Article

Two-Fluid Cosmological Models in Bianchi Type-V Space-Time in Higher Dimensions

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Abstract

Anisotropic, homogeneous two-fluid cosmological models using Bianchi type-V space-time have been presented. In the two-fluid models, one fluid represents the matter content of the universe and another fluid is chosen to model the CMB radiation. The radiation and matter content of the universe are in interactive phase. The behaviour of fluid parameters and kinematical parameters are also discussed.

Keywords: Bianchi type-V space-time, two-fluid, higher dimensions.

1. Introduction

The study of higher dimensional cosmological models is motivated mainly by the possibility of geometrically unifying the fundamental interactions of the universe. There has been considerable interest in solutions of Einstein's equation in higher dimensions in the context of physics of the early universe both for cosmologists and practical physicists. Indeed the present four-dimensional stage of the universe could have been preceded by a higher-dimensional stage, which at later times becomes effectively four-dimensional in the sense that extra dimensions became unobservable small due to dynamical contraction. Also, the higher dimensional theory is important at the early stages of the evolution of the universe.Chodos A and Detweller S (1980), Ibancz J and Verdaguer E (1986), Khadekar G.S and Gaikwad M (2001), Adhav K.S *et al* (2008) have studied the multidimensional cosmological models in Einstein's general relativity theory.

The isotropic and homogeneous space-times due to FRW are simple models of the present stage of expanding universe. The discovery of 2.73 K isotropic cosmic microwave background radiation (CMBR) motivated many authors to investigate FRW model with a two-fluid source

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[Coley A.A and Tupper B.O.J. (1986), Davidson W (1962), McIntosh C.B.G (1968)]. One fluid represents the matter content of the universe and another fluid is chosen to model the CMB radiation in the two-fluid model. The observed anisotropic of CMB at various angular scales make a point of fresh look in the investigation of two-fluid models, especially those which exhibit anisotropy.

The conformation of the universe is expanding with a positive acceleration, that means the existence of the total negative pressure for the universe which is the observation of high red shift type-Ia supernova and supernova cosmology project [Garnavich P.M. *et al* (1998), Perlumutter S. *et al* (1997, 1998, 1999), Riess A.G. *et al* (1998), Schmidt B.P. *et al* (1998)].As compared to the homogeneous and isotropic FRW models, Bianchi space-times provide spatially homogeneous and isotropic models of the universe. The Bianchi type cosmological models for perfect fluid are studied by Beesham A. (1994), Chakraborty S. and Roy A. (1997). The solutions of Einstein's field equations with varying G and Λ are obtained by Kalligas D. *et al* (1995), Arbab A.I. (2003), Beesham A. *et al* (2000) and Kilinc C.B. (2004). Vishwakarma (2000) was investigated, Bianchi type-I model with varying G and Λ.
The two-fluid source of Bianchi type-VIo model has been investigated by Coley and Dunn (1990). Bianchi type-II space-time with a two-fluid cosmological model has been examined by Pant D.N. and Oli S. (2002).

In the presence and absence of variable G and Λ, two-fluid Bianchi type-I models are studied by Oli S. (2008). Here we have investigated two-fluid models in Bianchi type-V space time in higher dimensions. Motivating with this work, we have investigated two-fluid cosmological model in higher dimensions. This work is the extension of Adhav K. S. *et al* (2011).

2. Field Equations

Bianchi type – V space-time is given by

$$ds^2 = dt^2 - A^2dx^2 - e^{-2\alpha x}\left(B^2dy^2 + C^2dz^2 + D^2du^2\right), \qquad (1)$$

where A,B,C and D are functions of t only and α is a constant.

Here the extra coordinate is taken to be space-like.

