Article

Nonlinear Theory of Elementary Particles: I. Choice of Axiomatics and Mathematical Apparatus of Theory

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Abstract

In the previous paper (http://prespacetime.com/index.php/pst/article/viewFile/14/11), which can be considered as an introduction to the nonlinear theory, we have shown that the Standard Model (SM) is not an axiomatic, but an algorithmic theory. In the proposed article the simplest (minimal) axiomatics is examined from the point of view of the possible forms of its mathematical representation.

Key-words: axiomatics, nonlinear theory, elementary particles, quantum field theory, physical mathematics.

1.0. The peculiarities of the Standard Model theory, which lie in the basis of the axiomatic theory

The modern theory of elementary particles is called the Standard Model Theory or simply Standard Model (SM). More specifically, SM consists of the quantum field theories of photons, leptons, intermediate bosons, hadrons and theirs interactions.

All these theories are wave theories, in the sense that all particles are described by wave equations relative to the appropriate wave functions of particles. The wave function was introduced as a phase wave, which, as L. de Broglie showed, always accompanies particle motion. The physical meaning of this wave was not discovered.

At the initial stage of development of quantum theory, it was shown that the wave function of the electron cannot be compared with the linear electromagnetic waves of Maxwell's theory. Since no other sense for this function could be established, scientists agreed that the quantum wave function has no physical sense. In order to use it in calculations, it was postulated that the square of the wave function determines the probability of finding the particle at a given point of space at a given moment. This interpretation was assumed to be the basis of quantum theory, since experimental verification showed its complete correctness.

In contemporary quantum field theory, the state of a system is described by elements of Hilbert space. Hilbert space is a generalization of linear Euclidean vector space to the infinite-dimensional case. In other words, Hilbert space is a special case of a linear space. Due to this fact, quantum field theory is considered to be a linear theory.

In order to expand the theory's applicability, attempts to introduce nonlinearity into quantum mechanics were made since a long time ago. These attempts did not achieve the desired result. But the development of quantum field theory led unexpectedly to the appearance of the nonlinearity.

According to modern ideas (Ryder, 1985), the observed substance of the Universe consists of photons, leptons and quarks. Besides electromagnetic interactions, there are strong and weak interactions. All of these interactions are described by the unified theory, which is a substantial generalization of Maxwell's theory. Instead of vectors of the usual electrical and magnetic fields $\vec{E}$ and $\vec{B}$, the modern theory contains several similar field vectors $\vec{E}_i$ and $\vec{B}_i$, and in a natural way, the waves of these vectors are strictly nonlinear.

The first such generalization of Maxwell's theory was made by C. Yang and R. Mills in 1954. All similar theories are therefore called the Yang-Mills theories. Let us emphasize that the nonlinearity is so deeply

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embedded into the nature of the Yang-Mills fields: "*The generalization of the Maxwell theory is the theory of the Yang-Mills fields or non-Abelian gauge fields. Its equations are nonlinear. In contrast to this, the equations of Maxwell are linear, in other words, Abelian*" (I. Nambu); (a detailed derivation of the Yang-Mills equations in the form of Maxwell's equations can be found in the book of Ryder (Ryder, 1985)).

Thus, Yang-Mills theory can be considered as the ***quantum nonlinear electromagnetic theory, in which the nonlinear electromagnetic field is not Maxwellian***. Therefore, nonlinearity and electromagnetics are the essential aspect of the contemporary theory of elementary particles. In connection with this arises the question, why is it possible to examine the contemporary quantum field theory within the framework of linear paradigm, as we have noted previously.

Actually, this approach unavoidably leads to the fact that the nonlinearity must play an essential role in the Universe (Ryder, 1985): "*Now it turns out that non-linear field theories possess extended solutions, commonly known as solitons, which represent stable configurations with a well-defined energy which is nowhere singular. May this be of relevance to particle physics? Since non-Abelian gauge theories are non-linear, it may well be, and the last ten years have seen the discovery of vortices, magnetic monopoles and 'instantons', which are soliton solutions to the gauge-field equations in two space dimensions (i.e. a 'string' in 3- dimensional space), three space dimensions (localized in space but not in time) and 4-dimensional space-time (localized in space and time). If gauge theories are taken seriously then so must these solitions be*".

The idea that elementary particles are electromagnetic solitons, or soliton-like objects, is an idea with a rich history. For instance, we can recall Kelvin's theory of vortex atoms, developed in the middle of the 19th century: here the particles are closed strings, and their mass depends on the frequency of their vibrations as in the modern string theory.

At the end of the 19th and at the beginning of the 20th centuries, realistic soliton-like models were proposed by H. Lorentz for the electron within the framework of a linear approximation (Lorentz, 1952). His proposal was that the electron consists of an electromagnetic field in the limited space volume. This theory was further supported by two substantial results: Lorentz derived the known Lorentz transformations, and found the relationship between the electron mass and energy, but with a coefficient 4/3. Therefore R. Feynman writes (Feynman, Leighton and Sands, 1964; chap. 28, "Electromagnetic mass"): "*We only wish to emphasize here the following points:* 1*) the electromagnetic theory predicts the existence of an electromagnetic mass, but it also falls on its face in doing so, because it does not produce a consistent theory – and the same is true with the quantum modifications*."

