Article

Computer Hardware of the Future: Will the Classical-wave Simulated "Long Qubyte" Trump the True-quantum Qubit?

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Abstract

Using the results of a recent earlier article, it is pointed out that homogeneously linear, strictly oscillatory, conservative classical dynamical systems that have N degrees of freedom automatically simulate N-state quantum systems. Because the simulating oscillatory classical system is not subject to the information-destroying constraints which are imposed on the simulated quantum system by its probability interpretation, that simulating classical system actually has a greater information-bearing capacity than the N-state quantum system which it simulates. In addition, it is easy to ensure that the simulating oscillatory classical system is robust against the "decoherence" problems which typically beset the simulated N-state quantum system. Though it therefore might seem tempting to classically simulate two-state quantum systems, namely the much-discussed qubits, it appears to be far more advantageous to use cavity electromagnetic standing-wave modes to simulate N-state quantum systems for which N is as large as possible. A single such cavity-mode simulated "long qubyte" might even suffice as the random access memory for an entire computer.

Keywords: Schrödinger equation, oscillatory classical system, normal modes, electromagnetism, N-state quantum system, classical simulation of quantum systems, information capacity, decoherence, qubit, plumb line, "qubyte", cavity standing-wave modes, holographic snapshot of wave-borne information.

Review of Schrödinger-equation rendition of oscillatory classical dynamics

It was pointed out in an earlier article [1] that homogeneously linear, strictly oscillatory, conservative classical equations of motion (or classical field equations) which have the form,

$$\ddot{q} + Kq = 0, \tag{1a}$$

with the conserved energy,

$$\mathcal{E}_K[q, \dot{q}] = (\,(\dot{q}, \dot{q}) + (q, Kq)\,)/(2\gamma^2), \tag{1b}$$

where the real-valued constant γ is positive and the real-valued elastic coupling-strength matrix (or linear operator) K is symmetric and positive-definite, are linearly equivalent to the Schrödinger equation,

$$i\hbar\dot{\psi} = H\psi, \tag{1c}$$

whose Hermitian Hamiltonian matrix (or linear operator) H is given by,

$$H = \hbar K^{\frac{1}{2}}, \tag{1d}$$

and whose complex-valued wave vector (or wave function) ψ is given by the following one-to-one linear mapping of the real-valued classical dynamical variable (or classical field) pair $[q, \dot{q}]$,

$$\psi = (K^{\frac{1}{4}}q + iK^{-\frac{1}{4}}\dot{q})/(2\gamma^2\hbar)^{\frac{1}{2}}, \tag{1e}$$

which has the explicit linear inverse,

$$[q, \dot{q}] = [((\gamma^2\hbar)/2)^{\frac{1}{2}}K^{-\frac{1}{4}}(\psi + \psi^*), -i((\gamma^2\hbar)/2)^{\frac{1}{2}}K^{\frac{1}{4}}(\psi - \psi^*)], \tag{1f}$$

and for which the time-invariant expectation value of the Hamiltonian matrix (or linear operator) $H = \hbar K^{\frac{1}{2}}$ is precisely equal to the conserved classical energy,

$$(\psi^*, H\psi) = (\psi^*, \hbar K^{\frac{1}{2}}\psi) = (\,(\dot{q}, \dot{q}) + (q, Kq)\,)/(2\gamma^2) = \mathcal{E}_K[q, \dot{q}]. \tag{1g}$$

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The reader can straightforwardly verify Eq. (1g) from the one-to-one linear mapping given by Eq. (1e), and can likewise verify that the time-dependent Schrödinger equation of Eq. (1c) with the Hamiltonian matrix (or linear operator) given by Eq. (1d) *follows* from the one-to-one linear mapping given by Eq. (1e) *together* with the classical equation or motion (or classical field equation) given by Eq. (1a). Conversely, that classical equation of motion (or classical field equation) given by Eq. (1a) can be verified to follow from the *inverse* linear mapping given by Eq. (1f) *together* with the time-dependent Schrödinger equation and its Hamiltonian matrix (or linear operator) that are given by Eqs. (1c) and (1d).

