Article

Hidden Dimensions Can Explain 'Superluminal' Neutrinos, and the Origin of Fermionic Mass

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Abstract

Recent findings by OPERA indicate that neutrinos may travel faster than the speed of light. At face value, this implies that Einstein's Theory of Relativity is either incorrect or requires an ad-hoc modification. But this result may be significant evidence for more dimensions than simply the four dimensions of Spacetime, which also has some interesting implications for particle physics. In addition; enough questions about the Standard Model Higgs boson have been raised by recent LHC data that it is wise to consider alternate mechanisms for fermionic mass. We explore how adding a family of 'scalar fermions' might address both of these issues.

Key Words: superluminal, neutrino, tachyon, PMNS matrix

Introduction

Recently, the OPERA Collaboration reported measuring a 6 sigma result of neutrinos travelling at 25 parts-per-million faster than the speed of light in a vacuum, *c* [1]. If this finding bears out, it could be clear evidence of extra dimensions, tachyonic particles, or both. Various possibilities, along these lines, have been explored in the Physics literature [2, 3, 4]. One possible dynamic that is especially interesting is that neutrinos could transform into, or from, some species of particle that is superluminal or tachyonic – and thus receive a boost. In prior papers [5, 6], we proposed 'scalar fermions' with tachyonic properties [7, 8] to supply an origin of fermionic mass and explain coupling differences between left-handed and right-handed neutrinos. This construction can also provide an explanation for the OPERA result.

Tachyons are theoretical particles allowed by Einstein's Special Theory of Relativity, with imaginary-valued proper masses, that always travel faster than the speed of light. The tachyons we posit behave like scalar particles in 4-D Spacetime, but behave like generalized fermions in a higher-dimensional embedding space in the 7-D Theory based on a 3-simplex of Electro-Color and a 4-simplex of Gravi-Weak relations represented in Tables 1 through 4. We expect that this additional family of particles was common in the early universe, perhaps playing a part in the inflationary process, but virtually non-existent after decoupling – because the ambient energy became too low to support their existence. Accordingly; we speculate that decoupling also provides a cutoff mechanism – which effectively hides higher dimensions from view.

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Table 1 – Dual 3-Simplices within a 3-D (g_3, g_8, g_{15}) Electro-Color Basis

Bosons	g_3	g_8	γ	
Charges → ↓ Colors	g_3	$\sqrt{3}g_8$	$\sqrt{6}g_{15} = -\frac{3}{2}Y'$	g'_8
Red r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
Green g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
Blue b	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
White w	0	0	$\frac{3}{2}$	$\frac{1}{2}$

Bosons	g_3	g_8	γ	
Charges → ↓ Colors	g_3	$\sqrt{3}g_8$	$\sqrt{6}g_{15} = -\frac{3}{2}Y'$	g'_8
Anti-Red $\bar{r}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
Anti-Green $\bar{g}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
Anti-Blue $\bar{b}$	0	1	$\frac{1}{2}$	$\frac{1}{2}$
Anti-White $\bar{w}$	0	0	$-\frac{3}{2}$	$-\frac{1}{2}$

Table 1 follows the gluon basis (g_3, g_8) and color definitions that Garrett Lisi used in his "Exceptionally Simple TOE" [9] – with the added definition of a Universal Hypercharge Y' that accounts for proposed Weak interactions of left- or right-handed helicity (see Table 2) and an effective third gluon basis charge g_{15} – which leads to a post-symmetry-breaking secondary conserved quantum number g'_8 and a GUT expectation value for the Weinberg (weak) angle of:

$$g'_8 = \left(\sqrt{3}\, g_8 + \sqrt{6}\, g_{15}\right)\!/3 = \left(\sqrt{3}\, g_8 - \tfrac{3}{2} Y'\right)\!/3 = 0, \pm\tfrac{1}{2}, \pm 1, etc. \tag{1}$$

$$\sin^2\theta_W = \left(\frac{g_{15}}{Y'}\right) = \left(\frac{-3}{2\sqrt{6}}\right)^2 = \frac{3}{8}. \tag{2}$$

The 3-simplex of Electro-Color is an important building block in this model, and it helps account for the total number of dimensions, but the physics behind so-called superluminal neutrinos is largely contained in Table 2, which is based on 4-simplices.