The Einstein's field equations for a two fluid source in natural unit (gravitational units) are written as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij} \tag{2}$$

The energy momentum tensor for a two fluid source is given by

$$T_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)} \tag{3}$$

where $T_{ij}^{(m)}$ is the energy momentum tensor for matter field and $T_{ij}^{(r)}$ is the energy momentum tensor for radiation field [5] which are given by

$$T_{ij}^{(m)} = (p_m + \rho_m) u_i^m \, u_j^m - p_m g_{ij} \tag{4}$$

$$T_{ij}^{(r)} = \frac{4}{3} \rho_r u_i^r \, u_j^r - \frac{1}{3} \rho_r g_{ij} \tag{5}$$

with

$$g^{ij} u_i^m \, u_j^m = 1, \qquad g^{ij} u_i^r \, u_j^r = 1 \tag{6}$$

the off diagonal equations of (2) together with energy conditions imply that the matter and radiation are both co-moving, we get,

$$u_i^{(m)} = (0,0,0,0,1), \qquad u_i^{(r)} = (0,0,0,0,1) \tag{7}$$

Using (1), (3), (4), (5) and (6) the field equations (2) reduces to :

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{D}}{BD} + \frac{\dot{C}\dot{D}}{CD} - \frac{3\alpha^2}{A^2} = -8\pi\left(-p_m - \frac{\rho_r}{3}\right) \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{D}}{AD} + \frac{\dot{C}\dot{D}}{CD} - \frac{3\alpha^2}{A^2} = -8\pi\left(-p_m - \frac{\rho_r}{3}\right) \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{D}}{D} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{D}}{AD} + \frac{\dot{B}\dot{D}}{BD} - \frac{3\alpha^2}{A^2} = -8\pi\left(-p_m - \frac{\rho_r}{3}\right) \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = -8\pi\left(-p_m - \frac{\rho_r}{3}\right) \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{D}}{AD} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{D}}{BD} + \frac{\dot{C}\dot{D}}{CD} - \frac{6\alpha^2}{A^2} = -8\pi(\rho_m + \rho_r) \tag{12}$$

$$\frac{3\dot{A}}{A} = \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D}. \tag{13}$$

On integrating (13), we get

$$A^3 = BCD \tag{14}$$

$$A^2 = (BCD)^{\frac{2}{3}}$$

Using (14), the field equation (8) to (12) reduce to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\ddot{D}}{D}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}\dot{D}}{BD}+\frac{\dot{C}\dot{D}}{CD}-\frac{3\alpha^2}{(BCD)^{\frac{2}{3}}}=-8\pi\left(-p_m-\frac{\rho_r}{3}\right) \tag{15}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\ddot{D}}{D}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{A}\dot{D}}{AD}+\frac{\dot{C}\dot{D}}{CD}-\frac{3\alpha^2}{(BCD)^{\frac{2}{3}}}=-8\pi\left(-p_m-\frac{\rho_r}{3}\right) \tag{16}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{D}}{D}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{D}}{AD}+\frac{\dot{B}\dot{D}}{BD}-\frac{3\alpha^2}{(BCD)^{\frac{2}{3}}}=-8\pi\left(-p_m-\frac{\rho_r}{3}\right) \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{3\alpha^2}{(BCD)^{\frac{2}{3}}}=-8\pi\left(-p_m-\frac{\rho_r}{3}\right) \tag{18}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{A}\dot{D}}{AD}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}\dot{D}}{BD}+\frac{\dot{C}\dot{D}}{CD}-\frac{6\alpha^2}{(BCD)^{\frac{2}{3}}}=-8\pi(\rho_m+\rho_r) \tag{19}$$

By comparing (15), (16), (17) and (18), we get

$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\dot{C}}{C}=\frac{\dot{D}}{D} \tag{20}$$

Solving (20), we get

$$A = B = C = D = at + k\,, \tag{21}$$

where a is arbitrary constant and k is constant of integration.

Using above, the space-time (1) can be written as

$$ds^2 = dt^2 - (at+k)^2 dx^2 - e^{-2\alpha x}\left[(at+k)^2 dy^2 + (at+k)^2 dz^2 + (at+k)^2 du^2\right].$$

With suitable choice of coordinates $at + k = T$ and constant, the above metric become

$$dS^2 = dT^2 - T^2\left[dX^2 + e^{-2\alpha x}\left(dY^2 + dZ^2 + dU^2\right)\right].$$

3. Some Physical and Kinematical Properties

We assume the relation between pressure and energy density of matter fluid through the "gamma-law" equation of state which is given by

$$p_m = (\gamma - 1)\rho_m, \qquad 1 \le \gamma \le 2$$

We get energy density of matter, energy density of radiation and total energy density as

$$\rho_m = \frac{15(\alpha^2 - a^2)}{(4-3\gamma)T^2} \tag{22}$$

$$\rho_r = \frac{(9-18\gamma)(\alpha^2 - a^2)}{(4-3\gamma)T^2} \tag{23}$$

$$\rho = \rho_m + \rho_r$$

$$\rho = \ = \frac{(24-18\gamma)(\alpha^2 - a^2)}{(4-3\gamma)T^2} \tag{24}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4),$$

where , $H_1 = \frac{\dot{A}}{A}, H_2 = H_3 = H_4 = \frac{\dot{A}}{A}$

are the directional Hubble Parameter in the direction of *x*, *y*, *z* and *u*-axes respectively.