Luckily, this defect of the Lorentz theory (i.e. a factor 4/3) can be corrected rather easily. A conscientious merger of this theory with special relativity assures that this factor disappears, since it is incompatible with the relativistic transformation properties. This was first pointed out by Fermi in 1922 (Fermi, 1922), and later rediscovered several times (Kwal, 1949; Rohrlich, 1960; and others; see for detail (Jackson, 1999)). Unfortunately, these papers were either never understood or soon forgotten. In any case, the factor 4/3 can still be found in some of today's texts, as in the above book of R. Feynman. However R. Feynman writes further: "2) *there is experimental evidence for the existence of electromagnetic mass; and* 3) *all these masses are roughly the same as the mass of an electron. So we come back again to the original idea of Lorentz – may be all the mass of an electron is purely electromagnetic, maybe the whole 0.511 MeV is due to electrodynamics. Is it or isn't it? We haven't got a theory, so we cannot say*."

In 1912, the German physicist Gustav Mie (Pauli, 1958; Tonnela, 1959) found a remarkable nonlinear generalization of Maxwell's theory in which electromagnetic waves were nonlinear. In Mie's theory, the electron appeared as soliton-like particle of small and finite dimension, in which the final electromagnetic energy was stored. Later, as an application of this theory, the well-known nonlinear Born-Infeld theory was developed (Pauli, 1958; Tonnela, 1959), and encouraging numerical results were obtained.

The obvious deficiencies in all these preliminary attempts of "solitonization" of the elementary particles were that they did not consider the requirements of quantum theory, expressed in the relationships of the quantization of Planck and de Broglie.

The nonlinear non-Maxwellian electromagnetic field lies in the basis of the proposed nonlinear theory of elementary particles (NTEP). We will show that nonlinearity is needed to understand all special features of wave functions of elementary particles within the framework of the Copenhagen interpretation of Quantum Mechanics. Furthermore, it will follow from this theory that nonlinearity is crucial for understanding all differences which separate classical physics from quantum.

On the basis of above-enumerated results we will select the following the simplest (minimal) axiomatics of the nonlinear theory of elementary particles.

2.0. Axiomatics of the non-linear theory of elementary particles

The axiomatic basis of the proposed theory is composed by 4 postulates, from which the first 3 are the postulates of contemporary field theory. Postulate 4 expresses the specific nonlinearity of theory, but it does not contradict to the results of contemporary physics.

1) Postulate of fundamentality of the electromagnetic field: ***Maxwell's equation for the field without sources:***

$$\frac{1}{c}\frac{\partial \vec{E}}{\partial t} - rot\vec{H} = 0, \qquad (1.2.1)$$

$$\frac{1}{c}\frac{\partial \vec{H}}{\partial t} + rot\vec{E} = 0, \qquad (1.2.2)$$

$$div\vec{E} = 0, \qquad (1.2.3)$$

$$div\vec{H} = 0, \qquad (1.2.4)$$

are fundamental independent equations of motion of fields.

Definition 1: A self-propagated in space, alternated electric and magnetic fields is called electromagnetic (EM) wave.

2) Postulates of Planck and de Broglie: ***the relationship between the energy, frequency and wavelength of photon is determined by the following formulas:***

$$\varepsilon_{ph} = h\nu = \hbar\omega, \qquad (1.2.5)$$

$$\lambda = \frac{h}{p_{ph}} = \frac{hc}{\varepsilon}, \qquad (1.2.6)$$

3) The postulate of superposition of wave fields: ***in the general case electromagnetic waves are the superposition of elementary wave fields, the simplest of which are photons.***

4) Postulate of the massive particles' generation: ***for generation of the massive particles the field of photon must undergo the rotation transformation.***

Here: $\vec{E}$ and $\vec{H}$ are the vectors of strength of electrical and magnetic fields; ε is energy, $\vec{p}$ is momentum, λ is wavelength, c is speed of light.

Let us note that the Maxwell equations of the first postulate are linear equations. Postulate 4 introduces nonlinearity into the theory, transferring the linear equations of Maxwell into the nonlinear non-Maxwellian equations of electromagnetic field. Our goal is to show that all massive elementary particles – intermediate bosons, leptons, quarks, etc - are the special nonlinear electromagnetic wave formations.

We do not consider the question, if this system of axioms is complete. Moreover, judging by the formulation of other axiomatic theories, it is possible to use another system of axioms. We considered the simplest (minimal) of them, which is necessary and sufficient for describing of all known fundamental elementary particles.

Below we consider the mathematical formulation of axiomatics of theory and physical sense of mathematical description of theory.

3.0. Mathematical apparatus of the nonlinear theory of elementary particles

Mathematical apparatus of NTEP does not differ in its form from the mathematical apparatus of SM. But within the framework of NTEP special consideration is given to physical content and physical interpretation of the elements of mathematics. Below we will examine briefly some important for the development of nonlinear theory cases.

3.1. Mathematical forms of writing of Maxwell's equations and their physical sense

First of all, let us note that the physical sense of mathematical expressions is an invariant relative to mathematical symbols and it is fixed only with the determination of their physical values. This means that the choice of mathematical symbols is arbitrary and is dictated only by the convenience and tradition.