The notation in Table 2 is a spin-off of the lead author's Hyperflavor and Lisi's Gravi-Weak notation, but Table 2 is radically different from Table 1, and deserves special attention. The tabular column under (Z^0, T_{3L}) should be familiar as the *Z* boson and the Weak Isospin quantum numbers for the standard left-handed Weak force. The two columns under (B_1^0, T_{3HF}) and (B_2^0, T_{8HF}) are the mediating bosons and quantum numbers for a proposed Hyperflavor-Weak force [10] that 1) allows us to define the right-handed neutrino as 'something' – otherwise the right-handed neutrino would have a set of zero quantum numbers that could likewise represent 'nothing'; and 2) introduces new, more massive *Z*' and *W*' bosons capable of producing weak helicity interactions between right-handed fermions, and capable of producing weak helicity interactions from right- to left-handed fermions, and vice versa. The tabular column under

Table 2 – Dual 4-Simplices‡ within a 4-D $\left(T_{3L}, \sqrt{3}\,T_{3HF}, \sqrt{6}\,T_{8HF}, \sqrt{10}\,T_G\right)$ Gravi-Weak Basis

Bosons	Z^0	B_1^0	B_2^0	G	F_3		
Charges → ↓ Fermions	T_{3L}	$\sqrt{3}\,T_{3HF}$	$\sqrt{6}\,T_{8HF}$	$\sqrt{10}\,T_G$	$\sqrt{15}\,T_F = -(3Y'+1)/4$	T_{3R}	T'_G
$f_{1L}^{\wedge} = (u_L, \bar{e}_L)$	½	−½	½	½	−½	0	0
$f_{1L}^{\vee} = (d_L, \bar{\nu}_{eL})$	−½	−½	½	½	−½	0	0
$f_{1R}^{\wedge} = (u_R, \bar{e}_R)$	0	1	½	½	−½	½	0
$f_{1R}^{\vee} = (d_R, \bar{\nu}_{eR})$	0	0	−3⁄2	½	−½	−½	0
$sf = (sq, s\bar{l})$	0	0	0	−2	−½	0	−½
$\bar{f}_{1L}^{\wedge} = (e_L, \bar{u}_L)$	−½	½	−½	−½	½	0	0
$\bar{f}_{1L}^{\vee} = (\nu_{eL}, \bar{d}_L)$	½	½	−½	−½	½	0	0
$\bar{f}_{1R}^{\wedge} = (e_R, \bar{u}_R)$	0	−1	−½	−½	½	−½	0
$\bar{f}_{1R}^{\vee} = (\nu_{eR}, \bar{d}_R)$	0	0	3⁄2	−½	½	½	0
$s\bar{f} = (sl, s\bar{q})$	0	0	0	2	½	0	½

(G, T_G) introduces potential Quantum Gravity quantum numbers capable of defining tachyonic matter as something distinct and different from normal or tardyonic matter. This 4-simplex of Gravi-Weak relations has another post-symmetry-breaking secondary conserved quantum number, the right-handed 'Weak Isospin':

$$T_{3R} = \left(\sqrt{3}\,T_{3HF} + \sqrt{6}\,T_{8HF}\right)/3 = 0, \pm\tfrac{1}{2}, \pm 1, etc. \tag{3}$$

If we treat these two spaces – a 3-D Electro-Color and a 4-D Gravi-Weak – as components of a Clifford bivector, then we should expect mixing of the two spaces. If we define:

$$T_F = -(3Y'+1)/\left(4\sqrt{15}\right) \tag{4}$$

then we may construct another secondary conserved quantum number:

$$\begin{aligned} T'_G &= \left(\sqrt{10}\,T_G + \sqrt{15}\,T_F\right)/5 = 0, \pm\tfrac{1}{2}, \pm 1, etc. \\ &= \tfrac{1}{2}\ \text{for}\ sl^{w}, -\tfrac{1}{2}\ \text{for}\ sq^{c}\ \text{and 0 for all known quarks and leptons.} \end{aligned} \tag{5}$$

We may now use these secondary conserved quantum numbers to define the Generalized Intrinsic

‡ The 5-fold symmetry of the 4-simplex yields the scale-invariant properties of the Golden ratio $\phi = \left(1+\sqrt{5}\right)/2$, and this has been used to explain mass ratios within the Ising model of magnetism.

