bodies, etc.) at different time points. Often as frame of reference is understood the system of coordinates, equipped with a clock”.

Under the conditionally solid body is meant a system of rigid bodies, whose positions relative to each other are the same. For example, in astronomy, such a system for the solar system is the Sun (as the center of the system) and two or three stars in the Universe, which are with very good approximation fixed relatively to each other (for example, the Sun, the Pole Star, etc.)

3. (PhED, 1962): "**Coordinates of point** are the numbers that define its position on a surface in space."

The system of coordinates (or reference system) is a method to specify uniquely the coordinates of the point. Usually, the coordinate system is defined by means of a point called the center of coordinate system, and one or more lines associated with this point and with each other. In the simplest case straight lines are selected. But there are also countless other selections of lines.
As straight lines trajectory are taken the trajectories of the rays of light (i.e., photon trajectories) at infinite distance from the material bodies (i.e., in very weak fields). Therefore, a straight-line (Cartesian) coordinate system plays in physics the role of the standard for all other coordinate systems.

The unambiguity of the coordinate system setting in a physical theory is determined by its binding to a real or notional reference frame system.

2. Pre-spacetime in terms of quantum field theory and general relativity

At this stage of research we give a naive definition of a pre-spacetime, as volume containing the moving material bodies and fields. This allows us to enter the relationships that define the space and time according to the above definitions. According to the objectives of the study, we, first of all, should answer two questions.

What is a pre-spacetime of Universe, i.e., a volume containing all the bodies, which we feel by our sense organs and the experimental devices? If we remove all material bodies and fields of the Universe, will it be empty, or will it be filled with something?

At the times of K. Maxwell, J.J. Thomson and H. Lorentz, scientists thought that this pre-spacetime of the Universe is not empty, and it is filled with some medium - electromagnetic ether.

(Lodge,1909): “Introduction. The problem of the constitution of the Ether, and of the way in which portions of it are modified to form the atoms or other constituent units of ordinary matter, has not yet been solved… Meanwhile there are few physicists who will dissent from Clerk-Maxwel’s penultimate sentence in the article "Ether," of which the beginning has already been quoted:

“Whatever difficulties we may have in forming a consistent idea of the constitution of the ether, there can be no doubt that the interplanetary and interstellar spaces are not empty, but are occupied by a material substance or body, which is certainly the largest, and probably the most uniform body of which we have any knowledge.”

What do we have in this regard in modern physics? According to the prominent Russian Academician A. Migdal (Migdal, 1982): "The history of the ether provided us with a good example of intertwining of old and new ideas.

In the XIX century ether was endowed with contradictory properties to explain the laws of propagation of light in vacuum and in moving bodies. The special theory of relativity resolved all the contradictions of ether. Moreover, the need has disappeared in the very concept of the ether. However, it later emerged that emptiness - former "ether" - is not only the carrier of electromagnetic waves; the continuous oscillations of the electromagnetic field ("zero-point oscillations"), creation and annihilation of electrons and positrons, protons and antiprotons and, in general, of all elementary particles occur in it. If e. g. two protons collide, these "virtual" particles can become real: from "emptiness" the bundle of particles is occur in it.

It appears that emptiness is a very complex physical object. In fact, physicists have returned to the concept of "ether", but without the controversy. The old concept was not taken from the archive - it arose again in the process of science development. We call the new ether "vacuum" or "physical emptiness." But the history of the ether did not end here.

The theory of relativity is based on the assumption that in our world there is no selected coordinate system and therefore there is no absolute velocity; we observe only the relative motion. However, the selected coordinate system was introduced in our universe with the discovery of cosmic microwave background (CMB). This is a system, in which the CMB photons' distribution over velocities is spherically symmetric (as the gas particles in a stationary box).

In the "new ether" there is absolute velocity. However, the conclusions of the theory of relativity are conserved with enormous precision in accordance with the "correspondence principle".
Indeed, the cosmic microwave background (CMB) has temperature anisotropy (see image 2.1)

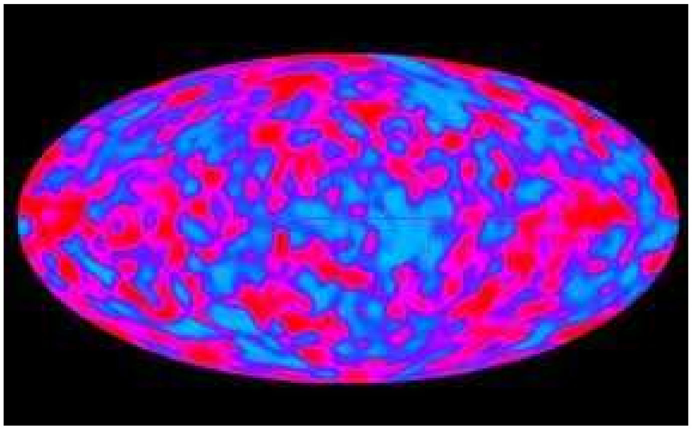

Fig. 2.1

This anisotropy allows us to introduce a single coordinate system for the entire Universe (roughly speaking, it allows us to fasten it to the inhomogeneities of the temperature distribution of microwave radiation). In this sense this system can be called "absolute". However, since the electromagnetic ether obeys the Lorentz transformations, as emphasized Oliver Lodge and Gustav Mie, all inertial coordinate systems are equal here, and the "absolute" frame of reference is one of them. Hence, indeed, the principle of relativity in such "absolute" space is not violated.

These conclusions of A. Migdal certainly do not contradict the fact that he wrote the creator of the theory of relativity, Einstein.