Case I : *Dust model* In order to investigate the physical behaviour of the fluid parameters we consider the particular case of dust i.e. when $\gamma = 1$.

The scalar of expansion, shear scalar and deceleration parameter are given by

$$\theta = 4H = \frac{4a}{T}$$

$$\sigma^2 = \frac{32a^2}{9T^2} \tag{25}$$

$$q = 0$$

The density parameters for matter and radiation respectively are given by

$$\Omega_m = \frac{15\left(\alpha^2 - a^2\right)}{4a^2}$$
$$\Omega_r = \frac{9\left(a^2 - \alpha^2\right)}{4a^2}$$
$$\Omega_o = \frac{3\left(\alpha^2 - a^2\right)}{2a^2} \quad ,$$

where Ω_o is the total density parameter.

Case II : *Radiation universe* $\left(when\ \gamma = \frac{4}{3}\right)$, On substituting $\gamma = \frac{4}{3}$, we get, the scalar of expansion, shear scalar and deceleration parameter as

$$\theta = 4H = \frac{4a}{T}$$
$$\sigma^2 = \frac{32a^2}{9T^2} \qquad (26)$$
$$q = 0$$

and the density parameters using $\gamma = 4/3$ in (22) and (23) are given by

$$\Omega_m = \infty$$
$$\Omega_r = \infty$$

Case III : *Hard universe* $\left(\gamma \in \left(\frac{4}{3}, 2\right)\ Let\ \gamma = \frac{5}{3}\right)$ the scalar of expansion, shear scalar and deceleration parameter in Hard universe are

$$\theta = 4H = \frac{4a}{T}$$
$$\sigma^2 = \frac{32a^2}{9T^2}$$
$$q = 0$$

For $\gamma = \frac{5}{3}$, (22) and (23) imply density parameters as

$$\Omega_m = \frac{-15\left(\alpha^2 - a^2\right)}{4a^2}$$
$$\Omega_r = \frac{21\left(\alpha^2 - a^2\right)}{4a^2} \qquad (27)$$

$$\Omega_o = \frac{3\left(\alpha^2 - a^2\right)}{2a^2}.$$

Here ρ_m is negative and total density ρ whereas ρ_r is positive.

Case IV: *Zeldovich Universe* $(\gamma = 2)$. In this case, we get, the scalar of expansion, shear scalar and declaration parameter as

$$\theta = 4H = \frac{4a}{T}$$
$$\sigma^2 = \frac{32a^2}{9T^2} \qquad (28)$$
$$q = 0$$

We get, the energy density of matter, energy density of radiation and total energy density as

$$\Omega_m = \frac{-15\left(\alpha^2 - a^2\right)}{8a^2}$$
$$\Omega_r = \frac{27\left(\alpha^2 - a^2\right)}{8a^2}$$
$$\Omega_o = \frac{3\left(\alpha^2 - a^2\right)}{2a^2}$$

Here ρ_m is negative, ρ_r and total density ρ all are positive.

4. Conclusion

The sign of deceleration parameter *q* indicates whether the model accelerates or not. The positive sign of q (> 1) corresponds to decelerating model whereas the negative sign (-1< *q*< 0) indicates acceleration and *q* = 0 corresponds to expansion with constant velocity. Here, in all cases, we get *q* = 0. This implies that these two fluid models are expanding with constant velocity. In all cases, we get, the ratio $\left(\frac{\sigma}{\theta}\right)^2 = \frac{2}{9} \neq 0$. Therefore, these models do not approach isotropy for large value of *T*. These models come out to be rotating as well as expanding ones, the rate of expansion decrease with time, which can be thought of as realistic models.
In absence of fifth dimension our results resembles to that of AdhavK. S. *et al* (2011).

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