Spin of a Gravi-Weak Fermion:

$$s_F = T_{3L} + T_{3R} + T'_G = \pm\tfrac{1}{2}, \pm\tfrac{3}{2}, \ldots \tag{6}$$

such that we appear to have an 8-D behavior (most likely an Octonion) comprised of two smaller connected spaces (perhaps bi-Quaternions) that mix via the Universal Hypercharge:

$$Y' = -\frac{2\sqrt{6}}{3} g_{15} = \frac{-\left(4\sqrt{15}\,T_F + 1\right)}{3} = 2\times\left(Q - s_F\right) \tag{7}$$

We summarize these secondary conserved quantum numbers in Table 3, and three generations of fundamental fermions in Table 4, where most notation should be obvious except for the introduction of 'scalar fermions' *sf* – including scalar quarks *sq* and scalar leptons *sl*.

These new 'scalar leptons' *sl* have the following properties:

- They are 'sterile' to color, electric and standard weak charges by definition of *Y´*;
- Their quantum numbers $T_{3L} = T_{3R} = 0$ seem to imply a scalar, but their quantum number $T'_G = \pm\tfrac{1}{2}$ (from Equation (5)) implies a higher-dimensional quantum gravitational effect that places these particles in the same particle multiplet as other fundamental Fermions; and
- They behave like tachyons because they are highly non-localized in 3-D space (unlike standard particles which are relatively localized) and carry Gravi-Weak quantum numbers, such that they seem to cause a weak entanglement of particles.

Table 3 – Secondary Conserved Quantum Numbers for Electro-Gravi-Weak

Charges → ↓ Fermions	T_{3L}	T_{3R}	T'_G	s_F	Q	Y'
u_L	½	0	0	½	⅔	⅓
u_R	0	½	0	½	⅔	⅓
d_L	–½	0	0	–½	–⅓	⅓
d_R	0	–½	0	–½	–⅓	⅓
sq	0	0	–½	–½	–⅓	⅓
e_L	–½	0	0	–½	−1	−1
e_r	0	–½	0	–½	−1	−1
ν_{eL}	½	0	0	½	0	−1
ν_{eR}	0	½	0	½	0	−1
sl	0	0	½	½	0	−1

Table 4 – 20-plets of First, Second and Third Generation Fermions

$$
\begin{array}{ccccccccccccccc}
f_{1L}^{\wedge} & f_{1L}^{\vee} & sf & f_{1R}^{\vee} & f_{1R}^{\wedge} & f_{2L}^{\wedge} & f_{2L}^{\vee} & sf & f_{2R}^{\vee} & f_{2R}^{\wedge} & f_{3L}^{\wedge} & f_{3L}^{\vee} & sf & f_{3R}^{\vee} & f_{3R}^{\wedge} \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow
\end{array}
$$

$$
\begin{array}{cc}
r & \rightarrow \\
g & \rightarrow \\
b & \rightarrow \\
w & \rightarrow
\end{array}
\begin{pmatrix}
u_L^r & d_L^r & sq^r & d_R^r & u_R^r \\
u_L^g & d_L^g & sq^g & d_R^g & u_R^g \\
u_L^b & d_L^b & sq^b & d_R^b & u_R^b \\
e_L^w & \nu_{eL}^w & sl^w & \nu_{eR}^w & e_R^w
\end{pmatrix}
\begin{pmatrix}
c_L^r & s_L^r & sq^r & s_R^r & c_R^r \\
c_L^g & s_L^g & sq^g & s_R^g & c_R^g \\
c_L^b & s_L^b & sq^b & s_R^b & c_R^b \\
\mu_L^w & \nu_{\mu L}^w & sl^w & \nu_{\mu R}^w & \mu_R^w
\end{pmatrix}
\begin{pmatrix}
t_L^r & b_L^r & sq^r & b_R^r & t_R^r \\
t_L^g & b_L^g & sq^g & b_R^g & t_R^g \\
t_L^b & b_L^b & sq^b & b_R^b & t_R^b \\
\tau_L^w & \nu_{\tau L}^w & sl^w & \nu_{\tau R}^w & \tau_R^w
\end{pmatrix}
$$