It is to be noted that a homogeneously linear, strictly oscillatory, conservative classical equation of motion (or classical field equation) can always be brought to the form given by Eq. (1a); that is discussed in some detail in the earlier article [1]. Here we simply point out that this type of classical equation of motion (or classical field equation) is sometimes presented as a *first-order* in time equation,

$$\dot{s} = Ws, \tag{2}$$

where the real-valued frequency matrix (or linear operator) W is antisymmetric and invertible (i.e., nonsingular). That is specifically the case, for example, for Maxwell's source-free electromagnetic field equations, which we showed in the earlier article to be linearly equivalent to the time-dependent Schrödinger equation for a single free photon [1]. It is, of course, apparent that in such cases one will obtain from Eq. (2) a form of Eq. (1a) that has K equal to $-W^2$.

Classical simulation of N-state quantum systems

If the homogeneously linear, strictly oscillatory conservative classical dynamics described by Eqs. (1a) and (1b) has N degrees of freedom, i.e., if K is an N-by-N matrix, then the Hermitian Hamiltonian $H = \hbar K^{\frac{1}{2}}$ of the linearly equivalent time-dependent Schrödinger equation of Eq. (1c) is *also* an N-by-N matrix, which implies that Eq. (1c) describes an N-state quantum system. Thus any homogeneously linear, strictly oscillatory conservative classical system which has N degrees of freedom *automatically* simulates an N-state quantum system.

However, from the perspective of their respective theoretical information-bearing capacities, the simulating oscillatory classical system which has N degrees of freedom turns out to actually be slightly *superior* to the simulated N-state quantum system. This is most easily appreciated by considering the simplest possible case, namely that where $N = 1$, which reduces the simulating oscillatory classical system to a single simple harmonic oscillator (i.e., K reduces to merely a positive real number which is the *square* of that simple harmonic oscillator's angular frequency ω). That simple harmonic oscillator can, in principle, be made to carry information by making use of both its oscillation *amplitude* and *phase*. This gives it a two-dimensional "information-bearing space", of which only some bounded subregion can actually be available in any particular practical implementation.

Now the simulated *one-state* quantum system's *wave vector* has only *one* complex-valued component, which is a function of time of the form $C\exp(-iEt/\hbar)$, where $E = \hbar\omega$ is the real-valued energy of that state. From a purely *mathematical* perspective, the complex-valued constant C provides *exactly the same* two-dimensional "information-bearing space" as is available from the *simulating* classical simple harmonic oscillator. Unfortunately, however, the *probability interpretation* of quantum theory does away with *any possibility* of reading off the value of C from physical measurement. Firstly, that interpretation attaches physical (probabilistic) meaning to *only absolute squares of inner products* of wave vectors. The value of the *phase* of the constant C obviously cannot be determined from such absolute squares. Secondly, full consistency of the probability interpretation requires that all inner products of wave vectors with *themselves* (which, because of the time-dependent Schrödinger equation, are always constant in time) must be *forced* to take the value unity. That obviously does away with the possibility that the *amplitude* of the complex-valued constant C could be utilized to carry information. We therefore see that the simulated *one-state* quantum system provides *no* information-bearing capacity whatsoever, albeit, as we noted above, its simulating *classical* simple harmonic oscillator system *does* provide an implementation-dependent bounded subregion of a *two-dimensional* "information-bearing space".

Going on from the above discussion of the nil information-bearing capacity of the one-state quantum system, we now consider the *two-state* quantum system, which, in the jargon of contemporary computing studies, is called a qubit. A qubit can indeed carry a substantial amount of information, certainly *very* much more than the binary digit or bit. The wave vector of a qubit has *two* complex-valued time-dependent components, and taking account of *both* its Hilbert-space normalization to unity *and* the fact that its overall constant phase factor cannot be physically measured, we can see that the qubit's "information-bearing space" is conveniently portrayed as the surface of a unit sphere. This clearly is vastly more impressive than the paltry set of *but two discrete points* which portrays the information capacity of the binary bit. However, the homogeneously linear oscillatory *classical* system which *simulates* a qubit has two degrees of freedom, and thus can be decomposed into two normal modes, each of which is an *independent* classical simple harmonic oscillator [1]. Now the amplitude and phase of *each* such independent classical simple harmonic oscillator provides a two-dimensional "information-bearing space", which implies that the oscillatory classical system of two degrees of freedom that *simulates* a qubit has a *four*-dimensional "information-bearing space", of which some bounded subregion would actually be available in a practical implementation. Thus even for the qubit, the four-dimensional information-bearing capacity of its *classical simulation* can be expected to in practice substantially outperform its *own* two-dimensional information-bearing capacity, albeit *both* of these sophisticated systems are in a different league entirely from that of the binary bit, whose information-bearing capacity encompasses just two discrete points.

