Article

Extra Dimensional FRW Cosmology with Variable G and Λ

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Abstract

We have considered a (n+2) dimensional cosmological model of the FRW universe with variable G and Λ. The cosmological parameters have also been obtained for dust, radiation and stiff matter. The state finder parameters are analyzed and have shown that these depends only on ω and ε. Physical and Kinematical parameters of the models are obtained.

Keywords: cosmology, variable G and Λ, EoS parameter, state finder parameters.

1. Introduction

The Einstein field equation has two parameters – the gravitational constant G and the cosmological constant Λ. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein field equations. In an evolving Universe, it appears natural to look at this "constant "as a function of time. Many extensions of Einstein's theory with time dependent G have also been proposed in order to achieve a possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity. From the point of view of incorporating particle physics into Einstein's theory of gravitation, the simplest approach is to interpret the cosmological constant Λ in terms of quantum mechanics and the physics of vacuum There is a significant observational evidence for the detection of Einstein's cosmological constant Λ or a component of material content of the Universe, which varies slowly with time and space and so act like. The exact physical situation at very early stages of the formation of our universe provoked great interest among researchers. Higher dimensional space-time has taken considerable research interest in an attempt to unify gravity with other forces in nature. This idea is particularly important in the field of cosmology since one knows that our universe was much smaller in its early stage than it is today. In this connection a number of attempts have been made to study the role of gravity with other fundamental forces in nature.

The most famous five dimensional theory proposed by Kaluza [1] and Klein [2], was the first theory in which gravitation and electromagnetism could be unified in a single geometrical structure. The Kaluza-Klein's idea to consider the coefficient of the fifth co-ordinate as constant was generalized by Thiry [3] and Jordan [4]. Marciano [5] has suggested that the experimental

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detections of time variations of the fundamental constants could be strong evidence of extra dimensions. To achieve unification of all interactions including weak and strong forces, many authors [6, 7] have extended the Kaluza-Klein formalism to higher dimensions. The investigations of super-string theory and super gravitational theory have created renewed interest among theoretical physicists to study the physics in higher dimensional space-time [8, 9]. Multidimensional space-time is believed to be relevant in the context of cosmology. Thorough study of Kaluza-Klein theory has been undertaken by Wesson [10]. A number of authors (refer [11–18] and references therein) have studied the physics of the universe in higher dimensional space-time.

Nowadays relativists are interested in theories with more than four dimensional space-times. Alvarez et al. [19], Randjbar-Daemi *et al.* [20], Marciano [21] suggested that the experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. The extra dimensions in the space-time contracted to a very small size of Planck length or remain invariant. Further, during contraction process, extra dimensions produce large amount of entropy which provides an alternative resolution to the flatness and horizon problem [22]. Misner [23] pointed out that viscosity plays an important role in the formation of galaxy. Viscosity also accounts for the large entropy per baryon observed in the present universe [24]. Banerjee *et al.* [25] constructed Bianchi type I cosmological models with viscous fluid in higher dimensional space-time. Chatterjee and Bhui [26] obtained exact solutions of the field equations in a five dimensional space time with viscous fluid. Singh et al. [27] obtained exact solutions of the field equations for a five dimensional cosmological model with variable G and bulk viscosity in Lyra geometry. Mohanty and Pradhan[28] constructed Robertson-Walker cosmological model with bulk viscosity with equation of state $p=(\gamma-1)\rho$ where $0\le\gamma\le 2$. Murphy[29] constructed isotropic homogeneous spatially flat cosmological model with bulk viscous fluid alone because the shear viscosity can not exist due to assumption of isotropy . Bali and Jain [30, 31] investigated some expanding and shearing viscosity fluid cosmological model in which the coefficient of shear viscosity is proportional to the rate of expansion of model. Further, they showed that in free gravitational field the model yields “degenerate and non degenerate Petrov Type-1 Universe.” Roy and Prakash [32, 33] investigated viscous fluid cosmological model of Petrov Type-I D and non degenerate Petrov Type I in which coefficient of shear and bulk viscosities are constants. Bali and Dave [34] constructed cosmological models in the presence and absence of bulk viscous fluid in which coefficient of bulk viscosity is constant.