Let us enumerate some mathematical forms of Maxwell's equations without the sources (for details see (Jackson, 1999; Akhiezer and Berestetskii, 1965; Schiff, 1955; Cronin, Greenberg, Telegdi, 1967; and others) with minimum explanations.

a) the scalar form

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = 0, \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) = 0 \\ \frac{1}{c}\frac{\partial E_y}{\partial t} + \left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) = 0, \\ \frac{1}{c}\frac{\partial H_y}{\partial t} - \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) = 0, \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = 0, \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = 0 \end{cases}, \qquad (1.3.1)$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0,$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0,$$

Let us note that in a number of cases (especially in case of the electrodynamics of waves of high frequencies) the field functions and the electrodynamic parameters of equations (1.3.1) are considered as complex functions. This makes it possible to solve more simply the problems of generation and propagation of EM waves.

The system of equations (1.3.1) is a most detailed writing of Maxwell's equations in a form, suitable for the solution of concrete practical problems. Further enumerated forms, with respect to this, are different kinds of compactification of record.

b) the vector form, expressed through the vectors of the strengths of the EM field:

$$\frac{1}{c}\frac{\partial \vec{\mathrm{E}}}{\partial t} - rot\vec{\mathrm{H}} = 0,$$
$$\frac{1}{c}\frac{\partial \vec{\mathrm{H}}}{\partial t} + rot\vec{\mathrm{E}} = 0, \qquad (1.3.2)$$
$$div\vec{\mathrm{E}} = 0,$$
$$div\vec{\mathrm{H}} = 0,$$

b) the vector form, which uses the potentials (scalar φ and vector $\vec{A}$):

$$\vec{\nabla}^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = 0$$
$$\vec{\nabla}^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0 \quad , \qquad (1.3.3)$$

The field strengths are expressed here through potentials by the relationships:

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial\vec{A}}{\partial t}, \; \vec{H} = rot\vec{A}$$

b') the 4-dimensional form, which uses the 4-potential $A_\mu = (\varphi, \vec{A})$:

$$\partial^2 A_\mu / \partial t^2 - \Delta A_\mu = 0, \qquad (1.3.4)$$

or through d'Alembert operator:

$$\Box A_\mu = 0, \qquad (1.3.4')$$

where the Greek index μ takes the values: 1,2,3,4.

c) the tensor form

Introducing the following tensor of electromagnetic field:

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu},$$

where the Greek indices $\lambda, \mu, \nu,$ take the values: 1,2,3,4, it is possible for all the equations of Maxwell to be written down in the form of two tensor equations:

$$\frac{\partial F_{\mu\nu}}{\partial x_\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x_\mu} + \frac{\partial F_{\lambda\mu}}{\partial x_\nu} = 0, \; \frac{\partial F_{\mu\nu}}{\partial x_\nu} = 0, \qquad (1.3.5)$$

or, more briefly:

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0, \; \partial_\nu F^{\mu\nu} = 0, \qquad (1.3.5')$$

where the rule of summation over the repetitive indices is used, as well as the reduced record of differentiation).

d) the bivector form:

$$\frac{1}{c}\frac{\partial \vec{F}}{\partial t} - rot\vec{F} = 0, \qquad (1.3.6)$$

where $\vec{F} = \vec{E} + i\vec{H}$, $\vec{F}^* = \vec{E} - i\vec{H}$ are the complex-conjugate bivectors.

f) the bivector operator- matrix form, adequate to Dirac's equation form:

$$i\frac{1}{c}\frac{\partial \vec{F}}{\partial t} - (\hat{\vec{S}} \cdot \hat{\vec{p}})\vec{F} = 0, \qquad (1.3.7)$$

where $\vec{F}$ is the bivector; $\hat{p} = -i\hbar\vec{\nabla}$ is the operator of momentum; $\hat{\vec{S}} = \{\hat{S}_1, \hat{S}_2, \hat{S}_3\}$ are, the so-called, spin matrices of photon, which are the generators of rotation in the 3D-space:

$$\hat{S}_0 = I, \hat{S}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \hat{S}_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \hat{S}_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where $I \equiv \hat{S}_0$ is a single 3 x 3 matrix. Comparing the cases d) and e) it is not difficult to see that they differ only by the form of the record of rotation of fields: $rot\vec{F} = (\hat{\vec{S}} \cdot \hat{\vec{p}})\vec{F}$.

It is possible to easily ascertain that the matrices *S* are subordinated to commutation relationships for the momentum $[\vec{S}, \vec{S}] = i\vec{S}$ and moreover, that $S^2 = S_1^2 + S_2^2 + S_3^2 = 2I$. From this follows that the equation (1.3.7.) describes the particle with spin 1, for which $S^2 = S(S+1)I$. This means that this particle spin S is equal to one some as for photon.