(Einstein, 1920): “More careful reflection teaches us, however, that the special theory of relativity does not compel us to deny ether. We may assume the existence of an ether,; only we must give up ascribing a definite state of motion to it, i.e. we must by abstraction take from it the last mechanical characteristic which Lorentz had still left it. We shall see later that this point of view is justified by the results of the general theory of relativity...

The special theory of relativity forbids us to assume the ether to consist of particles observable through time, but the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be on our guard against ascribing a state of motion to the ether...
But on the other hand there is a weighty argument to be adduced in favor of the ether hypothesis. To deny the ether is ultimately to assume that empty space has no physical qualities whatever. The fundamental facts of mechanics do not harmonize with this view...

Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable inertia, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it...”

Ether, about which Einstein speaks, was conceived as a continuous medium of unknown origin, whose properties are such that its fluctuations are electromagnetic waves (light, etc.). This ether is known as "electromagnetic ether" (briefly, "EM-ether"). Since the detection of the Lorentz transformations and then the construction of special relativity theory, Einstein pointed out that the existence of EM-ether does not contradict the theory of relativity (contrary to the mechanical ether of pre-Maxwellian times). Thus, Gustav Mie, who is known as the creator of the first unified theory of gravitation and electromagnetism, who based all his research on the special theory of relativity, emphasized this feature of the EM-ether (Mie, 1925): "EM ether, contrary to matter, can not itself be noticeable, since it is homogeneous and does not have any irregularities, which can be measurable."

Unfortunately, the mathematical description of the ether in classical physics has not been developed. But in the framework of quantum electrodynamics (QED) has been shown that the quantization of the electromagnetic field discloses the existence of medium like ether, which was named “physical vacuum". Comparison shows that the physical vacuum is equivalent to quantized electromagnetic ether (Kyriakos, 2010); (for details see below the analysis of this problem by Zel'dovich et al.).

But, the theory of physical vacuum has not yet been developed, (Dirac, 1957: "Until now, there are significant difficulties with the description of the physical vacuum ..."). Many well-known physicists have noted that the theory of vacuum is necessary to complete our knowledge about the world.

3. Space-time in general relativity

3.1. The cosmological constant (Zel'dovich and Novikov, 1971)

The general requirements usually placed on the equations of the theory of gravitation permit one to write a variational principle with the action in the form

$$S = -mc\int ds - \frac{c^3}{16\pi\gamma}\left[\int R dV + \int 2\Lambda dV\right], \qquad (2.3.1)$$

(where V is the four-dimensional volume)

The corresponding field equations have the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = \frac{\kappa}{c^2}\tau_{\mu\nu}, \qquad (2.3.2)$$

Here Λ is the so-called cosmological constant, and quantities that are proportional to it $(\Lambda dV, \Lambda g_{\mu\nu})$ are called cosmological terms. These field equations obviously satisfy the condition of local Lorentz-invariance and include the equations of motion in the same sense as the equations without Λ do $\tau^k_{i;k} = 0$, as before.

Einstein initially selected Λ in such a manner as to obtain a stationary cosmological solution with a mean density $\tau^0_0 = \rho c^2 = const$ that is different from zero; for this, it is necessary that $\Lambda = \frac{8\pi\gamma\rho}{3c^2}$. After the discovery of the cosmological redshift, Einstein preferred the equations with $\Lambda = 0$.

Both stationary and non-stationary cosmological solutions with $\Lambda \neq 0$ were investigated in detail before 1930; however, until 1967 there were no observational indications as to the necessity or even desirability of introducing Λ. Since 1967, observational data on quasars have suggested that A might not actually be zero, but might instead have a value of the order of $\Lambda \approx 10^{-55}$ cm^{-2}.
At present this hypothesis is not at all proved; in fact, it encounters difficulties in explaining the quasar observations. However, in the course of the discussions it has become apparent that the simplest assumption of $\Lambda = 0$, while not refuted, has not been distinctly and uniquely proved.
How can the physical meaning of the cosmological constant be understood? Why, in fact, is it interesting for physics as a whole?

One approach was prompted by the dimensions of $[\Lambda] = cm^{-2}$. In this approach one views Λ as the curvature of empty space. But the theory of gravitation links the curvature to the energy, momentum, and pressure of matter. Putting the terms with Λ onto the right side of the field equation, we obtain

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi\gamma}{c^4}\tau_{\mu\nu} - g_{\mu\nu}\Lambda, \qquad (2.3.3)$$

The assumption that $\Lambda \neq 0$ means that empty space creates a gravitational field identical with that in the theory with $\Lambda = 0$, but with matter filling all of space-time with a mass density $\rho_\Lambda = \frac{c^2\Lambda}{8\pi\gamma}$, an energy density $u_\Lambda = \frac{c^4\Lambda}{8\pi\gamma}$, and a pressure $P_\Lambda = -u_\Lambda$.

In this sense one can speak about the energy density and the pressure (stress tensor) of the vacuum.

Notice that our assumptions about P_Λ and u_Λ were formulated in such a manner that the relativistic invariance of the theory was not broken; P_Λ and u_Λ are the same in all coordinate systems moving relative to each other (Lorentz- transformed).

These quantities do not make themselves felt either in elementary-particle experiments or in atomic and molecular physics: the vacuum energy of the vessel in which an experiment takes place plays the role of a constant term which can be canceled in the law of energy conservation.
The only sort of phenomenon in which P_Λ and u_Λ manifest themselves is a gravitational one. In this case P_Λ and u_Λ "work" not only in empty space; they are, as is clear from the formulae (2.3.2), full and equal members of the field equations even when normal matter is present.
A practical measurement of the influence of P_Λ and u_Λ is not possible, either in laboratory experiments or in observations of planetary motions in the solar system or stellar motions in the Galaxy.

