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Einstein Rosen Bulk Viscous Cosmological Solutions with Zero Mass Scalar Field in Lyra Geometry

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Abstract

In this paper, we have investigated cylindrically symmetric Einstein-Rosen cosmological model with bulk viscosity and zero-mass scalar field in Lyra geometry .The cosmological models are obtained with the help of the special law of variation for Hubble's parameter proposed by Bermann [1] Some physical properties of the models are discussed.

Keywords: cylindrically symmetric Einstein-Rosen, Zero-mass scalar field, Bulk viscosity, Lyra geometry.

1. Introduction

Based on the cosmological principle, Einstein introduced the cosmological constant into his field equations in order to obtain a static model of the model of the Universe since without the cosmological term his field equations admit only nonstatic cosmological models for nonzero energy density. A year after that, Weyl [2] made one of the best attempts in this direction. He proposed a more general theory in which electromagnetism is also described geometrically.

However this theory, based on non-integrability of length transfer, had some unsatisfactory features and did not gain general acceptance. Later further modification of Riemannian geometry proposed by Lyra [3] bears close resemblance to Wely's geometry , by introducing a gauge function into the structure fewer manifolds which removes the non-integrability condition of the length of a vector under parallel transport and a cosmological constant is naturally introduced from the geometry. In subsequent investigations Sen [4], Sen and Dunn [5] introduced a new scalar-tensor theory of gravitation and obtained the field equations analogous to the Einstein's field equations based on Lyra's geometry which in normal gauge can be written in the form

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij} \phi_\alpha \phi^\alpha = -8\pi G T_{ij} \ , \qquad (1)$$

where φ_i is the displacement field and other symbols have their usual meaning as in Riemannian geometry. The displacement field φ_i can be written as

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$$\phi_i = (0, 0, 0, \beta), \tag{2}$$

where β is a constant (we use the gravitational units $8\pi G = c = 1$).

Halford [31] developed a cosmological theory within the framework of Lyra's geometry which gives rise to nonstatic perfect fluid models. The energy momentum tensor T^{ij} is not conserved in Lyra's geometry. Further, it was shown by Halford [30] that the scalar-tensor treatment based on Lyra's geometry predicts some effects, within the observational limit, as in Einstein theory. Soleng [11] has investigated that constant gauge function ϕ in Lyra's geometry either included a creation field and is equal to Hoyle creation field cosmology [32,33] or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. In the latter case the solutions are equal to the general relativistic cosmologies with a cosmological term.. Beesham [34] constructed four dimensional FRW cosmological models in Lyra manifold. Assuming the energy density of the universe equal to its critical value he showed that the models have $k = -1$ geometry.

Eminent researchers viz. Bhamra [6], Karade and Borikar [7], Kalyanshetti and Waghmode [8], Reddy and Innaiah[9], Reddy and Venkateswarlu [10] and soleng [11] have studied cosmological models based on Lyra's geometry with a constant displacement field vector. Singh and Singh [12,13] and Singh and Desikan [14] have studied Bianchi type-I, III, Kantowski-Sachs and a new class of models with a time dependent displacement field and have made a comparative study of Robertson-Walker models in Einstein's theory with a cosmological term and in the cosmological theory based on Lyra's geometry by using a constant deceleration parameter. Pradhan and Pandey [15] have discussed bulk viscous cosmological models in Lyra geometry. Interacting scalar fields in Lyra geometry have been studied by Pradhan and Vishwarkarma [16], Mohanty *et al* [17], Casana *et al* [18,19], Bali and Chandnani [20], Kumar and Singh [21], Rao *et al* [22] in various context. Recently Ibotombi Singh *et al* [23] discussed bulk viscous cosmological models with variable deceleration parameter in Lyra's geometry. Ram, Zeyauddin and Singh [28], Singh [29] and Rao, Vinutha and Santhi [27] have studied cosmological models based on Lyra's geometry in various contexts.