g) equations in the quantum form:

$$\begin{aligned} &\left(\hat{\alpha}_0^{(6)}\hat{\varepsilon} - c \cdot \hat{\vec{\alpha}}^{(6)}\hat{\vec{p}}\right)\psi = 0 \\ &\psi^+\left(\hat{\alpha}_0^{(6)}\hat{\varepsilon} + c \cdot \hat{\vec{\alpha}}^{(6)}\hat{\vec{p}}\right) = 0 \end{aligned}, \qquad (1.3.8)$$

where $\hat{\varepsilon} = i\hbar \partial/\partial t$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are operators of energy and momentum; $\hat{\alpha}^{(6)}$ are 6 x 6 - matrices of the following form:

$$\hat{\vec{\alpha}}^{(6)} = \begin{pmatrix} \hat{0} & \hat{\vec{S}} \\ \hat{\vec{S}} & \hat{0} \end{pmatrix}, \hat{\alpha}_0^{(6)} = \begin{pmatrix} \hat{S}_0 & \hat{0} \\ \hat{0} & \hat{S}_0 \end{pmatrix}, \hat{\alpha}_4^{(6)} \equiv \hat{\beta}^{(6)} = \begin{pmatrix} \hat{S}_0 & \hat{0} \\ \hat{0} & -\hat{S}_0 \end{pmatrix};$$

where $\hat{0}$ is 3 x 3- zero matrix; wave function $\psi = \begin{pmatrix} \vec{E} \\ i\vec{H} \end{pmatrix}$ and Hermitian-conjugated to it function $\psi^+ = \begin{pmatrix} \vec{E} & -i\vec{H} \end{pmatrix} = \begin{pmatrix} E_x & E_y & E_z & iH_x & iH_y & iH_z \end{pmatrix}$ are 6 x 1 and 1 x 6 matrices, respectively.

Let us note that these equations are similar to Dirac's equation without the mass term and describe two antisymmetric systems of Maxwell equations (which can be considered as equations in the right and left handed system of coordinates). It is not difficult to verify that the above 6x6 matrices give correct expressions for the energy density: $\psi^+\hat{\alpha}_0^{(6)}\psi = \vec{E}^2 + \vec{H}^2 = 8\pi\ U$, for the vector of Poynting (and for momentum density, respectively): $\vec{S}_P = \frac{1}{8\pi}\psi^+\hat{\vec{\alpha}}^{(6)}\psi$, and also for Lagrangian of the EM field: $\psi^+\hat{\alpha}_4^{(6)}\psi = \vec{E}^2 - \vec{H}^2 = 8\pi\ I_1$.

Let us emphasize that this form, although identical to the usual equations of Maxwell, has a richer sense. As we will show subsequently, it reflects the connection of electrodynamics with quantum mechanics.

3.2. The forms of the record of field equations and parameters

Besides enumerated above forms, there are also other forms: quaternion, biquaternion, algebraic and others. A great number of record forms of equation in different systems of coordinates (including curvilinear) should be added. Moreover, not only the equations of electromagnetism, but almost all equations of physics can be written down in a set of forms.

Are these forms physically significant? Or in other words: are some of them capable of giving additional results in comparison with other forms?

It is not difficult to understand that all enumerated above forms of the equations of EM field from a) to g) are almost (with some exceptions, which we will mention below) equal: no additional results can any of them give in comparison to others. As proof of this assertion serves the fact that from any form we can pass to the form a). Furthermore, in order to obtain of the real solution of a problem we must pass from any form to such form, in which all the equation quantities can be measured.

But frequently an exceptional sense is given now to the choice of one or another form. Here the position is complicated by the fact that many judgments and conclusions, which are recognized as obvious, but are not reinforced by proofs, were accumulated. This leads sometimes to conclusions, which are not justified in any way, and even to difficulties in the theories, which we will note below.

At the same time, the different mathematical forms of writing of equations are not useless: each form has some physical sense. If we characterize the usefulness of forms by one general property, it is possible to say that each form facilitates a solution of a specific problem. Actually, each form is invented and introduced into the practice in order to reflect some characteristic of a physical object. Frequently these characteristics (as EM field strengths and EM potentials, energy - momentum and frequency - wave vector, etc) are connected together one-to-one or with an accuracy to the values, which do not affect measurements. Therefore we can formulate equations, separating in each case some parameters and leaving others in a latent state.

For instance there is connection between the force and the energy. Due to this connection at least two forms of the equations - energy and force forms - exist. Depending on initial conditions and requirements of each task, it is more easy to solve it in one or the other equation form.

Below we will examine in greater detail the physical sense of some mathematical forms, which play an important role in the theory of elementary particles.

3.2.1. *Choice of the wave functions*

As is known (Akhiezer and Berestetskii, 1969; Levich, Vdovin and other, 1971; Gottfried and Weisskopf, 1984), the wave theory of photon is the quantized theory of the electromagnetic waves of classical electrodynamics.

In the literature of recent decades can be found the assertion that the quantum wave function of photon is 4-potential of electromagnetic field. Accordingly, in this case the equations of classical electrodynamics, expressed through the 4-potential of electromagnetic field, serve as base for the introduction of the quantum equation of photon.

Unfortunately, the authors, who use this approach, usually forget to mention that this is not the only choice of wave functions and equations of photon. Moreover, at the initial stage of development of the theory the strength of electromagnetic field served most frequently as wave functions of photon theory (Akhiezer and Berestetskii, 1969; Levich, Vdovin and other, 1971). Most clearly this possibility is manifested in the operator-matrix form f), where the matrices of photon are clearly present.