Let us dwell on the nature of Λ. One can take the viewpoint that a certain Λ and corresponding ρ_Λ, u_Λ and P_Λ are universal constants that do not require any further explanation. A different viewpoint is also conceivable: assume that in some zeroth-order approximation $\Lambda = \rho_\Lambda = u_\Lambda = P_\Lambda = 0$. Higher-order values that are different from zero and characterize the vacuum might be derivable from considerations of the theory of elementary particles. There is no such derivation at the present time, any more than there is a theory proving that $\Lambda = 0$. Below we present some ideas on how to create a theory of Λ.

3.2. Physical vacuum (Zel'dovich,1981)

“The vacuum is space in which there are no elementary particles. But does a vacuum theory exist? Can one say something meaningful about vacuum, i. e., about space containing nothing?
The present-day rich and complicated picture of the vacuum arises as a logical consequence of experiments and theories. One cannot say that the vacuum, i. e., empty space, is devoid of all properties "by definition".

We define vacuum as space without any particles. Such a definition coincides with the condition of a minimum of the energy density in the given volume of space. If the energy *E* of some region of space is greater than the minimal value ε_{min} for this region, then ε can be represented as the sum $\varepsilon_{min}+\Delta$, and the addition Δ can be regarded as the energy of the field or particles present in the given volume. Hence, a state with $\varepsilon > \varepsilon_{min}$ should not be called "vacuum.

"But the actual properties of the "minimal" state which is called the "vacuum" are dictated by the laws of physics, and we cannot insist that the minimum be zero or that the simplest possible situation be as simple as we wish.

The theory of electromagnetism leads to the conclusion that besides the static electric field surrounding charges there also exist specific solutions in the form of fields propagating freely in space and describing electromagnetic waves (radio waves, light, X- rays, gamma rays).

The modern view of electromagnetic waves emphasizes their similarity to a mechanical oscillator, i. e., a mass on a spring.

If one writes down the corresponding equations (which we shall not), it is found that the magnetic field plays the part of the spring, i. e., the energy of the magnetic field is analogous to the deformation of a spring, depending on the departure from the equilibrium position. The energy of the electric field is the analog of the kinetic energy of a moving particle. Thus, each definite oscillation mode of the electromagnetic field is analogous to the mechanical vibration of a mass on a spring.

Thus, in classical (but not quantum !) theory the vacuum concept is indeed rather simple — there is neither field nor energy.

Now quantum mechanics appears on the scene. The momentum and the coordinate of the mass cannot have definite values simultaneously. Applied to the electromagnetic field, this means (tot the magnetic and electric field cannot vanish simultaneously.

Quantum mechanics predicts that the possible values of the total energy of an oscillator are $\varepsilon_n=(n + 1/2)h\nu$ with arbitrary integral n, where h *is* Planck's constant and ν is the oscillator frequency. Thus, one can have only states with an energy value in the sequence:

$n = 0$, $\varepsilon_0 = (1/2)\, h\nu$, $n = 1$, $\varepsilon_1 = (3/2)\, h\nu$, $n = 2$, $\varepsilon_2=(5/2)h\nu$,...

If an oscillator can exchange energy with other objects, then it gives up or receives energy only in definite portions, which are multiples of $h\nu$.

In a transition $n = 1 \rightarrow n = 0$, the oscillator gives up $h\nu$; in a transition $n = 0 \rightarrow n = 2$, it acquires $2h\nu$, etc. But here we wish to draw attention to the mysterious "halves," i. e., the value $(1/2)h\nu$ of the oscillator groundstate energy.

Experiments with atoms and molecules confirm the presence of the "halves." It is impossible to use quantum mechanics and avoid this result.

By analogy, one could readily believe that the application of quantum theory to the electromagnetic field will necessarily lead to a similar result. Indeed, the electric field and the magnetic field cannot vanish simultaneously; the electromagnetic energy density cannot vanish. One can pose the question of the minimum of the energy in the same way that one can speak about the lowest (ground) state of an oscillator. It is clear however that this minimum is not zero.
To get any further, we must now make more precise what we mean by the modes of the electromagnetic waves and consider what are the quantities that occur in the expressions relating to electromagnetic wave. It is important that the appropriate variables, i. e., the analogs of the position and velocity of the mass, are not the magnetic and electric field at one point of space; for Maxwell's equations contain derivatives with respect to the spatial coordinates, and the evolution of fields at a given point depends on the values of the fields at other points of space.

This circumstance makes it necessary to consider individual waves, which are independent of each other.

Why is the possibility of describing the solution in the form of a system of independent equations for individual oscillators so important? One answer, seen immediumtely in the 19th century, is that if a set of particular solutions is known it is possible to construct a solution to the problem with arbitrary initial conditions. For we are concerned with a linear equation, and any sum of particular solutions is also a solution.