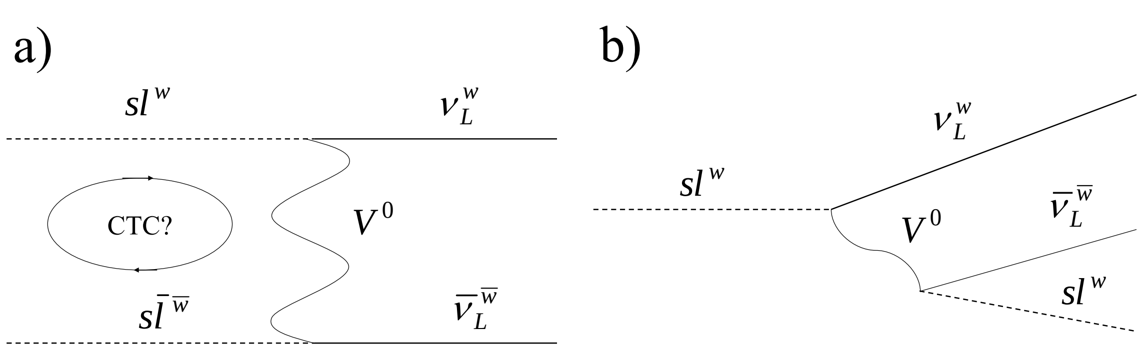
The Physics of 'Superluminal' Neutrinos

Now, we may define a left-handed neutrino ν_L^w with the set of 8 quantum numbers $\left(g_3, \sqrt{3}\, g_8, \sqrt{6}\, g_{15}, T_{3L}, \sqrt{3}\, T_{3HF}, \sqrt{6}\, T_{3HF}, \sqrt{10}\, T_G, \sqrt{15}\, T_F\right) = \left(0,0,\frac{3}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)$ and a 'scalar lepton' sl^w with $\left(0,0,\frac{3}{2},0,0,0,2,\frac{1}{2}\right)$. A Gravi-Weak vector boson V with the following quantum number differences:

$$\Delta = \left(0,0,0,\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{5}{2},0\right) \qquad (8)$$

could affect the transition from a tachyonic 'scalar lepton' into a tardyonic left-handed neutrino. This Gravi-Weak V boson accomplishes two distinct tasks: 1) it increases left-handed Weak helicity by one half, $\Delta T_{3L} = +\frac{1}{2}$, from neutral to left-handed helicity, and 2) it decreases the secondary Gravitational quantum number by one half, $\Delta T_G' = -\frac{1}{2}$, from 'anomalous tachyonic' mass to standard or 'tardyonic' mass. It is reasonable to expect such a weakly-interacting 'Quantum Gravity' event to violate our understanding of 'Relativistic Gravity'. Possible Feynman Diagrams of this process are shown in Figure 1. The left half of Figure 1.a) looks like half of a Closed Time-like Curve (CTC) [11]. Figure 1.b) may represent Gravi-Weak Cherenkov-like radiation.

Figure 1 – Feynman Diagrams for Interactions between a 'Scalar Lepton' and a Neutrino



A Generalized PMNS Matrix

The physics in the prior Sections was based on an 8-D Octonion model that decomposes into bi-Quaternions. To completely define three generations, we need to introduce at least two more dimensions that represent a $G2$ Lie Algebra 'triality' of generations based on close-packing of 1-spheres (circles) into a 2-D hexagonal Graphene-like lattice [12, 13]. These 1-spheres are geometrically representative of Complex Numbers in the same way that 3-spheres and 7-spheres are geometrically representative of Quaternions and Octonions, respectively [14]. This $G2$ of generations also shares the 7-fold symmetries of the Octonion 7-sphere, and decomposes as two generation-independent bosonic basis vectors (Q_3, Q_8) plus three generations – each containing four components, such that we have *ONLY* three generations of tardyonic fermion matter, and a fourth partial generation of tachyonic fermions where this 'fourth generation' is conferred from bosonic basis vectors to fermions via a global $U(1)$ symmetry of 'Universal Hypermass', X'.

$$G2 \rightarrow 2+3\times 4 \qquad G2 \text{ order of } 14 \tag{9}$$

This gives the correspondence between a broken 3-simplex of (3+1) colors and a broken 3-simplex of (3+1) generations with the following substitutions to Table 1:

$$\begin{aligned}
&\left(g_3, \sqrt{3}\,g_8, \sqrt{6}\,g_{15} = {}^{-3}\!/_{2} Y'\right) \rightarrow \left(Q_3, \sqrt{3}\,Q_8, \sqrt{6}\,X'\right)\\
&(r, g, b, \ldots, w) \rightarrow \left(1^{\text{st}}\text{ Gen}, 2^{\text{nd}}\text{ Gen}, 3^{\text{rd}}\text{ Gen}, \ldots, \text{Scalar Fermions}\right)\\
&\sqrt{6}\,X' = \left(10\sqrt{10}\,T_G + 1\right)/8
\end{aligned} \tag{10}$$

where we have eight 'Generaton' bosons, $Q_{1\text{-}8}$ that are analogous to eight gluons, and the octonion 7-sphere is connected with the complex $G2$ 1-sphere via the relationship between Gravitational Charge T_G (the fourth quantum charge in the Gravi-Weak 4-Simplex of Table 2) and Universal Hypermass X'. This requires a minimum of 10-D.