In this paper, we have investigated cylindrically symmetric Einstein-Rosen cosmological models corresponding to bulk-viscous fluid and zero-mass scalar fields in Lyra geometry with the help of Hubble's Law. We have discussed the physical models corresponding to Zel'dovich fluid, false vacuum and radiation respectively. This paper is organized as follows: Section [2] deals with the metric and field equations in Lyra's geometry. In section [3], we have obtained the solution of the field equations by using the special law of variation for Hubble's parameter, proposed by Bermann [1]. In section [4], the physical parameters are also discussed. The last section contains some concluding remarks.

2. The metric and field equations

We consider the cylindrically symmetric Einstein-Rosen metric in the form

$$ds^2 = e^{(2\alpha - 2\beta)}(dt^2 - dr^2) - r^2 e^{-2\beta} d\phi^2 - e^{2\beta} dz^2, \tag{3}$$

where α and β are functions of t only .we denote the coordinates r, ϕ, z, t as x^1, x^2, x^3, x^4 respectively.

The energy-momentum tensor due to the bulk-viscous fluid and zero-mass scalar field is written in the form

$$T_{ij} = (\rho + \bar{p})\, u_i\, u_j - \bar{p}\, g_{ij} + (\psi_{,i}\, \psi_{,j} - \frac{1}{2} g_{ij}\, \psi_{,m}\, \psi^{,m}), \quad (4)$$

together with

$$u^i u_i = 1, \quad (5)$$

and $$\bar{p} = p - \eta\, U^i_{;i}, \quad (6)$$

where u^i is the four velocity vector of the distribution, p is the isotropic pressure, $\bar{p}$ is the effective pressure, η is the coefficient of bulk-viscosity and ψ is the zero-mass scalar field. Hereafter, the semi-colon (;) denotes covariant differentiation.

The scalar field ψ satisfies the equation

$$\psi^{\,i}_{;i} = 0. \quad (7)$$

Using co-moving co-ordinates, the field equations (1) with the help of (3) and (.4) can be written as

$$\left(\beta_4^2 + \frac{3}{4}\beta^2\right) e^{-(2\alpha-2\beta)} = -[\bar{p} + \frac{1}{2}\psi_4^2 e^{-(2\alpha-2\beta)}], \quad (8)$$

$$\left(\alpha_{44} + \beta_4^2 + \frac{3}{4}\beta^2\right) e^{-(2\alpha-2\beta)} = -[\bar{p} + \frac{1}{2}\psi_4^2 e^{-(2\alpha-2\beta)}], \quad (9)$$

$$\left(\alpha_{44} + \beta_4^2 - 2\beta_{44} + \frac{3}{4}\beta^2\right) e^{-(2\alpha-2\beta)} = -[\bar{p} + \frac{1}{2}\psi_4^2 e^{-(2\alpha-2\beta)}], \quad (10)$$

$$\left(-\beta_4^2 - \frac{3}{4}\beta^2\right) e^{-(2\alpha-2\beta)} = [\rho + \frac{1}{2}\psi_4^2 e^{-(2\alpha-2\beta)}], \quad (11)$$

$$\frac{\alpha_4}{r} = 0, \quad (12)$$

$$\psi_{44} e^{-(2\alpha-2\beta)} = 0, \quad (13)$$

$$\bar{p} = p - 3\eta H, \quad (14)$$

where $$H = \frac{R_4}{R}, \quad (15)$$

is the Hubble's parameter and the subscript '4' after variables denotes ordinary differentiation with respect to t .

3. Solution of the field equations

Using equation (12), we have

$$\alpha = a, \quad (16)$$

where a is constant of integration.

From equations (9) and (10), we get

$$\beta = bt + c\,, \tag{17}$$

where b and c are constant of integrations.

From equations (13), we have

$$\psi = dt + e\,, \tag{18}$$

where d and e are constant of integrations.

Using equations (16),(17) and (18) in equation (8) , we obtain

$$\bar{p} = -\left\{ b^2 + \frac{d^2}{2} + \frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}. \tag{19}$$

where $m = -b$ and $n = a - c$.

Using equations (17),(18) in equation (11) , we have

$$\rho = -\left\{ b^2 + \frac{d^2}{2} + \frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}, \tag{20}$$

where $m = -b$ and $n = a - c$.