Recently, considerable interest have been evinced in theories of more than four dimensions, in which the extra dimension are compacted to small size in the course of evolution of the Universe [35].The cosmological study in higher dimensional space time are necessitated, even made urgent, by the growing belief that the nature of space time in the Universe are higher than four. Chattergee[36] studied massive strings in higher dimensional space time .Krori *et al.*[37] discussed Bianchi type-1 higher dimensional cosmological and concluded that, physically, string will be like geometric string, and matter and string coexist throughout the Universe . They mentioned that cosmic string with some specific orientation do not occur in Bianchi Type-V cosmology. Rahaman *et al.*[38] discussed some sting cosmological model in higher dimensional spherically symmetric space time based on Lyra’s geometry. Venkateswarlu[39] constructed

higher dimensional sting cosmological models in scale covariant theory of gravitation. Mukhopadhyay et al. investigated $(n+2)$dimensional dark energy with variable Λ and G.Recently Mohonty *et al*.[40] and Mohanty and Mahanta[41] constructed various higher dimensional string cosmological models and studied their geometrical and physical behaviors.
Jamil and Debnath [42] has studied cosmological model of FRW Universe with variable G and Λ.So it is not unnatural to make an attempt for exploring new physical futures by venturing into dimensions higher than usual 4D.Tade et. al. [43] has extended the work of Jamil and Debnath [42] to five –dimensional space time. This is the motivation behind the present work where a $(n+2)$ dimensional study.

2. The metric and field equations

We consider the $(n+2)$ dimensional FRW metric in the form

$$ds^2 = dt^2 - R^2(t)\left[dr^2 + r^2 (dx_n)^2\right], \tag{1}$$

where $R(t)$ represents the scale factor and

$$\begin{aligned}(dx_n)^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + ...\\ + \sin^2\theta_1 \sin^2\theta_2 ....\sin^2\theta_{n-1} d\theta_n^2\end{aligned}. \tag{2}$$

The Einstein field equations are given by

$$R_i^{\ j} - \frac{1}{2} g_i^{\ j} R = -8\pi G T_i^{\ j} + \Lambda g_i^{\ j}, \tag{3}$$

where G is the gravitational constant and Λ is the cosmological constant, which are time dependent.

The energy momentum tensor $T_i^{\ j}$ for perfect fluid is written in the form

$$T_i^{\ j} = (\rho + p) u_i u^j - p g_i^{\ j} \quad , \tag{4}$$

where u^i is the four velocity vector of the distribution satisfying $u_i u^j = 1$, ρ is the energy density and p is the pressure.

We assume the equation of state

$$p = (\omega - 1)\rho \qquad , \quad 1 \le \omega \le 2. \tag{5}$$

The Einstein field equations (3), with the metric (1) and the energy momentum tensor (4), can be written as

$$-n\frac{\ddot{R}}{R}-\frac{n(n-1)}{2}\frac{\dot{R}^2}{R^2}+\Lambda=8\pi Gp \ , \tag{6}$$

$$\frac{n(n+2)}{2}\frac{\dot{R}^2}{R^2}-\Lambda=8\pi G\rho \ , \tag{7}$$

where over dot denotes the differentiation with respect to time t . An additional equation for time changes of G and Λ is obtain by the divergence of Einstein tensor, *i.e.* $\left(R_i^{\ j}-\frac{1}{2}g_i^{\ j}R\right)_{;}=0$ which leads to $\left(8\pi GT_i^{\ j}-\Lambda g_i^{\ j}\right)_{;}=0$, yielding

$$8\pi G\left(\dot{\rho}+(n+1)(p+\rho)\frac{\dot{R}}{R}\right)+8\pi\dot{G}\rho+\dot{\Lambda}=0\,. \tag{8}$$

