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Bianchi Type-VI Inflationary Universe in General Relativity

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Abstract

In this paper, we have investigated the inflationary Bianchi type-VI cosmological model in the presence of mass less scalar field with a flat potential. To get an inflationary solution , we have considered a flat region of a constant potential V . Some physical and kinematical properties of the model are studied.

Keywords: inflationary universe, cosmology, general relativity, scalar fields.

1. Introduction

In recent years there has been lot of interest in inflationary models of the universe in general relativity. Inflationary models play an important role in solving number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. The standard explanation for the flatness of the universe is that it has undergone at an early stage of the evolution a period of exponential expansion named as inflation. It is well known that self interacting scalar fields play a vital role in the study of inflationary cosmology. Inflation , the stage of accelerated expansion of the Universe , first proposed in the beginning of the 1980s , nowadays receives a great deal of attention .Guth [1] proposed inflationary model in the context of grand unified theory (GUT), which has been accepted soon as the model of the early Universe. Barrow and Turner [2] showed that the large anisotropy prevents transition into an inflationary era according to Guth's original inflationary scenario.

In inflationary scenario, the de-Sitter expansion induced by vacuum energy density in general relativity. Several versions of the inflationary models are studied by Linde [3], La and Steinhardt [4], Abbott and Wise [5]. In particular, our universe is homogeneous and isotropic to a very high degree of precision. Such a universe can be described by the well-known Friedmann-Robertson-Walker (FRW) metric. In these models, flatness problem is well understood and solved, but this is not so clear about isotropy and homogeneity to solve such problem Using the concept of Higgs field ϕ with potential $V(\phi)$ has a flat region and the ϕ field evolves slowly but the universe expands in an exponential way due to vacuum field energy.

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In general relativity, scalar fields help in explaining the creation of matter in cosmological theories and can also describe the uncharged field. Scalar field is minimally coupled to the gravitational field. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmological model. Wald [6], Burd and Barrow [7], Barrow [8], Ellis and Madsen [9] and Heusler [10] studied different aspect of scalar fields in the evolution of the universe and FRW models. Bhattacharjee and Baruah [11], Bali and Jain [12] and Rahaman et al. [13] ,Singh et. al.[14] have studied the role of self-interacting scalar fields in inflationary cosmology. Recently, Reddy et al. [15], Reddy and Naidu [16] have studied inflationary universe models in four and five dimensions in general relativity. In recent years , Katore et. al. [17] ,Reddy and Naidu [18] have studied the cosmological models with constant deceleration parameter of the universe in the context of different aspects of different space-time. Reddy [19] has discussed Bianchi type – V inflationary universe in general relativity. Very recently, Katore et. al. [20],Katore et. al. [21] have discussed inflationary universe in general relativity. Therefore we propose to study of such inflation theory in general relativity.

In this paper, we have investigated Bianchi type VI cosmological model in the presence of mass less scalar fields with a flat potential in general relativity. To get a determinate solution, we have considered a flat region in which flat potential V is constant. We have also assumed a relation between metric potentials for this purpose.

2. Field equations & the model

We consider the spatially homogenous Bianchi type VI metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2qx}dy^2 + C^2e^{2qx}dz^2, \qquad (1)$$

where A, B, C, are the functions of t only and q is a non-zero constant .
In this case of gravity minimally coupled to a scalar field $V(\phi)$,the Lagrangian is

$$L = \int\left[R - \frac{1}{2}g^{ij}\phi_{,i}\phi_{,j} - V(\phi)\right]\sqrt{-g}\,d^4x\,, \qquad (2)$$

which on variation of L , with respect to the dynamical fields , to Einstein field equations

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \qquad (3)$$

With $T_{ij} = \phi_{,i}\phi_{,j} - \left[\frac{1}{2}\phi_{,k}\phi^{,k} + V(\phi)\right]g_{ij}$ (4)

The field equation is

$$\phi^{i}_{;i} = -\frac{dV}{d\phi}, \tag{5}$$

where comma and semicolon indicate ordinary and covariant differentiation respectively.

Other symbols have their usual meaning and units are taken so that

$8\pi G = C = 1.$

Now the field equations (3) for the metric (1) with the help of equation (4) are given by as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} + \frac{q^2}{A^2} = \frac{1}{2}\phi_4^2 - V(\phi), \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{q^2}{A^2} = \frac{1}{2}\phi_4^2 - V(\phi), \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{q^2}{A^2} = \frac{1}{2}\phi_4^2 - V(\phi), \tag{8}$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{B_4}{B}\frac{C_4}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{q^2}{A^2} = -\frac{1}{2}\phi_4^2 - V(\phi), \tag{9}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0, \tag{10}$$

and (5) , for the scalar field , takes the form

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = -\frac{dV}{d\phi}. \tag{11}$$

where the suffix 4 indicates differentiation with respect to t.