Having gone through the above preparatory exercises with the simplest two special cases, it now becomes straightforward to see that the N-state quantum system has an "information-bearing space" that is *compact* and possesses $(2N - 2)$ dimensions, whereas its oscillatory *classical simulation* of N degrees of freedom has a noncompact "information-bearing space" of $2N$ dimensions, of which only some bounded subregion would actually be available in a practical implementation.

It is also to be noted that true quantum qubits are extremely microscopic systems which are exquisitely sensitive to random small disturbances in their environment—that is the cause of their celebrated "decoherence" problems. The simulation of such systems by classical oscillatory ones can obviously be enormously more robust, and it is surprising just how workaday and humdrum a classical simulation of so esoteric an object as a qubit can actually be. A plumb line whose massive plumb bob makes only sufficiently small excursions from its equilibrium position moves in what is very nearly a horizontal plane and experiences what are very close to vector Hooke's Law type of restoring forces. Therefore a plumb line amounts essentially to an oscillatory classical system of *two* degrees of freedom, one which is far less susceptible to environmental decoherence than any true-quantum qubit which it simulates, and, as we have seen in the paragraphs above, one which in theory can *as well* have far greater information-bearing capacity than such a qubit.

Qubits versus classical cavity-mode simulated "long qubytes"

There are, of course, a great wealth of oscillatory classical systems known to modern technology, so the simulation of qubits by plumb lines is extremely unlikely to in any way be an optimal procedure! Even beyond that, spellbound fascination with qubit quantum *two-state* systems in recent years as *the* ostensible means of achieving immense computing power has inhibited exploration of using more *general* N-state quantum systems, especially those with *large* values of N, for this same purpose and, even much *more* regrettably, has as well inhibited exploration of using toward this end oscillatory *classical* systems of N degrees of freedom which *automatically simulate* N-state quantum ones, albeit with *far less* susceptibility to environmental decoherence as well as with more information-bearing capacity.

We therefore now eschew searching for "better qubits" in favor of seeking oscillatory classical systems which have as *many* accessible degrees of freedom as is at all possible. Such a system could be dubbed a classically-simulated "long qubyte". The *longer* the classically-simulated "qubyte" is, i.e., the *more* accessible oscillatory classical degrees of freedom it encompasses, the greater its information-bearing capacity: we have seen above that a classically-simulated "qubyte" of length N, i.e., one with N degrees of freedom, has an "information-bearing space" of $2N$ dimensions. This information-bearing capacity of a classically-simulated "long qubyte" vastly outstrips that achievable with binary bits, but it will nevertheless take much hard thinking and cleverness on the part of computing experts to come up with ways to make truly effective use of data presented in the analog amplitude/phase format which is natural to classical oscillatory systems.

Now certain oscillatory classical *fields*, e.g., those of electromagnetism, in principle have an *indefinitely large* number of degrees of freedom, albeit degrees of freedom of *sufficiently high frequency* will always tend to become *inaccessible* to any particular apparatus. Nonetheless, *standing electromagnetic wave modes confined within a cavity with reflecting walls* ought to offer immense classically-simulated "qubyte" length. As the typical frequencies of these standing wave modes are made higher (one would hope that visible frequencies and laser technology would eventually be used), so too would computing speeds ramp up, and the simulated "qubyte" would get longer as well—one can speculate that a *single* cavity standing-wave-mode simulated "long qubyte" might suffice as the random access memory for an entire typical computer. In addition, as an ancillary technology, taking utilizable "snapshots" of the cavity standing-wave modes, and thus of the information they carry, might eventually become feasible through holographic techniques.

Reference

[1] S. K. Kauffmann, Prespacetime Journal **2** (11), 1748 (2011).