The conservation of energy equation (8) yields

$$\dot{\rho}+(n+1)(p+\rho)\frac{\dot{R}}{R}=0 \ . \tag{9}$$

From equations (9) and (8),we get

$$8\pi\dot{G}\rho+\dot{\Lambda}=0\,. \tag{10}$$

Using equation (5) in equation (9), we obtain

$$\rho=\frac{A}{R^{(n+1)\omega}} \ , \tag{11}$$

where A is constant of integration. To determine the value of A we assume that $\omega=\omega_0$, $\rho=\rho_c$ due to the spatial flatness at $t=t_0$. It gives

$$A=\rho_c R_o^{\ (n+1)\omega_0} \ . \tag{12}$$

Using equation (12) in equation (11), we obtain

$$\rho=\rho_c\,\frac{R_0^{\ (n+1)\omega_0}}{R^{(n+1)\omega}} \ . \tag{13}$$

The field equation (6) and (7) imply that

$$8\pi Gp = nH^2\left[q - \frac{(n-1)}{2}\right] + \Lambda, \tag{14}$$

$$8\pi G\rho = \frac{n(n+1)}{2}H^2 - \Lambda \quad , \tag{15}$$

where $H = \frac{\dot{R}}{R}$ is the Hubble parameter and $q = -1 - \frac{\dot{H}}{H^2}$ is the deceleration parameter. Let us use the ersatz

$$\Lambda(t) = \varepsilon H^2 \quad , \tag{16}$$

where ε is constant.

Then from equations (5), (14), (15) and (16), we have

$$n\dot{H} + \omega\left(\frac{n(n+1)}{2} - \varepsilon\right)H^2 = 0. \tag{17}$$

Solving equation (17), we get

$$H(t) = \frac{n}{\left(\frac{n(n+1)}{2} - \varepsilon\right)\omega t - nB} \quad , \tag{18}$$

where B is constant of integration.

We determine the value of B as before, we get

$$B = \frac{\left(\frac{n(n+1)}{2} - \varepsilon\right)\omega_0 t_0 H_0 - n}{nH_0} \quad . \tag{19}$$

Then from equation (18) we have

$$H(t) = \frac{nH_0}{H_0\left(\frac{n(n+1)}{2} - \varepsilon\right)(\omega t - \omega_0 t_o) + n}. \tag{20}$$

From this equation, we have

$$t=\frac{n}{\left(\varepsilon-\frac{n(n+1)}{2}\right)\omega H_0}\left(1-\frac{H_0}{H}\right)+t_o\,\frac{\omega_0}{\omega}. \tag{21}$$

Using equation (7), we have

$$\rho=\frac{\left(\frac{n(n+1)}{2}-\varepsilon\right)H^2}{8\pi G}\ . \tag{22}$$

On integrating equation (20), we get

$$R(t)=C\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(\omega t-\omega_0 t_o)+n\right]^{\frac{n}{\omega\left(\frac{n(n+1)}{2}-\varepsilon\right)}}. \tag{23}$$

Now with the help of equation (23), equation (13) becomes

$$\rho(t)=\rho_c R_0^{(n+1)\omega_0}C^{-(n+1)\omega}\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(\omega t-\omega_0 t_o)+n\right]^{\frac{n(n+1)}{\left(\varepsilon-\frac{n(n+1)}{2}\right)}}. \tag{24}$$

Using equation (23) in equation (16), we obtain

$$\Lambda(t)=\frac{\varepsilon n^2 H_0^{\ 2}}{\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(\omega t-\omega_0 t_o)+n\right]^2}\ . \tag{25}$$

With the help of equation (24) and (23), equation (14) becomes

$$G(t)=\frac{n^2 H_0^{\ 2}\left(\frac{n(n+1)}{2}-\varepsilon\right)\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(\omega t-\omega_0 t_o)+n\right]^{\frac{2\varepsilon}{\frac{n(n+1)}{2}-\varepsilon}}}{8\pi\rho_c R_0^{(n+1)\omega_0}C^{-(n+1)\omega}}. \tag{26}$$

The deceleration parameter q is given by

$$q=-\frac{\dot{H}}{H^2}-1$$

$$=\frac{\left(\frac{n(n+1)}{2}-\varepsilon\right)\omega}{n}-1\ . \tag{27}$$

$$=\frac{(6-\varepsilon)\omega}{3}-1\,. \tag{26}$$

The sign of q indicates whether the model accelerates or not. The positive sign of q corresponds to decelerating models whereas the negative sign of $-1 \le q < 0$ indicates acceleration and $q = 0$ corresponds to expansion with constant velocity. Here for an accelerating Universe, $q \le -1$ are constraining by $\varepsilon < \frac{n(n+1)}{2}$.