Different initial conditions give a different set of quantities a_n and φ_n in the general expression $y = \sum a_n \cos(\omega_n t + \varphi_n)\sin \pi n x/l$.. There is however a deeper reason for using solutions of this type.
The point is that these solutions can be numbered and ordered. They can be arranged in a sequence with increasing value of the frequency. One can find the number of solutions with frequency less than a definite value or in a given interval of frequencies. In particular, for electromagnetic radiation in volume V the number of such solutions is $dN = V(8\pi\nu^2 \mathrm{d}\nu/c^3)$.
It is here understood that the frequency ν is such that the corresponding wavelength $\lambda = c/\nu$ is less than the linear dimension of the container $d \sim V^{1/3}$, and we consider an interval $d\nu$ that is not too narrow, so that $dN = 8\pi(V/\lambda^3)d\nu/\nu \gg 1$ (despite $d\nu/\nu \ll 1$). Accordingly, the total number of solutions with frequency less than the given ν (per unit volume) is $n = (8\pi/3)(\nu/c)^3 = 8\pi/3\lambda^3$."'
For a string, bell, and so forth there is a physical restriction, namely, the minimal wavelength of the vibrations cannot be less than the distance between the atoms. But in vacuum there is no definite minimal wavelength! Accelerator experiments study photons with an energy of about 10^{10} eV, and their wavelength is $\lambda \approx 10^{-14}$cm.

In cosmic rays, we observe photons of even higher energy and shorter wavelength. But more important is the argument of relativistic invariance: There is not and cannot be a limit to the photon energy or wavelength because these are quantities that depend on the motion of the observer. For an oncoming observer, the energy will be higher, the wavelength shorter.
The vacuum has an infinite number of vibration modes, or, more precisely, an infinite number of vibrations per unit volume of the vacuum. Theory must take into account this fact and must be able to overcome the difficulties— computational and conceptual, i. e., "physical" associated with this fact.

3.3. Vacuum energy density

We now turn to the above assertion ($\varepsilon_0 = (1/2)h\nu$) follows from quantum theory. In granting a modest $0.5h\nu$ to each individual wave, we soon discover that when all the waves are taken together they give an infinite energy density. If we were to restrict ourselves to a definite maximal frequency ν_m, we would obtain a result of the $u = a\int_0^{\nu_m} \frac{1}{2} h\nu \cdot \nu^2 d\nu = (ah/8)\nu_m^4$, where u is the energy density and a is a constant ($a = kc^{-3}$, where c is the velocity of light and k is a number of order unity). In the limit $\nu_m \to \infty$, the value of e also tends to infinity. If we set $\nu_m = \infty$ directly, we obtain a divergent integral.

This is the well-known divergence problem, the socalled "ultraviolet catastrophe" of quantum electrodynamics or, rather, it is part of this problem.2) And there is no simple escape; one cannot ignore or simply reject the problem. The nonvanishing fields in the absence of photons (the fields corresponding to the "halves" (½)$h\nu$ for all possible ν) are observed, and they modify the motion of electrons in atoms. The famous Lamb-Retherford experiment confirms this.

We note also a phenomenon associated with the idea of the zero-point energy - the (Casimir and Polder, 1948; Casimir, 1948; Barash and Ginzburg, 1975).

Casimir calculated a more subtle effect, namely, he found the dependence of the zero-point energy on the mutual position of the bodies, for example, on the distance between the plates of an uncharged capacitor. But the derivative of the energy with respect to the displacement is the force acting in the direction of the displacement. This quantity is finite and the corresponding integrals converge.

However, such a favorable situation does not occur in all phenomena, and the density of the zero-point energy does not always cancel. The most important manifestation of the nonzero vacuum energy density could be its influence on the gravitational force field and on the gravitational potential. The theory of gravitation contains the energy density of a body, including the energy density of the vacuum within the body and the surrounding space.

In this case, we are not speaking of energy differences, which could be zero. At first glance, we face an ineluctable contradiction. In principle, the contradiction could perhaps be avoided by taking into account the contribution of other particles. We shall merely emphasize here that this fundamental possibility has not yet been realized by modern science quantitatively and exactly!
More generally, the positive contribution of bosons could in principle be compensated by the negative contribution of the fermions.

Nevertheless, the most important theoretical question — that of the vacuum energy density— remains unanswered. Only astronomy gives definite strong restrictions (see Weinberg, 1980).
How would a finite energy density be manifested? In relativity theory, it is necessary that this energy density be the same for any observer. This leads to the condition that the pressure (tension) is the same in all directions and equal to $p = -u$, where p is the pressure and u is the energy density of the vacuum. As early as 1917, Einstein considered the possibility that the vacuum energy density could be nonzero. He used a different terminology and introduced the "cosmological constant" Λ , which is proportional to u. This name emphasized that such an energy density would have its strongest influence on cosmological phenomena.

4. On the possibility of calculating the gravitation constant from elementary-particle theory (Zel'dovich and Novikov, 1971)

As in Newtonian theory, so also in general relativity the gravitation constant γ is considered a universal constant to be determined by experilnent.

However, such an attempt has been made by Sakharov (Sakharov, 1967). This attempt is described below. So far this attempt has not led to any concrete results. Sakharov's formula for G contains another unknown quantity. Nonetheless, the novelty of Sakharov's approach to this deep problem of principle by itself justifies our discussing it.

Sakharov's starting point is the typical GTR viewpoint, which connects gravity with the concept of spacetime curvature. The essence of GTR is contained in the expression for the action

$$S = -mc\int ds - \frac{c^3}{16\pi\gamma}\int RdV \ , \qquad (2.4.1)$$

The first term in this equation is a sum over the trajectories of all particles.

By varying the trajectories of the particles in a space of fixed metric, we obtain the law of particle motion from the extremal condition on *S*. In curved space the action associated with a trajectory depends on the curvature, so that the first term in equation (2.4.1) takes into account the influence of the gravitational field on the particles' motion.

