Article

Kaluza-Klein Universe with Wet Dark Fluid in General Relativity

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Abstract

Kaluza- Klein Universe filled with dark energy from a wet dark fluid has been considered. A new equation of state for dark energy component of the Universe has been used. It is modeled on the equation of state $p=\gamma(\rho-\rho_*)$ which can describe a liquid, for example water. The exact solution to the corresponding field equations are obtained in quadrature form. The solution for constant deceleration parameter have been studied in detail for both power-law and exponential forms. The cases γ=1 and γ=1 have also been analyzed.

Keywords: General Relativity, Wet dark fluid, cosmological parameters.

1. Introduction

The exact physical modeled situations at very early stages of the universe have provoked great interest of researchers and it is a challenging problem. It is generally assumed that during the phase transition, the symmetry of the universe is broken spontaneously. Higher-dimensional space-time is an active research is an attempt to unify gravity with other forces in nature .This idea is particularly important in the field of cosmology since one knows that our universe was much smaller in its early stage than that today. The most famous by Kaluza [1] and Klein [2] was the first theory, in which gravitation and electromagnetism could be unified in a single geometrical structure. This idea has been worked by a large number of people, who have found models for various phenomenons in particle physics and cosmology in five or more dimensions. The evolution of various cosmological models with fluid containing viscosities has already attracted the attention of many authors.

Evenly the cosmology in Kaluza – Klein theory was introduced to unify Maxwell's theory of electromagnetism and Einstein's gravity theory. Kaluza- Klein theory has been revived in the modern physics such as super gravity by Duff et. al. [3] and in superstring theories by Green et. al.[4]. Many papers published on Kaluza Klein theory deal with cosmology. Ponce [5], Coley

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[6] have studied Kaluza Klein cosmological model with different matters. Recently some people argued that Kaluza Klein theory can be effective in dark constituent of universe. Qiang et al [7] have studied string cloud and domain wall with quark matter in N dimensional Kaluza Klein model.

The nature of the dark energy component of the universe (Riess et al. [8]; Perlmutter et al. [9]; Sahni [10]) remains one of the deepest mysteries of cosmology. There is certainly no lack of candidates: cosmological constant, quintessence (Ratra and Peebles [11]; Caldwell et al. [12]; Barreiro et al. [13]), k-essence (Armendariz-Picon et al.[14-15]; Gonzalez-Diaz [16]), phantom energy (Caldwell [17]; Carroll et al. [18]; Elizalde et al. [19]). Modifications of the Friedman equation such as Cardassian expansion (Freese and Lewis [20]; Freese [21]; Gondolo and Freese [22]) as well as what might be derived from brane cosmology (Deffayet et al. [23]; Dvali et al. [24]; Dvali and Turner [25]) have also been used to explain the acceleration of the universe.
Type Ia supernovae observational data suggest that the universe is dominated by two dark components containing dark matter (DM) and dark energy (DE). Dark matter, a matter without pressure, is mainly used to explain galactic curves and large-scale structure formation, while dark energy, an exotic energy with negative pressure, is used to explain the present cosmic accelerating expansion.

To understand the origin of dark matter & dark energy and its nature is one of the greatest astronomical cosmological problems of the 21st century. The nature of both components remains unknown, and in the near future we can hope that the Large Hadron Collider (LHC) will be able to provide hints on the nature of DM & DE. The understanding of cosmology has been revolutionized by recently observed astronomical phenomena.

The interacting dark energy models includes Chaplygin gas [Kamenshchik et al.[26],Bento et al[27]], Holographic dark energy models[Cohen[28], Horava[29], Thomas[30],Li[31],Zhang & Wu[32],Zhang[33],Gao et al [34] and Braneworld models [Sahni et al.[35]]. Freedman & Turner [36], Wang & Tegmark [37] established firmly that universe is actually undergoing acceleration, with repulsive gravity of some strange energy-form i.e. dark energy at work. Dark energy, a “mysterious substance” pressure of which is “negative” and accounts for nearly 70% of total matter-energy budget of universe, but has no clear explanation .In this work, we use Wet Dark Fluid (WDF) as a model for dark energy. This model is in the spirit of the generalized Chaplygin gas (GCG) (Gorini et al.[38]), where a physically motivated equation of state is offered with properties relevant for the dark energy problem. Here the motivation stems from an empirical equation of state proposed by Tait [39] and Hayward [40] to treat water and aqueous solution.