The expansion scalar takes the form

$$\Theta = nH = \frac{n^2 H_0}{H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(\omega t-\omega_0 t_o)+n}\ . \tag{28}$$

The density parameter can be obtained as

$$\Omega = \frac{8\pi G\rho}{\frac{n(n+1)}{2}H^2} = 1-\frac{2\varepsilon}{n(n+1)}\ . \tag{29}$$

In 2003, V. Sahni *et al.* [42] introduce a pair of parameters $\{r,s\}$, called state finder parameters. The trajectories in the $\{r,s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour .The above state finder diagnostic pair has the following form:

$$r = -\frac{\dddot{R}}{RH^3} \qquad \text{and} \qquad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)}\ ,$$

where $H = \frac{\dot{R}}{R}$ and $q = -\frac{R\ddot{R}}{\dot{R}^2}$ are the Hubble parameter and the deceleration parameter respectively. The new feature of the state finder is that it involves the third derivative of the cosmological radius. These parameters are dimensionless and allow as characterizing the properties of dark energy. Trajectories in the $\{r,s\}$ plane corresponding to different cosmological modes, for example ΛCDM model diagrams corresponds to the fixed point $s=0,\ r=1$.

The state finder parameter $\{r,s\}$ takes the form

$$r=-\frac{\dddot{R}}{RH^3}=\frac{1}{n^2}\left[n+\omega\left(\varepsilon-\frac{n(n+1)}{2}\right)\right]\left[n+2\omega\left(\varepsilon-\frac{n(n+1)}{2}\right)\right], \tag{30}$$

$$s=\frac{r-1}{3\left(q-\frac{1}{2}\right)}=\frac{2\omega\left(\frac{n(n+1)}{2}-\varepsilon\right)}{3n}. \tag{31}$$

We observe that for $\varepsilon=\frac{n(n+1)}{2}$, $\{r,s\}=\{1,0\}$ representing a state cosmological constant.

I] Universe containing only dust:

The matter (baryonic and non-baryonic) satisfies the EoS parameter $\omega=1$. Thus cosmological parameter takes the form

$$R(t)=C\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(t-\omega_0t_o)+n\right]^{\frac{n}{\left(\frac{n(n+1)}{2}-\varepsilon\right)}}. \tag{32}$$

$$\rho(t)=\rho_cR_0^{\;(n+1)\omega_0}C^{-(n+1)}\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(t-\omega_0t_o)+n\right]^{\frac{n(n+1)}{\left(\varepsilon-\frac{n(n+1)}{2}\right)}}. \tag{33}$$

$$\Lambda(t)=\frac{\varepsilon n^2H_0^{\;2}}{\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(t-\omega_0t_o)+n\right]^2}. \tag{34}$$

$$G(t)=\frac{n^2H_0^{\;2}\left(\frac{n(n+1)}{2}-\varepsilon\right)\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(t-\omega_0t_o)+n\right]^{\frac{2\varepsilon}{\frac{n(n+1)}{2}-\varepsilon}}}{8\pi\rho_cR_0^{\;(n+1)\omega_0}C^{-(n+1)}}. \tag{35}$$

$$q=\frac{n(n-1)-2\varepsilon}{2n}. \tag{36}$$

$$\Theta=\frac{n^2H_0}{H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(t-\omega_0t_o)+n}. \tag{37}$$

$$r=\frac{1}{n^2}\left[n+\left(\varepsilon-\frac{n(n+1)}{2}\right)\right]\left[n+2\left(\varepsilon-\frac{n(n+1)}{2}\right)\right]. \tag{38}$$

$$s=\frac{2\left(\frac{n(n+1)}{2}-\varepsilon\right)}{3n} \quad . \tag{39}$$

II] Universe containing only radiation:

The radiation satisfies the EoS parameter $\omega=\frac{4}{3}$. Thus cosmological parameter takes the form

$$R(t)=C\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)\left(\frac{4}{3}t-\omega_0 t_o\right)+n\right]^{\frac{n}{\left(\frac{n(n+1)}{2}-\varepsilon\right)}} . \tag{40}$$

$$\rho(t)=\rho_c R_0^{\;(n+1)\omega_0} C^{-\frac{4(n+1)}{3}}\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)\left(\frac{4}{3}t-\omega_0 t_o\right)+n\right]^{\frac{n(n+1)}{\left(\varepsilon-\frac{n(n+1)}{2}\right)}} . \tag{41}$$

$$\Lambda(t)=\frac{\varepsilon n^2 H_0^{\;2}}{\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)\left(\frac{4}{3}t-\omega_0 t_o\right)+n\right]^2} \quad . \tag{42}$$

$$G(t)=\frac{n^2 H_0^{\;2}\left(\frac{n(n+1)}{2}-\varepsilon\right)\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)\left(\frac{4}{3}t-\omega_0 t_o\right)+n\right]^{\frac{2\varepsilon}{\frac{n(n+2)}{2}-\varepsilon}}}{8\pi\rho_c R_0^{\;(n+1)\omega_0} C^{-\frac{4(n+1)}{3}}} . \tag{43}$$

$$q=\frac{4}{3n}\left(\frac{n(n+1)}{2}-\varepsilon\right)-1 \; . \tag{44}$$

$$\Theta=\frac{n^2 H_0}{H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)\left(\frac{4}{3}t-\omega_0 t_o\right)+n} . \tag{45}$$

$$r=\frac{1}{n^2}\left[n+\frac{4}{3}\left(\varepsilon-\frac{n(n+1)}{2}\right)\right]\left[n+\frac{8}{3}\left(\varepsilon-\frac{n(n+1)}{2}\right)\right] . \tag{46}$$

$$s=\frac{8}{9n}\left(\frac{n(n+1)}{2}-\varepsilon\right) \quad . \tag{47}$$

III] Universe containing only stiff matter:

The stiff fluid satisfies the EoS parameter $\omega=2$. Thus cosmological parameter takes the form

$$R(t)=C\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(2t-\omega_0 t_o)+n\right]^{\frac{n}{2\left(\frac{n(n+1)}{2}-\varepsilon\right)}}. \tag{48}$$

$$\rho(t)=\rho_c R_0^{(n+1)\omega_0}C^{-2(n+1)}\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(2t-\omega_0 t_o)+n\right]^{\frac{n(n+1)}{\left(\varepsilon-\frac{n(n+1)}{2}\right)}}\ . \tag{49}$$

$$\Lambda(t)=\frac{\varepsilon n^2 H_0^{\ 2}}{\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(2t-\omega_0 t_o)+n\right]^2}. \tag{50}$$

$$G(t)=\frac{n^2 H_0^{\ 2}\left(\frac{n(n+1)}{2}-\varepsilon\right)\left[H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(2t-\omega_0 t_o)+n\right]^{\frac{2\varepsilon}{\frac{n(n+1)}{2}-\varepsilon}}}{8\pi\rho_c R_0^{(n+1)\omega_0}C^{-2(n+1)}}\ . \tag{51}$$

$$q=\frac{2}{n}\left(\frac{n(n+1)}{2}-\varepsilon\right)-1\ . \tag{52}$$

$$\Theta=\frac{n^2 H_0}{H_0\left(\frac{n(n+1)}{2}-\varepsilon\right)(2t-\omega_0 t_o)+n}\ . \tag{53}$$

$$r=\frac{1}{n^2}\left[n+2\left(\varepsilon-\frac{n(n+1)}{2}\right)\right]\left[n+4\left(\varepsilon-\frac{n(n+1)}{2}\right)\right]. \tag{54}$$

$$s=\frac{4}{3n}\left(\frac{n(n+1)}{2}-\varepsilon\right)\ . \tag{55}$$

3. Conclusion

In this paper we have described a $(n+2)$ dimensional flat FRW model with variable G and Λ. The cosmological parameters, state finder parameters and deceleration parameter have been obtained for dust, radiation and stiff matter. If we put *n=2* , the results obtained by Jamil et.al .[42] can be obtained .If we put *n=3* , the results investigated by Tade et. al. [43] can be obtained. We hope that these finding may throw some light to understand the universe structure in higher dimensions. This study will throw some light on the structure formation of the universe, which has astrophysical significance.

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