By varying the metric in the expression for *S* we obtain the gravitational field equations. Symbolically,

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{2c}\tau_{\mu\nu} - \frac{c^3}{16\pi\gamma}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 0\,, \tag{2.4.2}$$

The first term in this expression comes from the first term in equation (2.4.1); the second arises from the curvature integral, i.e., from the second term in equation (2.4.1).

Roughly speaking, the first term describes the force with which the particles curve space. Such a description is in accordance with the principle of the equality of action and reaction; there is a reciprocity between the action of the curvature on the motion of the particles and the action of the particles on the curvature. This reciprocity manifests itself in the fact that both effects are obtained from one expression, $-mc\int ds$, where ds is calculated in Riemannian spacetime.

The terms containing R and $R_{\mu\nu}$ in formulae (2.4.1) and (2.4.2) can be interpreted as describing the elasticity of space - i.e., the "attempt" of space to remain flat, the resistance of space to being curved.

The constant c $\frac{c^3}{16\pi\gamma}$ associated with the elasticity of the vacuum is the quantity which we wish to calculate. This general-relativistic constant is huge in magnitude (recall that in Newtonian theory we dealt with the inverse of it, which was very small). So that we may deal with a dimensionless quantity, let us think about this elasticity of the vacuum in the following manner: The mass *m* of an elementary particle spread over its Compton wavelength, $\hbar$/mc, creates a very small curvature of space because the inelasticity of the vacuum, which resists the curvature, is so large.

Let us remind ourselves that here and in the discussion that follows, the curvature of space and general relativity are considered from a non-quantum, classical, deterministic viewpoint; whereas the motion of elementary particles through this classical curved space is governed by quantum-mechanical laws. We have not yet overstepped the bounds of conventional general-relativity theory; at most, we have only added decorative words to it.

Now we turn to the goal of Sakharov's discussion, the attempt to calculate the "elasticity" of the vacuum, $\frac{c^3}{16\pi\gamma}$.

The vacuum elasticity depends on the effects of the curvature of space on the quantum-mechanical motion of particles. One can say in this sense that the goal is to obtain the second term in equation (2.4.1) from the curved-space laws of motion, i.e., from the first term in equation (2.4.1), as written out for quantized particles. Recall that we are concerned here with virtual particles which are created alone (e.g., photons) or in pairs ($e^- e^+$), and with the vacuum, i.e., with space in which there are no real particles.

It follows from the general considerations of relativistic invariance and from dimensional considerations that the correction to the action must depend on the invariant R and must diverge. This means that a finite answer can be obtained only if we suppose that quantum theory breaks down above a certain cutoff momentum p_0.

The correction to the action is

$$\frac{kp_0}{\hbar}\int RdV ,$$

where k is a dimensionless numerical factor of the order of unity (We cannot even discuss the exact value of k today because we have no concrete picture of what happens for momenta equal to or larger than p_0. Even the sign of k is not clear). It is assumed that quantum effects alone completely determine the vacuum elasticity. Put the other way around, Sakharov presumes that the gravitation constant, which we usually find from experiment, can be calculated, at least in principle, from the condition

$$\begin{gathered}\frac{c^3}{16\pi\gamma}\int RdV \equiv \frac{kp_0}{\hbar}\int RdV, \gamma = k'\frac{c^3L^2}{\hbar} \\ L = \frac{\hbar}{p_0}, k' = \frac{1}{16\pi k}\end{gathered} , \tag{2.4.3}$$

The details of the calculations can be found in the original work of Sakharov. In order to obtain the observed value of the elasticity, one must put the cut-off momentum equal to an enormously large value-a value corresponding to the mass 10^{-5} g. Since we do not know of any elementary particle with such a mass, our theory for calculating one quantity, $\frac{c^3}{16\pi\gamma}$, containing the unknown γ depends upon another unknown quantity, p_0.

Thus, the formula remains the same as it always was:

$$m_g^2 c^2 / \hbar = c^3 / \gamma , \tag{2.4.4}$$

Our new progress lies in the realization that this formula should be read from right to left as a definition of γ.

Notice, stepping beyond the bounds of gravitation theory, that Sakharov's ideas can probably be extended to electrodynamics and to the theory of weak interactions (Zel'dovich 1967). One usually writes the action in electrodynamics as

$$S = -mc\int ds - \frac{e}{c}\int A_\mu dx^\mu - \frac{1}{8\pi}\int \left(E^2 - H^2\right)dV , \tag{2.4.5}$$

where the first term is the action for free, charged particles; the second is the action for the interaction between the charged particles and the electromagnetic field; and the third is the action of the free electromagnetic field. Sakharov's train of thought suggests that we take the first two terms as primary and obtain the third (the action of the field) as a result of quantum vacuum corrections, just as we did for the term $-\int RdV$ in the theory of gravity. In the remarkable work of Landau and Pomeranchuk (1955) one can find justifications for such an approach.

One can introduce a similar viewpoint into the theory of the intermediate charged W -bosons, which characterize the weak interactions according to $n = p + W^-, W^- = e^- + \overline{\nu}$.

In this case the theory leads to the conclusion that the mass of a photon must be zero, in contrast to the mass of a *W*- boson. One and the same value of p_0 (under certain assumptions about the spectrum of the fermions, masses) gives the correct magnitudes for the gravitation constant, for the charge of an elementary particle, and for the weak-interaction constant.

5. Geometry of empty space and non-empty curved space

Let us consider the behaviour of the vacuum in the case of non-empty curved space.
What happens to the empty space when the material body is entered it?