The equation of state for WDF is very simple,

$$p_{WDF} = \gamma(\rho_{WDF} - \rho_*), \tag{1.1}$$

It is motivated by the fact that it is a good approximation for many fluids, including water, in which initial attraction of the molecules makes negative pressure possible .The parameters γ and ρ_* are taken to be positive and we restrict ourselves to $0 \le \gamma \le 1$. Note that if C_s denotes the adiabatic sound speed in WDF , then $\gamma = C_s^2$ [Babichev et al [41]].To find the WDF energy density , we use the energy conservation equation

$$\dot{\rho}_{WDF} + 3H(p_{WDF} + \rho_{WDF}) = 0, \tag{1.2}$$

Using the equation of state (1) and relation $3H = \frac{\dot{v}}{v}$ in above equation (2) , we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho_* + \frac{D}{v^{(1+\gamma)}}, \tag{1.3}$$

where D is the constant of integration and v is the volume expansion.

WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a piece those red shifts as a standard fluid with an equation of state

$$p = \gamma\rho .$$

We can show that if we take D>0, this fluid will not violate the strong energy condition $\rho + p \ge 0$. Thus we get

$$\begin{aligned} p_{WDF} + \rho_{WDF} &= (1+\gamma)\rho_{WDF} - \gamma\rho^* \\ &= (1+\gamma)\frac{D}{v^{(1+\gamma)}} \ge 0 . \end{aligned} \tag{1.4}$$

Setare [42-45], Setare & Saridakis [46-47] have developed the idea of holographic dark energy .Recently the original agegraphic dark energy (OADE) and new agegraphic dark energy (NADE) models were proposed by Cai [48] and Wei & Cai [49-50]. Karami et al[51] introduced a polytrophic gas model of DE as an alternative model to explain the accelerated expansion of the universe. Gupta & Pradhan [52] proposed new candidate known as cosmological nuclear-energy as a possible suspect (candidate) for the dark energy. Mukhopadhyay et al[53] studied higher dimensional dark energy investigation with variable Λ and G . However, so far, the nature of dark energy is still unclear. The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW model by Holman and Naidu [54].The Bianchi type-I universe filled with dark energy from a wet dark fluid has been studied by Singh and Chaubey [55].Very recently, Katore et. al. [57] have investigated plane symmetric wet dark universe in General Relativity.

We consider Kaluza Klein model with matter term with dark energy as a dark fluid satisfying equation of state (1.1). The solution has been obtained in the quadrature form. We also discuss the models for $\gamma = 0$, $\gamma = 1$ and the models for power law and exponential law in details.

2. Metric and field equations

Consider five dimensional Kaluza-Klein space time

$$ds^2 = dt^2 - R^2\left(dx^2 + dy^2 + dz^2\right) - A^2 d\psi^2, \tag{2.1}$$

where R and A are functions of t only .

The Einstein field equations for metric (2.1) in the form

$$R_{ij} - \tfrac{1}{2} g_{ij} R = -k T_{ij}, \tag{2.2}$$

where k is gravitational constant.