Thus, we may represent these tachyons as a fourth partial generation in the same way that we represent leptons as a fourth partial color ('violet' in Pati-Salam Theory [15] or 'white' in Table 1). This also allows us to model interactions between nearly-degenerate, nearly-massless, Electro-Color neutral, tardyonic neutrinos and tachyonic scalar fermions in the same way that we model Neutrino Flavor Oscillations. These tachyonic 'scalar fermions' originate in the 8-D octonion symmetries, but are stable on a 2-D membrane – most likely the 2-D hexagonal Graphene-like G2 lattice – where they obey Anyonic [16] statistics.

Table 5 – Generalized PMNS [17, 18] Matrix between Left-handed Neutrinos & 'Scalar Leptons'

$$\begin{pmatrix} |sl\rangle \\ |\nu_{eL}\rangle \\ |\nu_{\mu L}\rangle \\ |\nu_{\tau L}\rangle \end{pmatrix} = \begin{pmatrix} U_{s0} & U_{s1} & U_{s2} & U_{s3} \\ U_{e0} & U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 0} & U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 0} & U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} |\nu_0\rangle \\ |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

where $|\nu_0\rangle$ is the negative mass-squared eigenstate that we obtain by diagonalizing the masses between the three neutrino flavors and the scalar lepton, all *s* notation refers to the scalar lepton, *sl* (not the strange quark), and all other notation is identical to the 'standard' 3×3 PMNS Matrix.

Of course, four dimensional rotations entail both left- and right-isoclinic rotations, so a similar Mixing Matrix – with different matrix entries – would be required to represent interactions between Right-handed Neutrinos[§] and 'Scalar Leptons'. Another interesting point is that the 'scalar leptons' carry the same electric charge *Q* as do neutrinos (and thus the similar applications via the PMNS matrix), and the 'scalar quarks' carry the same electric charge as do down-type quarks. So a similar type of mixing may occur between down-type quarks and 'scalar quarks' via the CKM matrix [19, 20], but this mixing effect is expected to be difficult to observe with 'scalar quarks' because 1) quarks are generally more massive than neutrinos, such that this interaction is less favored by kinematics, and 2) any observation of such mixing is complicated by color confinement. These charge similarities between scalar leptons vs. neutrinos and scalar quarks vs. down-type quarks could be the primary reason that down-type quarks mix via the CKM Matrix and neutrinos mix via the PMNS Matrix, but other fermions *DO NOT* mix.

Much of the criticism against the OPERA results is that they seem incompatible with observations of SN1987a. However, if certain neutrino flavors mix better with tachyons than do other neutrino flavors, then we may actually see at least two bursts of neutrinos – those neutrino flavors that mix well with tachyons (say higher-energy muon neutrinos if $|U_{\mu 0}| > |U_{e0}|$) would arrive before the gamma burst and before our actual observation of the Supernova, and those neutrino flavors that fail to mix with tachyons (say lower-energy electron neutrinos) would arrive after the gamma burst. Such a scenario may have occurred at the Mont Blanc Liquid Scintillation Detector, which detected a burst of 5 neutrinos prior to the visual observation of SN1987a [21], and implied 'superluminal neutrinos' years ago [2, 3, 4]. Another possible difference we may observe in these experiments is that the passage of neutrinos through matter may enhance

[§] Note that the right-handed Hyperflavor-Weak interactions of Table 2 are reduced by the larger anticipated masses of Hyperflavor-Weak *Z´* and *W´* bosons.

neutrino flavor mixing. Some recently published papers claim that 1) superluminal speed for neutrinos is 'impossible' within the Standard Model because bremsstrahlung radiation into electron-positron pairs will dissipate their energy [22], or that 2) the OPERA collaboration made a trivial GPS relativistic correction error. The easiest position to defend is that of a 'skeptical observer', but we should develop models that can explain 'superluminal neutrinos' just in case these OPERA results stand up to tests of scrutiny and independent confirmation.

Consider a light-weight tachyon that reaches relativistic energies easily. Such a tachyon would slow down to slightly faster than the speed of light[**]. Now suppose that we have the Generalized PMNS Matrix of Table 6 that allows tardyonic neutrinos (that are slightly slower than the speed of light) to interact with, mix quantum states with, and oscillate between tachyonic scalar fermions (that are slightly faster than the speed of light) – as long as the mass-squared gap between these states is kinematically accessible (the difference between, say, a few negative MeV^2 vs. a few positive MeV^2). This transition is off the mass shell because $m^2c^4 = E^2 - p^2c^2$ such that different values of mass-squared (even if the mass-squared gap is small) will require different momenta and a virtual particle propagator such as the V boson of Figure 1.

