Article

"Physical Intuition": What Is Wrong with It?

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Abstract

It appears not to be known that subjecting the axioms to certain conditions, such as for instance to be physically meaningful, may interfere with the logical essence of axiomatic systems, and do so in unforeseen ways, ways that should be carefully considered and accounted for. Consequently, the use of "physical intuition" in building up axiomatic systems for various theories of Physics may lead to situations which have so far not been carefully considered.

1. Prologue

The fact that in our times the various scientific theories of Physics are formulated in terms of Mathematics is by a free choice of physicists, certainly not imposed in any way on them, and least of all by mathematicians. The conscious and clear expression in this regard goes back at least as far as Galileo Galilei, according to whom the book of Nature is written in the language of Mathematics. And one can note that this statement refers not only to Physics, but to all disciplines which in those times were seen as constituting Philosophia Naturalis. Not much later, Newton gave the title "Principia Mathematicae Philosophia Naturalis" to his most important work. And nearer to our own days, in 1960, the Nobel laureate physicist Eugene Wigner addressed this very same issue in his paper "The Unreasonable Effectiveness of Mathematics in Natural Sciences". So much for the state of affairs according to which modern Physics ended up being formulated in terms of Mathematics.

As for Mathematics itself, ever since Euclid and his Geometry, more than two millennia ago, the rigorous way to present a theory of Mathematics means to present it as an axiomatic system.

And so it comes to pass that, no wonder, there are any number of axiomatic presentations, or attempts to such a presentation, in theoretical Physics, the first among them, as far as we happen to know nowadays, is the mentioned "Principia" of Newton.

Related to the above, the main issue addressed in this paper is :

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- How come that the "physical intuition" based search for physically meaningful axioms can create problems in the axiomatization of various theories of Physics ?

This issue seems so far to have escaped the awareness of most of physicists ...
Yet the answer to this question should be obvious to everybody who happens to have even a first grasp of the essence of the axiomatic method as such. Indeed, the axiomatic method is in its essence within the domain of Mathematical Logic, thus it does not involve considerations from other disciplines of science. The aim of this method, as already envisaged and attained by Euclid in his Geometry, is to organize a set $\mathcal{T}$ of *theorems* which are supposed to formulate a given theory, and do it in the following way. Certain theorems, constituting a set $\mathcal{A}$, preferably a small subset of $\mathcal{T}$, are considered to be true, while the rest of the theorems in $\mathcal{T}$ are supposed to be obtainable as purely logical consequences of the theorems in $\mathcal{A}$. And whenever such a program can be achieved, then the theorems in $\mathcal{A}$ are called the *axioms* of the theory given by the theorems $\mathcal{T}$.

Clearly, for a given set $\mathcal{T}$ of theorems there may in general exist more than one set of axioms $\mathcal{A}$.

However, equally clearly, one simply *cannot* impose on the set $\mathcal{A}$ of axioms absolutely arbitrary requirements, no matter how much one's intuition would impel one to do so. Indeed, the two requirements any set of axioms $\mathcal{A}$ can be asked to satisfy are :

- $\mathcal{A}$ has to be a subset of $\mathcal{T}$, that is, $\mathcal{A} \subseteq \mathcal{T}$
- $\mathcal{T}$ has to be a logical consequence of $\mathcal{A}$

And what is crucial to understand is that any other conditions required on a set of axioms $\mathcal{A}$ may make it impossible for such a set to exist, or alternatively, may lead to another set of theorems $\mathcal{T}$. Details in this regard are presented in sections 2 and 3 below.

In order to better focus on the relevance of this issue within the context of Physics, we shall mention a certain aspect in the axiomatization of Special Relativity, [4-9,25,26], and then note that, during the last decades, one of the active research fields in theoretical Physics has been the axiomatization of Quantum Mechanics. This research aims to replace the original von Neumann axiomatic system of quanta, formulated in the early 1930s and presently still taught in many places as Quantum Mechanics 101, with a system in which the axioms are supposed to have a clear and obvious *physical meaning*. And needless to say, it is precisely with this aim to find a system of physically meaningful axioms for quanta that the so called "physical intuition" is not only supposed to come into play, but it is simply expected, and in fact, required to do so.

And one could certainly ask : what may possibly be wrong with finding a system of physically meaningful axioms for a theory of Physics, such as for instance, Quantum

Mechanics ?

Well, so far, apparently this question has never been asked, simply because the answer to it is automatically considered to be so obvious by all those involved in the axiomatization of various theories of Physics. And their answer, of course, is : there can absolutely nothing be wrong !

And then, as recently noted in [26,25], and also mentioned above related to the way axiomatic systems work, it may indeed can come as a surprise to find out that :

- By requiring a condition such as "to be physically meaningful" on the axioms of a theory of Physics can lead to certain strange effects in the respective theory.

2. A Brief Review of Axiomatization in Mathematics

For convenience, let us recall the way axiomatic systems are conceived in Mathematics, a way which in its essence, even if not in its specific formulation, goes back at least as far as the Geometry of Euclid in ancient Egypt more than two millennia ago. Namely, in terms of modern Mathematical Logic, this way, presented briefly, is as follows.