The energy-momentum tensor of the source is given by

$$T_i^j = \left(\rho_{WDF} + p_{WDF}\right) u_i u^j - p_{WDF} \delta_i^j \ , \tag{2.3}$$

Where u^i is the flow vector satisfying

$$g_{ij} u^i u^j = 1. \tag{2.4}$$

Here p_{WDF} and ρ_{WDF} is pressure and energy density of WDF and related by equation of state

$$p_{WDF} = \gamma\left(\rho_{WDF} - \rho_*\right) \ . \tag{2.5}$$

In a co-moving system of coordinates, from equation (2.3) we find

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p_{WDF} \quad \text{and} \quad T_5^5 = \rho_{WDF} . \tag{2.6}$$

Now using equation (2.6) in equation (2.2) we obtain

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = -k p_{WDF} \qquad , \tag{2.7}$$

$$\frac{3\ddot{R}}{R}+\frac{3\dot{R}^2}{R^2}=-kp_{WDF} \quad , \tag{2.8}$$

$$\frac{3\dot{R}^2}{R^2}+\frac{3\dot{R}\dot{A}}{RA}=-k\rho_{WDF} \ . \tag{2.9}$$

Here k is the gravitational constant and overhead dot denotes differentiation with respect to t .

Subtracting equation (2.7) from equation (2.8) , we get

$$\frac{d}{dt}\left[\frac{\dot{R}}{R}-\frac{\dot{A}}{A}\right]+\left[\frac{\dot{R}}{R}-\frac{\dot{A}}{A}\right]\left[\frac{3\dot{R}}{R}+\frac{\dot{A}}{A}\right]=0 \ . \tag{2.10}$$

Let V be a function of t defined by

$$V=R^3A. \tag{2.11}$$

Then

$$\frac{\dot{V}}{V}=\frac{3\dot{R}}{R}+\frac{\dot{A}}{A} \quad , \tag{2.12}$$

$$\frac{\ddot{V}}{V}=\frac{3\ddot{R}}{R}+\frac{6\dot{R}^2}{R^2}+\frac{6\dot{R}\dot{A}}{RA}+\frac{\ddot{A}}{A}. \tag{2.13}$$

From equations (2.10) and (2.12) which leads to

$$\frac{R}{A}=d_1\exp\left\{x_1\int\frac{1}{V}dt\right\}, \tag{2.14}$$

where d_1 and x_1 are constants .

Now,

$$V=R^3A\Rightarrow A=\frac{V}{R^3} \quad , \tag{2.15}$$

$$\Rightarrow R=d_1^{\frac{1}{4}}V^{\frac{1}{4}}\exp\left\{\frac{x_1}{4}\int\frac{1}{V}dt\right\} \ . \tag{2.16}$$

Equations (2.15), (2.16) lead to

$$A = \frac{1}{d_1^{3/4}} V^{1/4} \exp\left\{ \frac{-3x_1}{4} \int \frac{1}{V} dt \right\}. \tag{2.16a}$$

Therefore, we write R and A in the explicit form as

$$R = D_1 V^{\frac{1}{4}} \exp\left\{ X_1 \int \frac{1}{V} dt \right\}, \tag{2.17}$$

$$A = D_2 V^{\frac{1}{4}} \exp\left\{ X_2 \int \frac{1}{V} dt \right\}, \tag{2.18}$$

where $D_i (i = 1,2)$ and $X_i (i = 1,2)$ satisfy the relation $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

Using equations (2.7), (2.8) and (2.9) , we get

$$\frac{3\ddot{R}}{R} + \frac{6\dot{R}^2}{R^2} + \frac{6\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = \frac{4k}{3}(\rho_{WDF} - p_{WDF})$$

In view of equation (2.13), we get

$$\frac{\ddot{V}}{V} == \frac{4k}{3}(\rho_{WDF} - p_{WDF}). \tag{2.19}$$

The conservational law for the energy-momentum tensor gives

$$\dot{\rho}_{WDF} = -\frac{\dot{V}}{V}(\rho_{WDF} + p_{WDF}) . \tag{2.20}$$

From equations (2.19) and (2.20), we have

$$\dot{V} = \sqrt{2\left[\frac{4kV^2}{3} \rho_{WDF} + C_1 \right]} \quad , \tag{2.21}$$

where C_1 being an integration constant.