The result (Zel'dovich, 1981) “can be interpreted as a change in the gravitational constant: $\gamma m + 0.01\gamma m = \gamma \cdot 1.01m = \gamma' m$. It is γ' that we observe and measure, so that for the whole of macroscopic physics the "old" unobservable value of γ is unimportant, and, these problems being interrelated, we do not face the problem of establishing what is the real contribution of the vacuum, which we took above arbitrarily, for illustration, to be 0.01.

A similar procedure was used for the first time in the fifties in quantum electrodynamics in connection with the electric charge: a free charge produces a vacuum polarization charge, perturbing the motion of charged particles (electrons, etc.) in the Dirac sea of states with negative energy, which are everywhere present in the vacuum. With each electric charge there is associated a transformation $e \rightarrow e'$ like the transformation $\gamma \rightarrow \gamma'$ for the gravitational constant. This procedure is called "charge renormalization."

It can be carried out even if the ratio e/e' is infinite. The theoreticians developed schemes for calculating all observable effects using only the observed value e'. However, it is necessary to emphasize a difference between electrodynamics and the theory of gravitation. In electrodynamics, one can study the interaction of two elementary particles at a very short distance.

We can study the gravitational interaction only at the macroscopic level, and therefore the experimental investigation of the gravitational vacuum polarization is at present outside the scope of the possible. We must content ourselves with an analysis of the theoretical conclusions.»
From astrophysical data we know that the vacuum energy in flat space is very small. The part of the vacuum energy in curved space proportional to the energy of the matter producing the curvature is manifested in a change in the previously unknown gravitational constant, and in this sense is unobservable.

The notion of a possible change in the gravitational constant due to vacuum polarization does not change the form of the equations but changes their meaning. Sakharov has conjectured that the gravitational constant is entirely determined by vacuum polarization.

The equations of gravitation can be perspicuously interpreted as a manifestation of the elasticity of space-time”.

6. The “bending” of space-time by electromagnetic field

As we know, the light bending (Kim and Lee, 2011a) by a massive object is one of the prominent features of the general relativity and is a useful tool in astrophysics through the gravitational lensing. A question may be raised whether there is an electrodynamic version of the bending: that is, whether an electric charge can bend light toward, or outward of it.

At classical level the linearity of the electrodynamics precludes bending of light, and therefore any bending must involve a nonlinear interaction from quantum corrections. The Euler-Heisenberg interaction that arises from the box diagram (for example see http://www.hep.ucl.ac.uk/opal/images/gg-scat.gif) in quantum electrodynamics can provide such a nonlinear interaction.

6.1. Extremely strong electromagnetic field

To associate this task with the task of bending of the light beam in general relativity, we will briefly dwell on the concept of the extreme - critical - electromagnetic field.

It is known that the electromagnetic wave field is massless. In this sense, it can not cause a gravitational field (however, according to general relativity, the electromagnetic wave is subject to the gravitational field). On the other hand, as we know, during the passage of electromagnetic waves in the electric field of a heavy elementary particle the photoproduction of massive particles takes place, in particular, of the electron and positron. It is clear that these particles possess a gravitational field. The question is what should be the value of the EM field for the production of massive particles to take place.

As we know (Ternov and Dorofeev, 1994), according to the uncertainty relation $\Delta t \cdot \Delta\varepsilon \geq \hbar$ short-time violation of the energy conservation law is possible, and the virtual electron-positron pair can be created from vacuum, which can exist for a period of time $\tau = \hbar/\Delta\varepsilon \cong \hbar/mc^2$. During this time, the particles can disperse to a distance of no more than $\Delta r = c\tau = \hbar/mc$, i.e., to a distance of order of the Compton wavelength $\lambda = \hbar/mc^2 \sim 10^{-10}$ cm. This is the so-called quantum electron radius, which characterizes the region of possible spatial localization of the electron in quantum theory. (The other - equivalent - interpretation of the uncertainty relations and the following from they conclusions is given in the article (Kyriakos, 2011a)

Now, if the external electric field can produce work ~ mc^2 on an electron at a distance Δr, the pair creation from vacuum is a real process. For this event the field value must be of the order of the critical value E_c:

$$e_0 E_c \hbar/mc = mc^2, \; E_c = m^2c^3/e_0\hbar$$

Under these conditions, the vacuum becomes unstable. E_c is the critical value of external field, at which it becomes possible two, conditionally speaking, phase transitions: on the one hand, massless particle (photon) is transformed into a massive particle; on the other hand, the neutral particle is converted into charged particles. If we talk about the fields of the particles, the critical value of the external field is a field, in which the particle has simultaneously an electric and gravitational fields.

Correspondingly to the above considered critical electric field, there is also a critical magnetic field, if the rotational energy of the electron $\hbar\Omega$ (where $\Omega = e_0 H/mc$ is the cyclotron frequency) is equal to the electron rest energy mc^2 : $\hbar\Omega = \hbar e_0 H_c \hbar/mc = mc^2$, $H_c = m^2c^3/e_0\hbar$ = 4,414 x 10^{13} Oersted.

However, due to the gyromagnetic properties, the magnetic field does not produce work (the Lorentz force is perpendicular to the particle trajectory). For this reason, even in case of excitation by the critical field, the vacuum remains stable. This represents a particular interest for the study of processes in such an extreme field.

Problems of development of quantum electrodynamics in a strong electromagnetic field in the last years have been the focus of attention. During the development of this area it was necessary to push the boundaries of the study of physical phenomena in the region of values of the fields, which were unavailable in the past.