One starts with a setup of a *formal deductive system*. Namely, let A be an *alphabet* which can be given by any nonvoid finite or infinite set. Then a procedure is given according to which one constructs - by using the symbols in A - a set $\mathcal{F}$ of *well formed formulas*, or in short wff-s. Next, one chooses a set $\mathcal{R}$ of *logical deduction rules* which operate as follows

$$(2.1) \qquad \mathcal{F} \supseteq P \stackrel{\mathcal{R}}{\longmapsto} Q \subseteq \mathcal{F}$$

that is, from any set P of wff-s which are the *premises*, it leads to a corresponding set Q of wff-s which are the *consequences*.

And now come the *axioms* which can be any subset $\mathcal{A} \subseteq \mathcal{F}$ of wff-s.

Once the above is established, the respective *axiomatic theory* follows easily as being the *smallest* subset $\mathcal{T} \subseteq \mathcal{F}$ with the properties

$$(2.2) \qquad \mathcal{A} \subseteq \mathcal{T}$$

$$(2.3) \qquad \mathcal{T} \supseteq P \stackrel{\mathcal{R}}{\longmapsto} Q \subseteq \mathcal{T}$$

in which case the wff-s in $\mathcal{T}$ are called the *theorems* of the axiomatic system $\mathcal{A}$.

Of course, one should not forget that the set $\mathcal{T}$ of theorems depends essentially not only on the axioms in $\mathcal{A}$, but also on the logical deduction rules $\mathcal{R}$. Consequently, it is appropriate to write

$$(2.4) \qquad \mathcal{T}_{\mathcal{R}}(\mathcal{A})$$

for the set $\mathcal{T}$ of theorems.

Here are some of the relevant questions which can arise regarding such axiomatic systems :

- are the axioms in $\mathcal{A}$ *independent* ?
- are the axioms in $\mathcal{A}$ *consistent* ?
- are the axioms in $\mathcal{A}$ *complete* ?

Independence means that for no axiom $P \in \mathcal{A}$, do we have $\mathcal{T}_{\mathcal{R}}(\mathcal{A}) = \mathcal{T}_{\mathcal{R}}(\mathcal{B})$, where $\mathcal{B} = \mathcal{A} \setminus \{P\}$. In other words, the axioms in $\mathcal{A}$ are minimal in order to obtain the theorems in $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$. This condition can be formulated equivalently, but more simply and sharply, by saying that for no axiom $P \in \mathcal{A}$, do we have $P \in \mathcal{T}_{\mathcal{R}}(\mathcal{B})$, where $\mathcal{B} = \mathcal{A} \setminus \{P\}$.

As for consistency, it means that there is no $P \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$, such that for its negation *non* P, we have *non* $P \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$.

Completeness, in one possible formulation, means that, given any additional axiom $P \in \mathcal{F} \setminus \mathcal{A}$ which is independent from $\mathcal{A}$, the axiom system $\mathcal{B} = \mathcal{A} \cup \{P\}$ is inconsistent.

In the case of axiomatic systems for various theories of Physics, one is interested in the independence and consistency of the respective axioms. Independence means that the set of axioms is *minimal*, thus no axiom is a consequence of the other ones, therefore, no axiom can be eliminated without losing certain theorems. Consistency means that one cannot obtain contradictory consequences of the axioms.

The completeness of an axiomatic system for a given theory of Physics apparently has not yet been considered in the literature.

3. Back Now to Physically Meaningful Axioms

Let us see now in some more detail what can happen in case we start requiring certain conditions on the axioms of an axiomatic system. And to be specific, let us see in the general terms of section 2 above what may happen when we try to replace a set of axioms with another set which is supposed to satisfy certain conditions.

Namely, let be given a set $\mathcal{A} \subseteq \mathcal{F}$ of axioms. Then, as in section 2 above, we have the corresponding theorems $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$ of the axiomatic system $\mathcal{A}$.

Now, assume that we are not happy with the axioms in $\mathcal{A}$, and therefore, we want to replace them with another set $\mathcal{B} \subseteq \mathcal{F}$ of axioms.

Of course, in case such a replacement is not supposed to lead to another theory, then we must have

(3.1) $\quad \mathcal{T}_{\mathcal{R}}(\mathcal{B}) = \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

Let us now look more closely to what can happen in such a process of replacement of axioms.

The fact that we are not happy with the initial axioms in $\mathcal{A}$ means that we want to choose the axioms not from the whole set $\mathcal{F}$ of wff-s, but only from a special subset $\mathcal{F}_{Phys} \subset \mathcal{F}$, say for example, the subset $\mathcal{F}_{Phys}$ of so called "physically meaningful" wff-s in $\mathcal{F}$.
In other words, in addition to condition (3.1), and in fact, prior to it, we also require the condition that

(3.2) $\quad \mathcal{B} \subseteq \mathcal{F}_{Phys}$

And now, it is obvious that, in general, conditions (3.1) and (3.2) may happen to be incompatible.