Rewriting equation (2.20) in the form

$$\frac{\dot{\rho}}{\rho_{WDF} + p_{WDF}} = -\frac{\dot{V}}{V}, \tag{2.22}$$

and taking into account that the pressure and the energy density obeying an equation of state of type $p_{WDF} = f(\rho_{WDF})$, we conclude that ρ_{WDF} and p_{WDF} , hence the right-hand side of equation (2.19) is a function of V only .

$$\ddot{V} = \frac{4k}{3}(\rho_{WDF} - p_{WDF})V = F(V) . \tag{2.23}$$

From the mechanical point of view , equation (2.23) can be interpreted as equation of motion of single particle with unit mass under force $F(V)$ .Then

$$\dot{V} = \sqrt{2(\in - U(V))} . \tag{2.24}$$

Here $\in$ can be viewed as energy and $U(V)$ as the potential of the force F .

Comparing equations (2.21) and (2.24) , we find $\in = C_1$ and

$$U(V) = \frac{4kV^2}{3}\rho_{WDF} . \tag{2.25}$$

Finally, we write the solution to equation (2.21) in quadrature form

$$\int \frac{dV}{\sqrt{2\left(C_1 + \frac{4k}{3}\rho_{WDF}V^2\right)}} = t + t_0 , \tag{2.26}$$

where the integration constant t_0 can be zero , since it only gives a shift in time.

From equations (1.3) and (2.26) we obtain

$$\int \frac{dV}{\sqrt{\frac{8k\gamma\rho_*}{3(1+\gamma)}V^2 + \frac{8kC}{3}V^{1-\gamma} + 2C_1}} = t + t_0 . \tag{2.27}$$

3. Some particular cases

Case I : $\gamma = 0$ (Dust)

Equation (2.27) reduces to

$$\int \frac{dV}{\sqrt{\frac{8kC}{3}V + 2C_1}} = t \quad , \tag{3.1}$$

which gives

$$V = \frac{2kC}{3}t^2 - \frac{3C_1}{4kC} . \tag{3.2}$$

Case I (a): When $t > \frac{3\sqrt{C_1}}{2\sqrt{2}kC}$

From equations (2.17),(2.18) and (3.2) , we get

$$R = D_1\left[\frac{2kC}{3}t^2 - \frac{3C_1}{4kC}\right]^{1/4} \exp\left\{-X_1 \frac{\sqrt{2}}{\sqrt{C_1}} \coth^{-1} \frac{2\sqrt{2}kC}{3\sqrt{C_1}} t\right\}, \tag{3.3}$$

$$A = D_2\left[\frac{2kC}{3}t^2 - \frac{3C_1}{4kC}\right]^{1/4} \exp\left\{-X_2 \frac{\sqrt{2}}{\sqrt{C_1}} \coth^{-1} \frac{2\sqrt{2}kC}{3\sqrt{C_1}} t\right\}, \tag{3.4}$$

where $D_i (i = 1,2)$ and $X_i (i = 1,2)$ satisfy the relation $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

From equations (1.3) and (3.2), we get

$$\rho_{WDF} = C\left[\frac{2kC}{3}t^2 - \frac{3C_1}{4kC}\right]^{-1} . \tag{3.5}$$

From equations (1.1) and (3.5), we get

$$p_{WDF} = 0 . \tag{3.6}$$

The physical quantities of observational interest in cosmology are the expansion scalar θ , the mean anisotropy parameter A , the shear scalar σ^2 and the deceleration parameter q . They are defined as

$$\theta = 4H\ , \tag{3.7}$$

$$A = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 , \tag{3.8}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 3H^2\right) = \frac{4}{2}AH^2 , \tag{3.9}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{3.10}$$

Using equations (3.7)-(3.10), we can express the physical quantities as

$$\theta = \frac{16k^2C^2t}{8k^2C^2t^2 - 9C_1} , \tag{3.11}$$

$$A = \frac{9X^2}{4k^2C^2} \cdot \frac{1}{t^2} , \tag{3.12}$$

$$\sigma^2 = \frac{72X^2k^2C^2}{\left(8k^2C^2t^2 - 9C_1\right)^2} , \tag{3.13}$$

$$q = 1 + \frac{9C_1}{8k^2C^2t^2}\ . \tag{3.14}$$

where $X^2 = 3X_1^2 + X_2^2$ is a constant .