More Thoughts on Neutrino Flavors

Some possible properties of scalar leptons as a 'fourth' neutrino flavor:

- Neutrinos as carriers of spin, with the three families representing the possibility for up to three simultaneous rotations, look like a Quaternion, except they lack a real-valued, or scalar, component. A scalar lepton with imaginary mass and three neutrinos with real mass more closely resemble the Quaternion algebra with the Minkowski metric $(-1,+1,+1,+1)$ of Spacetime. This may explain how 'imaginary' and 'real' masses interact – otherwise, we should expect these particles to be forever separated by the light cone;
- The light-weight electron neutrino is as close to the fermion version of an Unparticle [23, 24] (which must be massless to retain scale invariance) as we have in this scenario – perhaps this nearly-Unparticle mass state is weakly perturbed by the axion [25] or the vacuum; and
- 2-D Magnetic Ising models are induced by phase transitions that lead to scale invariant mass ratios based on the Golden ratio [26, 27]. The Golden ratio arises in 5-fold symmetries such as the 4-simplex of Table 2 with the solution of the equation $\phi^2 - \phi - 1 = 0$, and yields

[**] Tachyonic speeds range from near infinity (for low-energies) to slightly above c (for relativistic energies).

$\{\phi, -\phi^{-1}\} = (1 \pm \sqrt{5})/2 = \{+1.618..., -0.618...\}$. Suppose that the scalar lepton's mass-squared eigenstate obtains the $2 \times \cos(3\pi/5) = -0.618...$ solution (an Icosahedral C_5 rotation†† through $72^\circ = 2\pi/5$ vs. Zamolodchikov's $2 \times \cos(\pi/5) = +1.618...$ solution) relative to the neutrinos' mass-squared eigenstates. If the Icosahedral group I_h of rank-10 and order-120 is involved in this process, then it implies a 10-D TOE that decomposes as:

$$\begin{array}{lllll} SU(11) & \to SU(7) & \times G2 & \times U(1)_{X'} & \times U(1)_{Y'} \\ \text{Icosahedral} & \to \text{Octonion} & \times \text{Graphene} & \times \text{Hypermass} & \times \text{Hypercharge} \end{array} \quad (11)$$

which is consistent with the 60-plet of Fermions in Table 4.

Geometric Attributes of Higher Dimensions

When considering constructions involving dimensions beyond those of conventional spacetime, it is wise to remember that there are geometric considerations. The theory we have presented requires a minimum of 10 dimensions, but having more dimensions changes some of the familiar geometric relations in interesting ways, and this alters the definitions of size, distance, and even locality. Although one might naively imagine that if one can add as many dimensions as you like, things will become more and more spacious as their number increases, this is not the case. The 4-sphere or 5-ball has the greatest content or hypervolume [28, 29], of all the spheres or balls. Ergo; while things become progressively more spacious for up to five spatial dimensions, adding more makes a space more compact instead, with the 24-D Leech lattice being the most compact regular arrangement. The theoretical curve actually has its maximum at around 5.257-D (Fig. 2 below), which means that number is the most spacious dimensionality possible.

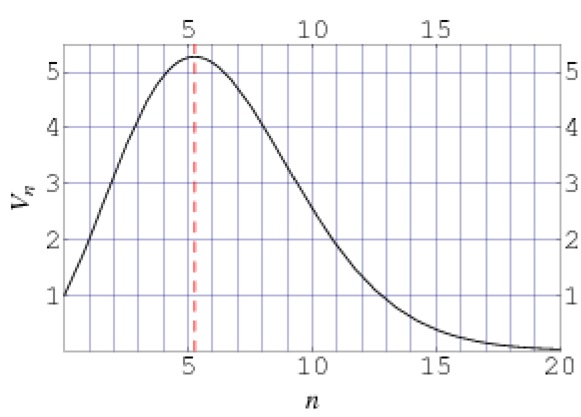


Figure 2 – Hypervolume content is maximal for the 5-D 5-ball, and falls off thereafter.

†† The full Icosahedral group I_h has a rank-10 and effective 10-D within a 3-D space. An order-5, effective rank-4 C_5 rotation operator changes 4 quantum numbers – as does the V boson [Equation (8)].