Now using the barotropic equation of state

$$p = (\gamma - 1)\rho, \text{ where } \gamma \ (0 \le \gamma \le 2) \text{ is a constant.} \tag{21}$$

Using equations (20) and (21) , we obtain the physical quantities p and η as

$$\rho = -(\gamma - 1)\left\{ b^2 + \frac{d^2}{2} + \frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}, \tag{22}$$

and

$$\eta = \frac{1}{3H}\left\{ (2-\gamma)b^2 + (2-\gamma)\frac{d^2}{2} + (2-\gamma)\frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}. \tag{23}$$

Thus cylindrically symmetric Einstein-Rosen cosmological model with the help of equations (16) and (17) in Lyra Manifold can be written (after suitable choice of coordinates and constants of integration) as

$$ds^2 = e^{-2T}(dt^2 - dr^2 - r^2 d\phi^2) - e^{2T} dz^2. \tag{24}$$

Physical Models

Here we discuss three physical models corresponding to $\gamma = 0, 2, \frac{4}{3}$ of the equation of state given by (24).

Case (I) False vacuum model:

For $\gamma = 0$, the distribution reduces to a special case with equation of state $\rho + p = 0$ which is referred to in the literature as 'false vacuum' or 'ρ vacuum' (Cho, [24]). The physical significance of this fluid in general relativity in non-viscous case has been studied by Mohanty and Pattanaik[25], while Mohanty and Pradhan [26] have investigated the viscous isotropic case.

In this case the physical quantities take the explicit form:

$$\rho = -p = \left\{ b^2 + \frac{d^2}{2} + \frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}, \tag{25}$$

and

$$\eta = \frac{1}{3H}\left\{ 2b^2 + d^2 + \frac{3}{2}\beta^2 \right\} e^{-2(mt+n)}. \tag{26}$$

Case (II) Zel'dovich fluid model :

For $\gamma = 2$, we have $\rho = p$ which represents Zel'dovich fluid or 'Stiff fluid' distribution and we get

$$\rho = p = -\left\{ b^2 + \frac{d^2}{2} + \frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}, \tag{27}$$

and

$$\eta = 0. \tag{28}$$

Case (III) Radiating model:

For $\gamma = \frac{4}{3}$, the distribution reduces to the special case with equation of state $\rho = 3p$, which represents disordered radiation and the physical quantities in this case take the form:

$$\rho = -\left\{ b^2 + \frac{d^2}{2} + \frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}, \tag{29}$$

$$p = -\frac{1}{3}\left\{ b^2 + \frac{d^2}{2} + \frac{3}{4}\beta^2 \right\} e^{-2(mt+n)}, \tag{30}$$

and

$$\eta = \frac{1}{3H}\left\{ \frac{2}{3}b^2 + \frac{d^2}{3} + \frac{1}{2}\beta^2 \right\} e^{-2(mt+n)}. \tag{31}$$

4. Some physical properties

The physical quantities that are important in cosmology are spatial volume v^3, the expansion scalar θ, shear scalar σ^2 which have the following expressions for the model (24) as given below:

Spatial volume= re^{-2T}, . (32)

Scalar expansion $\theta = U^i_{;i}$,

$$= e^{-T}. \quad (33)$$

Shear scalar $\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij}$,

$$\sigma^2 = \frac{2}{3}e^{-2T}. \quad (34)$$

The model (24) has no finite singularity. The spatial volume tends to zero as $T \to \infty$.The scalar of expansion is always negative. Thus the space-time (24) gives a contracting model of the universe .Here $\left(\frac{\sigma^2}{\theta^2}\right)$ does not tend to zero for large values of T which implies that the model is anisotropic and does not approach isotropy.