3. Inflationary model

We are interested, here, in inflationary solutions of the field equations (6)–(11).
Stein-Schabas [22] has shown that Higgs field ö with potential $V(\phi)$ has flat region and the field evolves slowly but the universe expands in an exponential way due to vacuum field energy. It is assumed that the scalar field will take sufficient time to cross the flat region so that the universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the

horizon size. Thus, we are interested here, in inflationary solutions of the field equations. The flat region is considered where the potential is constant.

We also assume that $A = B^n$. (12)

In order to derive exact solutions of the field equations (6)-(11) easily, we use the following scale transformation

$$A = e^{n\beta}, B = e^{\beta}, C = e^{\gamma}, dt = ABCdT\ . \tag{13}$$

The field equations (6)-(11) reduces to

$$\beta'' - n\beta'^2 - (n+1)\beta'\gamma' + \gamma'' + q^2 e^{2(\beta+\gamma)} = \left(\frac{1}{2}\phi'^2 - v_0\right)e^{2[(n+1)(\beta+\gamma)]}, \tag{14}$$

$$\gamma'' - n\beta'^2 - (n+1)\beta'\gamma' + n\beta'' - q^2 e^{2(\beta+\gamma)} = \left(\frac{1}{2}\phi'^2 - v_0\right)e^{2[(n+1)(\beta+\gamma)]}\ , \tag{15}$$

$$\beta'' - n\beta'^2 - (n+1)\beta'\gamma' + n\beta'' - q^2 e^{2(\beta+\gamma)} = \left(\frac{1}{2}\phi'^2 - v_0\right)e^{2[(n+1)(\beta+\gamma)]}, \tag{16}$$

$$n\beta'^2 + (n+1)\beta'\gamma' - q^2 e^{2(\beta+\gamma)} = \left(-\frac{1}{2}\phi'^2 - v_0\right)e^{2[(n+1)(\beta+\gamma)]}, \tag{17}$$

$$\beta' - \gamma' = 0\ , \tag{18}$$

$$\phi'' = 0, \tag{19}$$

where a superscript prime(') indicates differentiation with respect to T.
The set of equations (14) – (19) admit an exact solution

$$A = (k_1 T + k_2)^{\frac{-n}{2}}, \tag{20}$$

$$B = C = (k_1 T + k_2)^{\frac{-1}{2}}, \tag{21}$$

$$\phi = (k_3 T + k_4)\ , \tag{22}$$

where k_3, k_4 are constants of integrations and k_1, k_2 are arbitrary constants.
After a proper choice of co-ordinates and constants, the inflationary cosmological model corresponding to the solution can be written as

$$ds^2 = -dT^2 + \tau^{-n} dX^2 + \tau^{-1} e^{-2qX} dY^2 + \tau^{-1} e^{2qX} dZ^2\ . \tag{23}$$

4. Some Physical and Kinematical Properties

The model (23) represents a Bianchi type VI inflationary cosmological model in general relativity in the presence mass less scalar field with flat potential. For the model (23) , the physical and kinematical variables which are important , in cosmology , are

Spatial volume: $V=\sqrt{-g}$

$$= \frac{1}{(\tau)^{\left(\frac{n+2}{2}\right)}} \quad , \tag{24}$$

Expansion scalar: $\theta = \frac{1}{3}U^{i}_{;i}$

$$= \frac{-1}{6}\left(\frac{n+2}{\tau}\right) \quad , \tag{25}$$

Shear scalar: $$\sigma^2 = \frac{7}{648}\left(\frac{n+2}{\tau}\right)^2, \tag{26}$$

By the straight forward evaluation of deceleration parameter

$$q = -3\theta^{-2}\left[\theta_{;\alpha}u^{\alpha} + \frac{1}{3}\theta^2\right] = -\left[\frac{18k_1}{n+2} + 1\right], \tag{27}$$

For the model (23) , it is observed that q turns out to be negative which confirms the fact that model (23) represents inflation (Fienstein and Ibanez [23]).

It should be noted that the scalar field have no initial singularity. It can be observed that for large τ , the parameters θ, σ vanish and diverge when $\tau \rightarrow 0$.

Also for large value of τ, the ratio $\left(\frac{\sigma^2}{\theta^2}\right) \neq 0$ and hence the model (23) does not approach isotropy.

5. Conclusions

The inflationary Bianchi type-VI cosmological model in the presence of mass less scalar field with a flat region of constant potential is presented. . The model does not approach isotropy at late times. The role of the deceleration parameter seems to specify the expansion of the Universe .The positive value of the deceleration parameter indicates that the model decelerates in the standard way. But in the present observation the model inflates because the deceleration parameter is negative. The inflationary model obtained here has considerable astrophysical significance. For example, classical scalar fields are essential in the study of the present day cosmological models. In view of the fact that there is an increasing interest, in recent years, in scalar fields in general relativity and alternatives theories of gravitation in the context of inflationary Universe and they help us to describe the early stages of evolution of the Universe .

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