This fact enters into the question of how additional hidden dimensions help explain the appearance of superluminal neutrinos, in several ways. If the spacetime we observe is actually a subspace of a space with dimension greater than 5, some of those dimensions *must* be compact, while others are likely extended. This observation relates strongly to the idea that some entities are localized, while others are more spread out. People usually equate compactness or extent with the size of objects, but this dichotomy also enters into questions of locality vs. non-locality. One can contrast X-Ray or gamma ray photons – which are tightly localized – with radio frequency photons – which are quite spread out – for example.

This same concept, when extended to higher dimensions, can easily explain how a particle can be tachyonic – by virtue of its extendedness. Earlier; we stated that scalar fermions are spread out, rather than localized, which makes them tachyonic. Why is this? In effect, they are pinned to the edge of the universe, so spreading carries them forward of their location otherwise. Tachyons point at infinity, so their rest state trajectory takes them outside the 'light cone' of the observable universe. However; relativistic tachyonic scalar leptons would be only slightly faster than light and therefore near the edge of that cone. That relativistic neutrinos and relativistic scalar leptons are skirting the same edge makes the transition between the two species accessible, which is why scalar leptons can function as a fourth neutrino flavor, and mix with the other three types.

Another way to look at this is to examine what happens with conventional particles, such as electrons or protons (and their anti-particles), when they are accelerated to relativistic speeds. There is a spreading of the beam cross section, in particle accelerators, as the particles themselves become spread out and flattened. Specifically; they are flattened in the direction of motion, and spread out 90 degrees from that axis. Perhaps; because scalar fermions are native to a higher-dimensional space, they will instead spread in the direction of motion (which is orthogonal to the spreading in conventional particles) but is this only when they have relativistic energies? Instead; the reverse may in some sense be true, because they are tachyonic, and it is only when they are relativistic that they spread forward slowly enough to mix with tardyonic neutrinos.

Thermo-Geometric Instabilities of the Octonion 7-Sphere

A question arises of how we can generalize the concept of spreading, and how the dynamics of spreading relate to the evolution of Spacetime geometry – due to thermodynamic considerations. The 'Spontaneous Inflation' theory of Carroll and Chen [30] asserts that Entropy can act as the driver of universal inflation, because the end result of entropy's action is a universe which is as spread out as possible. Indeed; Leff and Lambert assert that all thermodynamic entropy can be thought of as forms of spreading [31, 32, 33], since concentrations of energy and/or substance become more dispersed into the available space (and/or substance) within a system, over time. However; when we weave in the question of how additional dimensions change this picture, the solution is by no means straightforward, due in part to the fact that different geometries allow us to maximize the volume or the area of spaces and figures.

In the prior Section, we noted that the 4-sphere or 5-ball has the greatest unit volume content of all of the balls or spheres, and that this relates to the volumetric spaciousness of spaces. Likewise, the 7-ball or 6-sphere has the greatest unit hypersurface area of all of the balls or spheres. Note specifically; the dimensionality for which the effective surface area of an object or space is maximal, is a different number from the dimensionality for the greatest effective volume. Figure 3 shows the variation of unit hypersurface area with number of dimensions n, and maximizes for n = 7.25695. Thus, the unit 7-sphere (which is geometrically representative of 8-dimensional or Octonion algebra) actually has less effective surface area than does the unit 6-sphere. We assert that this makes the 7-sphere a metastable object, despite its parallelizability, given that entropy and thermodynamics would tend to push out the boundaries of a system in the absence of constraints – thus maximizing the volume or area of that system – and that spreading is therefore justifiably a driver of cosmological geometrogenesis.

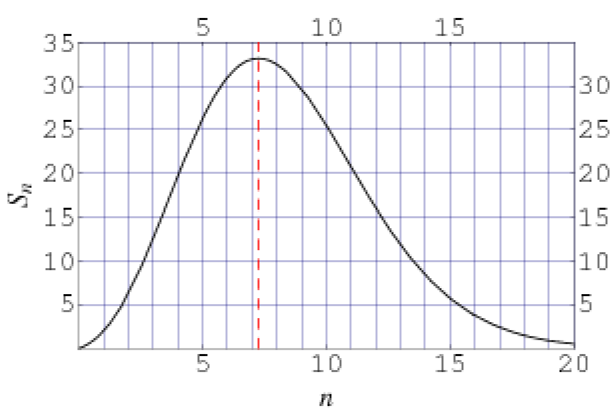


Figure 3 – Hypersurface area content is maximal for the 7-D 6-sphere, and falls off thereafter