As we noted above, in the choice of one or the other form cannot destroy the theory, since the potentials of electromagnetic field can be expressed through its strengths and vice versa. But as is known, in this case the correspondence between the field strengths and its potentials is determined with an accuracy to some arbitrary functions. On one hand, this makes it possible to select the different gauge transformation of potentials, which facilitates solution of problems. But on the other hand, this makes the theory ambiguous. Unfortunately, in the contemporary theory (Kyriakos, 2010), in which the gauge transformation plays the basic role, this leads to a significant complication of the theory.

In the proposed theory we reject this ambiguity, and as wave functions we use only the strength vectors of electromagnetic field. In this case it is possible to avoid some difficulties, which appear in the theory of Standard Model.

3.2.2. *Physical sense of the complexity of the functions*

The complex form of record plays a very important role in the quantum theory of elementary particles. As is known, complexity is the inherent property of the equations of quantum mechanics and theory of elementary particles. In the quantum field theory the physical characteristics of field (energy, momentum and so forth) are determined by bilinear forms from the wave functions. A probabilistic interpretation would encounter great difficulties, if the wave functions of elementary particles were real. But namely because of this interpretation, the complexity of equations and wave functions does not have a physical sense in the framework of modern quantum theory.

On the other hand, since the wave function describes elementary particles, i.e. the real physical objects, it would be very strange, if the complexity of wave functions did not have a physical sense.

A complex quantity can be expressed in various mathematical forms, each of which reflects some physical property. The record: $z = x + iy$ (where x and y are real numbers, and symbol i shows independence of x and y), in the geometric sense reflects the position of material point on the plane in the system of rectangular coordinates x and y . In this case the value z can be compared with the radius-vector of point (x , y) with length $r = |z| = \sqrt{x^2 + y^2}$.

If we express the actual *x* and imaginary *y* parts of the complex value through the module $r = |z|$ and argument ϕ so that $x = r\cos\phi$, $y = r\sin\phi$, then any complex number *z*, except zero, can be written down in the trigonometric form: $z = r(\cos\phi + i\sin\phi)$.

The physical sense of the *exponential* form, connected with the *trigonometric* form through the Euler formula $e^{i\phi} = \cos\phi + i\sin\phi$, consists in the rotation of vector r to the angle ϕ. If rotation is accomplished with the angular velocity ω, the indicated form of record describes the rotation of vector r with the angular velocity ω.

Comparing trigonometric and exponential forms, it is not difficult to conclude that the complex form describes also two mutually perpendicular oscillations along the axes x and y with the angular frequency ω. In connection with this, let us note an additional important property of the exponential form.

The general equation of circle in the Cartesian coordinates (with the center at the origin of coordinates) is written in the form of the **nonlinear** equation $x^2 + y^2 = R^2$, where R is radius of a circle. At the same time, this circle can be described with the system of two **linear** parametric equations: $x = R\cos\omega t$, $y = R\sin\omega t$, or in the form of one linear parametric equation: $z = R \cdot e^{i\omega t}$.

An important conclusion follows from this: *the complex form of record reflects the identity of the* ***nonlinear*** *circular motion (rotation) to the sum of* ***two linear*** *harmonic oscillations along mutually perpendicular axes*.

As it is known the possibility of complex record of the equations and their parameters is inherent in Maxwell's theory. Thus this allows to use it as basis for construction of quantum theory as complex theory.

Actually, in the basis of electromagnetic theory lies the description of the motion of three vectors: electrical $\vec{E}$, magnetic $\vec{H}$ and Poynting's vector $\vec{S}_P = \frac{1}{4\pi}\left[\vec{E} \times \vec{H}\right]$, which reflect the connection between the electromagnetic and mechanical values of theory. These three vectors are mutually perpendicular. In the tasks of emission and propagation of waves they are described by harmonic functions. Moreover, in the majority of physical tasks these functions can be expanded in the Fourier series of the trigonometric or exponential functions

$$f(t) = \sum_{\omega=-\infty}^{+\infty} f_\omega e^{i\omega t} = \sum_{\omega=-\infty}^{+\infty} f_\omega (\cos \omega t + i \sin \omega t).$$

The same occurs in the quantum field theory. The solutions of the equations of quantum field theory are the harmonic waves, recorded in the complex form:

$$\psi = B \exp\left(-\frac{i}{\hbar} (\varepsilon\ t - \vec{p}\vec{r}) \right), \qquad (1.3.9)$$

where $B = be^{i\phi}$; amplitude b is the number, and ϕ is the initial phase of wave. Function (1.3.9) is the eigenfunction of the operator of energy-momentum, where ε and $\vec{p}$ are the eigenvalues of energy and momentum respectively. Moreover, at any point of space this wave describes harmonic oscillation with respect to time, which is possible to interpret also as the rotation:

$$\psi = C \exp\left(-\frac{i}{\hbar} \varepsilon t \right) = C \exp(-i\omega t) \qquad (1.3.10)$$

where $\varepsilon = \hbar \omega$. This interpretation can be connected with the role that has the gauge transformation theory in physics of elementary particles. As is known, the gauge transformations of the wave function, which are the transformations of its phase, make it possible to describe all interactions of elementary particles. *From a physical point of view these transformations consist in the rotation of own field particles.*

We can assume that in connection with quantum theory of elementary particles these interpretations of complex function correspond to the behavior of the objects of these theories. In other words *we can assume that by means of the complex form of record the* ***nonlinear*** *nature of the objects of the quantum field theory can be described in the* ***linear*** *form*.