5. Conclusion

Interacting bulk-viscous fluid and zero-mass scalar fields play a vital role in understanding the early stages of evolution of the universe. In the present work, we have investigated cylindrically symmetric Einstein-Rosen cosmological models with bulk-viscosity and zero-mass scalar field in a scalar tensor theory of gravitation proposed by Sen [4] based on Lyra geometry. While solving the field equations in Lyra geometry, we have used special law of variation for Hubble's parameter proposed by Bermann [1]. We have discussed the physical models corresponding to Zel'dovich fluid, false vacuum model and radiating model. It is observed that the investigated models are free from singularities and are expanding. There seems a good possibility of Lyra's manifold to provide a theoretical foundation for relativistic gravitation, astrophysics and cosmology. However, the importance of Lyra's manifold for astrophysical bodies is still an open question. In fact, it needs a fair trial for experiment.

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References:

1. Berman, M.S: Nuovo Cimento 74B, 182 (1983)
2. Weyl,H.:S-B. Preuss. Akad. Wiss. Berlin,p-465 (1918)
3. Lyra,G.:Math.Z.,54,52.(1951)
4. Sen,D.K.:Z.Phys.,149,311(1957)
5. Sen,D.K. and Dunn,K.A.:j. Math. Phys.,12,578(1971)
6. Bhamra,K.S.:Ausyr. J.Phys.,27,541(1974)
7. Karade,T.M. and Borikar ,S.M.: Gen. Rel. Grav.,9,431(1978)
8. Kalyanshetti,S.B. and Waghmode,B.B.:Gen. Rel. Grav. 14,823,(1982)
9. Reddy,D.R.K and Innainh,P.:Phys. Rev. D.,37,3406(1988)
10. Reddy,D.R.K. and Venkateswarlu,R.:Astrophysics Space Sci. , 136,183(1987)
11. Soleng,H.H.:Astrophysics Space Sci. , 139,13(1987)
12. Singh,T and Singh,G.P.:J.Math. Phys. ,32,2456(1991a)
13. Singh,T and Singh,G.P.:Int. J. Theor. Phys.,31,1433(1992)
14. Singh,G.P. and Desikan,K.:Pramana J. Phys.,49,205(1997)
15. Pradhan,A. and Pandey,H.R.:arXiv:gr-qc/0307038vl(2003)
16. Pradhan,A. and Vishwakarma,A.K.:Int. J. Mod. Phys.,49,32(2004)
17. Mohanty,G.,Mahanta,K.L. and Sahoo,R.R.,:Astrophysics Space Sci. ,306,269(2006)
18. Casana,R.,Pimental,B.M. and deMelo,C.A.M.: Class Quantum Gravity,24,723(2007)
19. Casana,R.,Pimental,B.M. and deMelo,C.A.M.:Astrophysics Space Sci. ,305,125(2006)
20. Bali,R. and Chandnani,N.K.:Astrophysics Space Sci. , 318,325(2008)
21. Kumar,S. and Singh,C.P.:Int. Mod. Phys.,A23,813(2008).
22. Reddy,D.R.K.,Naidu,R.L. and Rao.,A.S.:Int.J.Theor.Phys.,47,1016(2008)
23. Singh,I.,Devi,R.,Singh,S. and Devi,S.: Astrophysics Space Sci. ,321,233(2009)
24. Cho,Y.M.:Phys.Rev.Lett.,68,3133(1992)
25. Mohanty,G. and Pattanaik,R.R.:Astrophysics Space Sci. , 161,253,(1989)
26. Mohanty,G. and Pradhan,A..:Astrophysics Space Sci. , 181,1,(1991).
27. Rao, V.U.M., Vinutha, T., Santhi, M.V.: Astrophys. Space Sci. **314**, 213 (2008)
28. Ram, S., Zeyauddin, M., Singh, C.P.: Int. J. Mod. Phys. A **23**, 4991 (2008)
29. Singh, J.K.: Astrophys. Space Sci. **314**, 361 (2008)
30. Halford, W.D.: J. Math. Phys. **13**, 1699 (1972)
31. Halford, W.D.: Aust. J. Phys. **23**, 863 (1970)
32. Hoyle, F.: Mon. Not. R. Astron. Soc. **108**, 372 (1948)
33. Hoyle, F., Narlikar, J.V.: Proc. R. Soc. Lond. Ser. A **277**, 1 (1963)
34. Beesham,A.:Aust. J. Phys. 41,833(1988).