And in such a case, the natural way to proceed is to *weaken* the requirement (3.1), by replacing it with

(3.3) $\quad \mathcal{T}_{\mathcal{R}}(\mathcal{B}) \supseteq \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

In other words, instead of (3.1) and (3.2) which are incompatible, we have now (3.2) and (3.3). This means that the new set $\mathcal{B}$ of axioms certainly recovers all the theorems $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$ in the axiomatic system $\mathcal{A}$ which was replaced. However, we risk to have *additional* theorems in $\mathcal{T}_{\mathcal{R}}(\mathcal{B})$, which were not among the theorems in $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$. Thus in (3.3), we may in fact have

(3.4) $\mathcal{T}_{\mathcal{R}}(\mathcal{A}) \subsetneqq \mathcal{T}_{\mathcal{R}}(\mathcal{B})$

due to the restriction (3.2).

4. The Physically Meaningful Axioms of Special Relativity

The above situation in (3.3) appears to happen in Special Relativity, [4-9,25,26]. Indeed, the usual two axioms, [3], which have a clear physical meaning are :

- Galilean Relativity : the Laws of Physics are the same in all inertial reference frames.
- Constancy of the Speed of Light : in all inertial reference frames the speed of light has the same value c.

However, as shown in [4-9,25,26], in order to obtain the Lorentz Transformations of inertial reference frames, which as is known, contain the essence of Special Relativity, one only needs the following weaker axioms :

- the homogeneity and isotropy of space,
- the homogeneity of time,
- the Axiom of Reciprocity which means that, given two inertial reference frames S and S', and a speed $v \in \mathbb{R}$, the laws of Physics are the same whether S' moves related to S with speed v, or with speed $-v$,
- the upper limit of all physical speeds, which already results from the above three axioms for all inertial reference frames, [9,5], is the speed c of light in void.

Here we note that the homogeneity and isotropy of space and the homogeneity of time are assumed as well when one axiomatizes Special Relativity with mentioned two usual axioms.

It follows that if we denote by $\mathcal{A}$ the above four axioms, while by $\mathcal{B}$ we denote the mentioned two usual axioms of Special Relativity, plus the two above axioms about space and time, then

(4.1) $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$

already contains the Lorentz Transformations of inertial reference frames.

Thus the following questions arise :

- are the axioms in $\mathcal{B}$ independent ?

and in case they are not, as suggested by [4-9,25,26], then :

- what is the point in stating the two usual axioms of Special Relativity in such a redundant manner, just in order to have an obvious physical meaning ?

Here one can, of course, note that the axioms $\mathcal{A}$ themselves have already an obvious physical meaning. Not to mention that both sets of axioms have the same number, namely, four, of axioms, thus none of them is shorter than the other one.

5. A Memento for the Axiomatization of Quantum Mechanics

Recently, a number of impressive attempts have been seen in the literature which present an axiomatization of Quantum Mechanics that can replace the classical one given by von Neumann in the early 1930s, and which is quite widely perceived as not being formulated in terms sufficiently meaningful physically. A remarkable feature of some of such recent axiomatization is that they manage to come up with the axioms formulated in terms of *information*.
Since we try to avoid pointing out for special attention one or another of such axiomatizations, we refer the reader to the Quantum Physics section of the well known web site arXiv, where a cursory browsing can provide with a good amount of related literature.

What is obvious related to such axiomatizations is the following. The axiomatization of von Neumann has stood its ground for more than eight decades by now, and it did so both theoretically and experimentally, the main objections, and quite numerous at that, being related not to the axioms themselves, as to the classical Copenhagen Interpretation which, rightly or wrongly, tends to be closely associated with it.
Consequently, a main reason for replacing the von Neumann axioms should perhaps be, if at all, their alleged association with the Copenhagen Interpretation, rather than the lack of clear physical meaning of the respective axioms.
Needless to say, to the extent the axioms of Quantum Mechanics are desired to be formulated in terms of information, that in itself may be sufficient reason to do so, taking into account that information is, so far, by far the most subtle concept used in Physics, far more subtle indeed than mass, position, momentum, energy, and so on.

However, in view of the facts mentioned in sections 2 - 4 above concerning axiomatic systems, one should perhaps ask the following simple looking question, a question which seemingly never got a consciously enough formulated answer, and an answer which was sufficiently analyzed in order to reveal more fully its possible relevance in Physics, or if one prefers, its physical relevance :

- Which are in general the truly significant advantages in replacing one set of axioms with another one in the case of a given theory ? And specifically in Quantum Mechanics, which are such advantages if the new axioms are formulated in physically meaningful ways, let alone, in terms of information ?

Needless to say, none of the arguments above should be construed in any way as objecting against new axiomatizations of existing theories of Physics, or for that matter, of any other scientific theories.

What is to be considered more carefully, however, is that changing one set of axioms for a given theory with another set of axioms should, first of all, have a clearly expressed and significant reason, and second, subjecting the new set of axioms to certain requirements, such as for instance to be physically meaningful, may interfere with the logical essence of axiomatic systems, and do so in unforeseen ways, ways that should be carefully considered and accounted for.

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