According to postulate 4 the transition from linear classical electrodynamics to the nonlinear theory of elementary particles is connected with the transformation of rotation. At the same time, rotation transformations play large role in the contemporary theory of elementary particles - Standard Model. The question arises: is there some special feature in the quantum description of rotation in comparison with the classical description.

3.2.3. *The complex form and rotation description*

Further we will examine the descriptions of rotation characteristics in the classical and quantum theory in two cases (Rayder, 1985). This will allow us to draw a number of important conclusions.

1) Description of arbitrary rotation in the three-dimensional space

1a) ***Description with the use of mathematical apparatus of classical mechanics***

In general form for the arbitrary rotation in the space it is possible to write it down as:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = (R) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

where R is 3 x 3 orthogonal matrix of rotation. Arbitrary rotation in the space is composed of rotations around each of the axes x, y, z, each of which is described by suitable matrix. Matrix R is the product of these three matrices, which form the group, designated as $O(3)$. Using trigonometry, it is possible to find these matrices:

$$R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix},\ R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix},\ R_y(\chi) = \begin{pmatrix} \cos\chi & 0 & -\sin\chi \\ 0 & 1 & 0 \\ \sin\chi & 0 & \cos\chi \end{pmatrix}$$

The three angles (in this case, Euler angles) are called parameters of rotation. Matrices do not commutate between themselves. It is possible to express moment of rotation by means of the indicated matrices.

1b) *Description with the use of mathematical apparatus of quantum field theory*

In the group theory, which is the mathematical apparatus of the quantum field theory, in order to obtain the rotation to the final angle the infinitesimal rotations are used. Their matrices are expressed as follows:

$$R_z(\delta\theta)=1+iJ_z\delta\theta\,,\;\; R_x(\delta\phi)=1+iJ_x\delta\phi\,,\;\; R_y(\delta\chi)=1+iJ_y\delta\chi\,,$$

where J_z, J_x, J_y are called generators of rotations, which are determined as follows,:

$$J_z=\frac{1}{i}\frac{dR_z}{d\theta}\bigg|_{\theta=0}=\begin{pmatrix}0&-i&0\\ i&0&0\\ 0&0&0\end{pmatrix}, J_x=\begin{pmatrix}0&0&0\\ 0&0&-i\\ 0&i&0\end{pmatrix}, J_y=\begin{pmatrix}0&0&i\\ 0&0&0\\ -i&0&0\end{pmatrix}$$

(note that it is possible to express the angular momentum of classical bodies through the rotation generators).

The matrix of rotation to the final angle can be also expressed as the complex function:

$$R_z(\theta)=e^{iJ_z\theta},\; R_x(\phi)=e^{iJ_x\phi},\; R_y(\chi)=e^{iJ_y\chi}$$

It is easy to ascertain that this corresponds to the above matrices:

$$e^{iJ_z\theta}=1+iJ_z\frac{\theta}{1!}-iJ_z^2\frac{\theta^2}{2!}-iJ_z^3\frac{\theta^3}{3!}.+...=\begin{pmatrix}1&0&0\\ 0&1&0\\ 0&0&1\end{pmatrix}+\theta\begin{pmatrix}0&1&0\\ -1&0&0\\ 0&0&0\end{pmatrix}+$$

$$\frac{\theta^2}{2!}\begin{pmatrix}-1&0&0\\ 0&-1&0\\ 0&0&0\end{pmatrix}+\frac{\theta^3}{3!}\begin{pmatrix}0&-1&0\\ 1&0&0\\ 0&0&0\end{pmatrix}+...=\begin{pmatrix}\cos\theta&\sin\theta&0\\ -\sin\theta&\cos\theta&0\\ 0&0&1\end{pmatrix}=R_z(\theta)$$

and so forth.

2) Description of arbitrary rotation in the spinor space

It is not difficult to show (Rayder, 1985), that the $SU(2)$ -transformation above the spinor $\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}$ is completely similar to the $O(3)$ -transformation above the vector $\begin{pmatrix}x\\ y\\ z\end{pmatrix}$. In other words, all mathematical formulas occur for $SU(2)$ -transformation, which are given above for $O(3)$ - transformation, recorded in the mathematical form of both classical and the quantum theory. If we designate the matrix of the $SU(2)$ - transformation through U, then we have the following correspondence between spinor and vector fields relative to rotation of any axis $\vec{n}$ to the angle θ:

$$U=e^{i\vec{\sigma}\vec{n}\theta/2}=\cos(\theta/2)+i\vec{\sigma}\vec{n}\sin(\theta/2)\leftrightarrow R=e^{i\vec{J}\vec{n}\theta}$$

where $\vec{\sigma}$ are the Pauli spinor matrices. From this correspondence follows that the transformation describes the rotation of spinor to half of angle, when vector is turned to the complete angle. rotates. In the quantum theory the wave functions of fermions actually possess such special feature (see the theory of electron).