With this understanding of the geometrical behavior of extra dimensions, we may now model the thermodynamic behavior of higher-dimensional spaces. An important Thermodynamic state variable is the Gibbs energy G [34]. The Gibbs energy is a measure of the amount of non-mechanical work that a system can perform in an isothermal and isobaric application, and may be defined as:

$$G(T,p) = H - TS \tag{12}$$

where $H = U + pV$ = Enthalpy is the internal energy of the system plus the energy to maintain a constant pressure between the system and its environment; and S = Entropy is the amount of energy available to perform work in a thermodynamic process. Thermodynamic processes generally prefer to minimize Enthalpy while simultaneously maximizing Entropy, such that minimum Gibbs energy designates equilibrium. Gibbs energy is an extensive Thermodynamic quantity, but its partial derivative with respect to the number of states yields the intensive quantity, Chemical Potential: $\mu = \left(\frac{\partial G}{\partial N}\right)_{T,p}$ in an isothermal and isobaric application.

We anticipate that minimum Enthalpy will occur for the 5-D 5-ball because maximal volume allows greater dispersion of the internal energy, and allows a lesser pressure between the system and its environment. Likewise, we anticipate that maximum Entropy will occur for the 7-D 6-sphere with maximal hypersurface area due to Beckenstein-Hawking Black Hole Entropy:

$$S_{BH} = \frac{kA}{4l_P^2} \qquad (A \text{ is surface area}) \tag{13}$$

We thus anticipate a minimum Gibbs energy to occur for a hyperball of dimensionality 5, 6 or 7 – depending on specific values of temperature and pressure. Thus, the octonion 8-D 7-sphere – though parallelizable and stable against rotational perturbations such as the Hairy Ball Theorem – contains a fundamental Thermo-Geometric instability that will cause its decay – first into either a 5-, 6- or 7-ball (none of which are parallelizable), and finally into a parallelizable quaternion 3-sphere and decay products. Simply put; all of the higher-dimensional spaces associated with these figures tend to evolve, or devolve, into subspaces resembling our observable Spacetime. Then, without the activation energy of the pre-decoupling fireball acting as an energy bath, we are constrained (by quantum-mechanical barriers) from any possibility of observing events in higher-order dimensions – even though we continue to abide within the higher-dimensional embedding space. This explains why we do not directly experience Octonion-Physics.

Conclusion

Much of the material in this paper comes out of the lead author's attempts to construct theories of particle physics which require few or no adjusting factors to be put in by hand, as they arise directly out of geometric considerations and mathematical symmetries. A recent paper by Gerard 't Hooft [35] stresses the need to construct such theories, if we hope to ever have a complete understanding of particle physics. The scalar fermions we have discussed are posited to exist because they occupy stations on the 'periodic table of particles' of the author's theory. However; since in this construction, scalar leptons can mix with neutrinos, and since these particles have the property of being tachyonic, this could easily explain the results obtained by the OPERA collaboration between scientists at CERN and Gran Sasso.

Normally, we might expect tachyons to either: 1) induce a broken symmetry, and be absorbed as degrees-of-freedom of that broken symmetry – as with Higgs Theory, or 2) fly off to infinity (or the nearest Black Hole) at an infinite speed. The setting for these tachyons is fundamentally different from Higgs Theory because they belong to the Fermion particle multiplet, as opposed to Higgs which are Bosons and provide the longitudinal degrees of freedom necessary for the *W* and *Z* Bosons. As such, these scalar Fermions may help explain the origin of fermionic mass[‡‡] but *ARE NOT* the sneutrinos of Supersymmetry (SUSY). We do not mean that our theory rules out the possibility for supersymmetry to exist, but rather that adding scalar fermions might be another valid way to extend known symmetries, and thus fill out the 'periodic table'.

Some Weak interactions permit CP Violation, and thus, any successful TOE model must contain complex representations in order to describe CP Violation. Appropriately enough, this model implies real and imaginary masses in addition to complex representations. A violation of Lorentz Invariance – even via Weak interactions – also implies a violation of CPT symmetry [36]. We do not experience these tachyons directly, as they 'live' in a higher-dimensional space (or on the lower-dimensional boundary between spaces), but they may exist nonetheless. Therefore; the mathematical model we have presented might reveal an important component of our understanding of reality, by alerting us to the presence of a partial family of particles that lives mainly in hidden dimensions.

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