6.2. Vacuum polarization by an external field in quantum electrodynamics

The problem of electrodynamics with a strong electromagnetic field goes back to the early works of Heisenberg and Euler (Heisenberg and Euler, 1936) (see also review (Kyriakos, 2011b)), dedicated to the calculation of vacuum polarization effects, and Sauter (Sauter, 1931), who analyzed the known "Klein paradox" associated with the production of electron-positron pairs in a strong electric field. As this was first pointed by Dirac, even if the external field does not lead to the production of pairs, it affects the electrons and positrons of vacuum, producing a redistribution of charges of vacuum and changing its energy (vacuum polarization).

This phenomenon is due to the fact that under the influence of the external field the energy levels of vacuum electrons are shifted, comparatively with the energy levels of vacuum in the absence of an external field. This change in energy leads to a change in the equations of electromagnetic field, and the Lagrange function is also changed. In this case the Maxwell function:

$$L_M = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right), \tag{2.6.1}$$

is now the first term in the expansion of the full function in powers of constant, characterizing the interaction of electrons with the vacuum field:

$$L = L_M + L_1 + \ldots,$$

where

$$L_1 = \frac{1}{8\pi^2}\int s^{-1} ds e^{-im^2 s}\left[(es)^2 f_2 ctg(esH) - 1 + \frac{2}{3}\left((es)^2 f_1\right)\right], \tag{2.6.2}$$

and f_1 and f_2 are well-known invariants of the electromagnetic field:

$$f_1 = \frac{1}{2H_c^2} e^{\mu\nu\sigma\rho} F_{\mu\nu} F^{\mu\nu} = \frac{\vec{H}^2 - \vec{E}^2}{H_c^2}$$

$$f_2 = \frac{1}{8H_c^2} e^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} = \frac{\vec{H}\vec{E}}{H_c^2}$$

From here follows in particular that the correction to the Lagrangian does not depend on the parameters of a plane electromagnetic wave, i.e., the external field of plane waves does not polarize the vacuum.

From equation (2.6.2) in the special case of weak fields ($\frac{H}{H_c} << 1,\ \frac{E}{E_c} << 1$) Heisenberg and Euler found:

$$L_1 = \frac{1}{360\pi^2 H_c^2}\left\{\left(\vec{H}^2 - \vec{E}^2\right)^2 + 7\left(\vec{E}\vec{H}\right)^2\right\}, \tag{2.6.3}$$

These are the first members of the corrections in the expansion in powers of f_1 and f_2, characterizing the energy shift of the classical electromagnetic field. In the other limiting case of extremely strong field from formula (2.6.2), with logarithmic accuracy we can find that

$$\frac{H}{H_c} << 1, E = 0, L_1 = \frac{e^2}{24\pi^2} H^2 \ln\frac{H}{H_c}\ , \tag{2.6.4a}$$

$$\frac{E}{E_c} << 1, H = 0, L_1 = -\frac{e^2}{24\pi^2} E^2 \ln\frac{E}{E_c} + i\frac{1}{8\pi^3} e^2 E^2 \sum_{n=1}^{\infty}\frac{1}{n^2} e^{-n\pi\frac{E_c}{E}}\ , \tag{2.6.4b}$$

(Amendment for correction of a higher order see (Ritus, 1975)).

Note that the complexity of L_1 means the quasi-stationarity of the vacuum state. The reason of violations of the stationarity is the possibility of the real production of electron-positron pairs from the vacuum when subjected to a strong field.

In the paper (Kim and Lee, 2011a) it is shown that an electric charge bends light toward it through the Euler-Heisenberg interaction, and compute the bending angle and trajectory of light in a Coulombic field. The bending of light by Euler-Heisenberg interaction is not new and has been investigated by several authors, particularly on astronomical objects. For instance, (Denisov, et al., 2001) studied light bending in the dipole magnetic field of a neutron star and (De Lorenci, et al., 2001) studied the light bending by a charged black hole (see also (Kim and Lee, 2011b)). . We develop a simple geometric way of computing the bending angle and trajectory based on the Snell's law.

The box diagram of quantum electrodynamics gives rise to a low energy effective Lagrangian of Euler-Heisenberg (1936)

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2\hbar^3}{90 m^4 c}\left\{\left(F_{\mu\nu} F^{\mu\nu}\right)^2 + \frac{7}{4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right\}, \tag{2.6.5}$$

In the presence of a background electric field the nonlinear interaction modifies the dispersion relation for the electromagnetic wave and results in a modified speed of light that reads

$$\frac{\upsilon}{c}=1-\frac{a\alpha^{2}\hbar^{3}}{90m^{4}c}\left(\vec{\upsilon}^{0}\times\vec{E}\right), \qquad (2.6.6)$$

where $\vec{\upsilon}^{0}$ denotes the unit vector in the direction of propagation. Because the speed of light depends on the field strength the light bends in presence of a nonuniform field. The bending can be studied in geometric optics by noting that the index of refraction of the background field is given in leading order by

$$n=1+\frac{a\alpha^{2}\hbar^{3}}{45m^{4}c^{5}}\left(\vec{\upsilon}^{0}\times\vec{E}\right)^{2}, \qquad (2.6.7)$$

The infinitesimal bending of photon trajectory over $\delta\vec{r}$ can be obtained from the Snell's law…
The comparing of the result with the cross section for Compton scattering shows that the bending effect can easily dominate the Compton scattering, hence may be observable. Nevertheless, the observation may require extreme precision.