It appears also that $\vec{J}$ and $\vec{\sigma}$ can be written down also as differential operators (not only as matrix). They will, of course, obey the same commutation relations. So we will obtain for the rotation in the 3-dimensional space (3x3 matrices)

$$\hat{\vec{J}}=\left[\hat{\vec{r}}\times\hat{\vec{p}}\right],$$

where $\hat{\vec{r}}=\{x,y,z\}$, $\hat{\vec{p}}=-i\hbar\vec{\nabla}$. Applied to the wave function, this operator will give the value of angular momentum.

Thus, as we see, there is no difference between the quantum and classical description of the rotation and rotation transformation. Depending on a goal, we can use various forms of description. In this case, description in the form of classical physics can give to us the visual physical interpretation of mathematics, which is used in the quantum theory.

3.2.4. *Physical sense of the four-dimensional forms*

There is the opinion that the 4-space-time record of the equations of Maxwell and other equations makes fundamental sense and contains much additional information. In particular, with 4-dimensional form of electrodynamics is connected the covariance of equations of EM field relative to the transformations of Poincare-Lorentz group. The existence of a 4-dimensional form of record is understood sometimes as an independent existence of 4-dimensional space-time. Frequently, the writing of the laws of physics in four-dimensional form identifies with the relativity. The assertion that only the four-dimensional form of record is correct is even encountered.

The 4-dimensional form of record actually has a great physical sense, but not that, which is enumerated above. The best thing about this was said by R. Feynman (Feynman, Leighton, Sands, 1964)):

"This equation, together with the conservation of charge, gives us the fundamental law of the electromagnetic field:

$$\partial^2 A_\mu / \partial t^2 - \Delta A_\mu = j_\mu / \varepsilon_o ,\ \nabla_\mu j_\mu = 0 \qquad (25.29)$$

There, in one tiny space on the page, are all of the Maxwell equations - beautiful and simple. Did we learn anything from writing the equations this way, besides that they are beautiful and simple? In the first place, is it anything different from what we had before when we wrote everything out in all the various components? Can we from this equation deduce something that could not be deduced from the wave equations for the potentials in terms of the charges and currents? The answer is definitely no. The only thing we have been doing is changing the names of things - using a new notation...

What then is the significance of the fact that the equations can be written in this simple form? From the point of view of deducing anything directly, it doesn't mean anything. Perhaps, though, the simplicity of the equations means that nature also has a certain simplicity…

It is therefore absolutely obvious that a simple notation that just hides the complexity in the definitions of symbols is not real simplicity. It is just a trick…

However, there is more to the simplicity of the laws of electromagnetism written in the form of Eq. (25.29)*… The fact that the electromagnetic equations can be written in a very particular notation which was designed for the four-dimensional geometry of the Lorentz transformations - in other words, as a vector equation in the four-space - means that it is invariant under the Lorentz transformations…*

The fact that the Maxwell equations are simple in this particular notation is not a miracle, because the notation was invented with them in mind. But the interesting physical thing is that every law of physics must have this same invariance under the same transformation".

Obviously, there is no need to rewrite the theory in the 4-dimensional designations in order to prove its Lorentz-covariance. The covariance of Maxwell's equations relative to the transformations of Poincare-Lorentz was proven before the 4-dimensional form of record was introduced into physics. Moreover, Maxwell's equations are relativistic-invariant not only in the 4- dimensional record, but in any correct record.

Nevertheless, the question remains why most common forms of physical equations can be recorded in the 4-dimensional form (or differently: which physical reason exists for the fact that "*every law of physics must have this same invariance under the same transformation*"?). It is possible to say that Lorentz transformations appear as the reflection of some unified beginning of nature. Then the question can be reformulated as: what unified beginning of nature does the existence of the Lorentz transformations reflect?

From a formal point of view "four-dimensionality of world" consists in the fact that all "correct", i.e. relativistic equations of nature must contain, conditionally speaking, both time and space coordinates in the same degree.

Then the question arises: is there such unified physical quantity, which does contain simultaneously both time and space coordinates and makes equations relativistic, and the world "four-dimensional"?

It occurs that such value exists. Let us name this function as theta (ϑ) and let us enumerate the requirements, which it must satisfy:

1) for describing the material particles,"theta" must enter a de Broglie phase wave (1.3.9):

2) it must describe gauge transformation;

3) it must contain the pair of canonical variables "time - space coordinates" and it must not have a sense with respect to separate components of this pair;

4) it must be physical, i.e. it must contain values, which can be measured directly;

5) it must be dimensionless in order to be invariant relatively to the choice of units.

Obviously, only the phase of wave (1.3.9) is such value "theta":

$$\vartheta = \frac{1}{\hbar}\left(\varepsilon\ t - \vec{p}\vec{r}\right) = \left(\omega\ t - \vec{k}\vec{r}\right), \qquad (1.3.11):$$

It satisfies all requirements enumerated above and offers explanation of many facts. Actually, comparing the 4-vectors we see that, in spite of apparent variety, they are connected with each other with the completely specific relationships, out of which this association has no sense. Moreover, it is not difficult to show that all canonical dual variables can be expressed through the function "theta".

Now it is possible to answer the question, why the transformations of Poincare-Lorentz determine, in a certain sense, the physics of our world: the de Broglie waves, which describe matter, must not change from rotations and displacement of the coordinate system

The Lorentz transformations corresponds to invariance with respect to the rotation in the 4-dimensional space, which are described by the mathematical expressions, similar to the ones given above.

As is known, on the basis of Noether theorem, the conservation laws follow from the invariance relative to transformations. In particular, from the invariance relative to the transformations of Poincare follow the conservation laws of energy and momentum. But it is not difficult to obtain these laws also from the function of theta.

The invariance of the function of theta relative to shift in the time means that $\partial\vartheta/\partial t = const1$, while the invariance relative to shift in the space means that $grad\ \vartheta = const2$. Actually, it follows from (3.11):

$$\frac{\partial\ \vartheta}{\partial\ t} = \varepsilon / \hbar\,,\quad grad\ \ \vartheta = -\vec{p} / \hbar\,, \qquad (1.3.12)$$

From this follows that $\varepsilon = \hbar \cdot const1$ and $\vec{p} = \hbar \cdot const2$, or in the case of EM wave: $\varepsilon = \hbar \cdot \omega$ and $\vec{p} = \hbar \cdot \vec{k}$ ($\vec{k} = \vec{k}^{\,0}/\lambdabar$ is the wave vector, $\lambdabar = \lambda/2\pi$ is the shortened wavelength).

As we see, although the values (3.12) compose the 4-vector, the presence of the conservation laws is not connected to this fact.

Moreover, the function "theta" gives the possibility of describing not only different parameters of particles, but it can also serve as the basis of the construction of theory.

In the case of EM wave we obtain:

$$\left(\frac{1}{c}\frac{\partial\ \vartheta_{pl}}{\partial\ t}\right)^2 - \left(grad\ \ \vartheta_{pl}\right)^2 = 0\,, \qquad (1.3.13)$$

i.e. Hamilton-Jacobi equation, which describes the propagation of the front of plane EM wave. It is also obvious that this equation is simultaneously a dispersion equation of EM wave: $\omega^2 - c^2\vec{k}^2 = 0$. As is known in the case of nonlinear waves this relationship takes the form:

$$\omega^2 - c^2\vec{k}^2 = \omega_0^2\,, \qquad (1.3.14\,)$$

where ω_0 is constant. Comparing with the equation of energy-momentum conservation, which describes massive particle (for example, electron):

$$\varepsilon^{\,2} - c^2\vec{p}^{\,2} = m^2c^4\,, \qquad (1.3.15)$$

it is not difficult to ascertain that it corresponds namely to nonlinear waves. Moreover, this equation (1.3.15), expressed through the function "theta":

$$\left(\frac{1}{c}\frac{\partial\ \vartheta}{\partial\ t}\right)^2 - \left(grad\ \ \vartheta\right)^2 = \hbar^2 m^2 c^2\,, \qquad (1.3.16)$$

corresponds to Hamilton-Jacoby equation, if the action according to Hamilton was chosen in the form

$$S_H = \hbar^2 \vartheta, \qquad (1.3.17)$$

Let us recall that the ψ -function of the Schroedinger equation is introduced by him (Schroedinger, 1926) by the expression, which coincides to (1.3.9). Namely, he assumed:

$$\psi = \psi_0 \exp\left(i\frac{S_H}{\hbar}\right), \qquad (1.3.18)$$

From above follows that the invariance relative to phase actually corresponds to the invariance of the Lagrangian and action relative to phase (gauge) transformations. Furthermore, we see that the function of "theta", i.e. wave phase, actually makes it possible to formulate a theory in the 4-dimensional form. But it is not difficult to understand that physics in the four-dimensional form gives no new results in comparison with physics, which uses separately these values, since they are always measured independently one from the other.

The only thing that the 4-dimentional approach confirms is that the material objects are built from waves.

3.2.5. *Physical sense of the choice of coordinates for writing of the equations*

The conservation laws are known from Newton's times. In our time the presence of the conservation laws is connected with the symmetry, since the symmetry is equivalent mathematically to conservation of some value. This connection (Noether theorem) was discovered only at the end of XIX centuries and has been used since the beginning of XX centuries. Thus, although the connection of symmetry with the conservation laws is interesting itself, this fact should not have a superfluous sense.

However, using this connection, we can explain the advantages of the appropriate choice of the coordinate system. It known for a long time that the choice of coordinates, which corresponds to the symmetry of task, considerably facilitates the solution of problem. It follows from above that the choice of the coordinate system to be symmetrical for this task automatically considers the appropriate laws of conservation, and in other words reduces the number of unknown parameters of the problem.

For example, the motion of two bodies in the gravitational field (Kepler's task, solved by Newton) are characterized by twelve variables - three coordinates and three projections of momentum. In this case Newton's equations are subordinated to 10 laws of conservation. Using the suitable coordinate system, that automatically considers some of these laws, it is possible to reduce the number of unknown variables and to facilitate the solution of problem.

Conclusion

Consecutively using the indicated axiomatics and the mathematical forms in the construction of the theory of elementary particles, we will show in the following publications that this approach makes it possible to create the theory of the nonlinear quantized waves (i.e. elementary particles), in which all special features of particles appear regularly. Within the framework of this approach the majority of difficulties of Standard Model can be resolved, without the use of additional hypotheses.

The author will be grateful to the readers for useful criticism and additions, which can improve and refine the content.

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