6.3. "Electromagnetic" pre-spacetime

Many works are devoted to the study of effects of light propagation in extremely strong electromagnetic fields (see references in the above-mentioned articles). They lead to the following conclusions:

1. The external field of plane waves does not polarize the vacuum.

2. Under the influence of electromagnetic field of sources, the vacuum changes its properties, i.e., is polarized.

3. Because of this polarization the bending of light rays takes place, similar to the one that occurs in a gravitational field.

4. The bending under the action of the electromagnetic field is bigger, than the bending by meansof the gravitational field at comparable energy field densities.

5. The latter does not contradict our assumption that the gravitational field is a residual part of the electromagnetic field.

6. It follows that the vacuum polarization (i.e., pre-spacetime) under the action of the electromagnetic field changes the geometry of space-time.

7. The nature of pre-spacetime

All the existing field theories (theory of gravity, electromagnetic classical theory and quantum field theory) lead to the following interpretations of existing experimental and theoretical knowledge:

The characteristics of the physical vacuum (PhV) play the role of the basis for constructing a geometric space-time (pre-spacetime).

PhV without material particles and fields is unexcited. This means that in the absence of interaction of PhV lies in a basic steady state.

Introduction of material bodies and fields transforms PhV into an excited state. This means that inside the material body and around it, the PhV loses homogeneity and stationarity. In other words it can be said that PhV is perturbed.

The intensity and magnitude of the perturbation of PhV depends on the properties of material bodies and fields, and above all, of their energy-momentum.

For a charged particle this perturbation is an electromagnetic perturbation. The intensity of the perturbation is determined by the electric charge, and in the first place, it obeys Coulomb's law. Due to the fact that this force is strong, the perturbation is very intense. However, since the particle size is small, the perturbation area of PhV is small. It is assumed that in the Universe there are large charged bodies - neutron stars, black holes, etc. and the electromagnetic fields of these objects cover a large area. But this is not experimentally proven.

Most of the large bodies of the Universe are neutral. All of the bodies - both charged and neutral - excite the gravitational field. Its intensity is determined by the gravitational charge, in fact, by mass. Therefore, the gravitational perturbation of PhV from the elementary particle is neglected. But the perturbation of PhV from the planets and stars are very intense, and extend for a considerable distance. Therefore, it is considered that the main source of perturbation of PhV in interstellar space is gravity.

Due to the perturbation of PhV, the individual particles (bodies) interact with each other. This interaction occurs as follows: PhV perturbation caused by a single particle is superimposed on the PhV perturbation caused by another particle, and thus they change their condition and movement. If the perturbation of PhV is relatively small, then the interaction obeys the superposition principle, and it is described by linear equations. If the perturbation is large, the principle of superposition is not valid and the interaction is described by nonlinear equations. But, fortunately, it turned out that perturbation can often be represented as the sum of interactions, whose intensity decreases rapidly. Because of this, it is possible to consider this sum of interactions in accordance with a procedure, called "perturbation theory".

8. Geometrization of pre-spacetime

Thus, we can say that prespacetime, i.e., a basis for the introduction of mathematical space and time, is connected to the PhV and its perturbations. Our analysis allows us to answer the question: how the procedure, which we call geometrization of prespacetime, takes place. To visualize this process, we introduce the concept of the refractive index of physical vacuum.

According to modern concepts, the physical vacuum is a continuous medium consisting of an unknown primary matter (pre-matter). It is described mathematically as some pre-field consisting of a set of oscillators. Using for clarity the analogy with optical media, we can characterize this medium by specific refractive index of PhV. In this case, when we talk about some external fields of bodies (electromagnetic, strong, weak, or gravitational), we can represent the interaction of bodies as follows: one body changes the refractive index of the medium, and on other body changes its state and movement in accordance with the value of this index, and vice versa.

A ray of light is a line on which we build coordinate lines in empty space. In PhV in the non-excited state, the light ray propagates in straight lines. This basic steady state of PhV corresponds to the pseudo-Euclidean geometry, in which the Lorentz transformations take place. If in a limited volume, physical vacuum is excited, i.e., its homogeneity is violated, the light ray will propagate in this part of PhV in curve line. Being in this place, we can not build a rectilinear coordinate system. But we can (in a good approximation) build a rectilinear coordinate system in a place, located away from the excited volume of PhV, and then interpolate it in the region of excitation. For example, we could not detect the bending of light rays in the gravitational field of the Sun without taking into account the motion of light rays in the absence of the Sun.

Thus, any curvilinearity is disclosed with respect to the main reference frame, built in straight lines-rays. Due to this it is disclosed that in this volume of the PhV there is a curvature of light rays, and hence there is a field of force, and matter, which generates it. In this sense, the curvilinearity is not evidence that the space-time geometry is Riemannian geometry. But we can declare it as Riemannian, if we consider only the excited part of PhV. It is clear that the Riemannian geometry is a way of describing the geometry of the excited PhV.

Therefore, when in general relativity it is mentioned that space-time is Riemannian, it is purely a conditional expression. Space and time - by themselves - are some of the mathematical relationships that are bound to PhV. They can not bend. This is the material carrier of this relationship - PhV, which is bent (or rather, turned into a new excited state). This bending is detected by comparing the trajectories of photons in a given location relative to the unexcited PhV. All the changes of time and space occur due to changes in the properties of a given piece of PhV. The changes can be seen as global, if we do not go beyond the excited volume of PhV. But at the same time, they are only local with respect to the total volume of PhV in the Universe.

The space, expressed by the lengths and direction of the lines, varies due to different types of PhV deformations. Time is measured by the frequency of the processes occurring in the bodies of deformed PhV, and therefore differs by time for the bodies in non-deformed PhV. And, as is clear in this case the changes in space and time are interrelated.

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