For large t , the model tends to be isotropic.

Case I (b) : When $t < \dfrac{3\sqrt{C_1}}{2\sqrt{2}KC}$

From equations (2.17),(2.18) and (3.2) , we get

$$R = D_1\left[\frac{2kC}{3}t^2 - \frac{3C_1}{4KC}\right]^{1/4} \exp\left\{-X_1\frac{\sqrt{2}}{\sqrt{C_1}}\tanh^{-1}\left(\frac{2\sqrt{2}kC}{3\sqrt{C_1}}t\right)\right\}, \tag{3.15}$$

$$A = D_2\left[\frac{2kC}{3}t^2 - \frac{3C_1}{4kC}\right]^{1/4} \exp\left\{-X_2\frac{\sqrt{2}}{\sqrt{C_1}}\tanh^{-1}\left(\frac{2\sqrt{2}kC}{3\sqrt{C_1}}t\right)\right\}. \tag{3.16}$$

where $D_i\,(i=1,2)$ and $X_i\,(i=1,2)$ satisfy the relation $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

From equations (1.3) and (3.2) , we get

$$\rho_{WDF} = C\left[\frac{2kC}{3}t^2 - \frac{3C_1}{4kC}\right]^{-1}. \tag{3.17}$$

From equations (1.1) and (3.17), we get

$$p_{WDF} = 0. \tag{3.18}$$

Using equations (3.7)-(3.10), we can express the physical quantities as

$$\theta = \frac{16k^2C^2t}{8k^2C^2t^2 - 9C_1}, \tag{3.19}$$

$$A = \frac{9X^2}{4k^2C^2}\cdot\frac{1}{t^2}, \tag{3.20}$$

$$\sigma^2 = \frac{72X^2k^2C^2}{\left(8k^2C^2t^2 - 9C_1\right)^2}, \tag{3.21}$$

$$q = 1 + \frac{9C_1}{8k^2C^2t^2}. \tag{3.22}$$

where $X^2 = 3X_1^2 + X_2^2$ is a constant .

For large t , the model tends to be isotropic.

Case II : $\gamma = 1$ (Zel'dovich fluid)

Equation (2.27) reduces to

$$\int \frac{dV}{\sqrt{\frac{4k\rho_*}{3}V^2 + \left(\frac{8kC}{3} + 2C_1\right)}} = t \ , \tag{3.23}$$

which gives

$$V = \sqrt{\frac{4kC + 3C_1}{2k\rho_*}} \sinh\left(\sqrt{\frac{4k\rho_*}{3}}t\right). \tag{3.24}$$

From equations (2.17),(2.18) and (3.24) , we get

$$R = D_1\left[\frac{4kC + 3C_1}{2k\rho_*}\right]^{1/8}\left(\sinh\sqrt{\frac{4k\rho_*}{3}}t\right)^{1/4} \exp\left\{X_1\sqrt{\frac{3}{8kC + 6C_1}} \log\tanh\left(\sqrt{\frac{4k\rho_*}{3}}t\right)\right\}, \tag{3.25}$$

$$A = D_2\left[\frac{4kC + 3C_1}{2k\rho_0}\right]^{1/8}\left(\sinh\sqrt{\frac{8\rho_*}{3}}t\right)^{1/4} \exp\left\{X_2\sqrt{\frac{3}{8kC + 6C_1}} \log\tanh\left(\sqrt{\frac{4k\rho_0}{3}}t\right)\right\}, \tag{3.26}$$

where $D_i(i = 1,2)$ and $X_i(i = 1,2)$ satisfy the relation $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

From equations (1.3) and (3.24) , we get

$$\rho_{WDF} = \frac{\rho_*}{2} + \frac{kC\rho_*}{4kC + 3C_1} \operatorname{cosech}^2 \sqrt{\frac{4kC\rho_*}{3}}t \, . \tag{3.27}$$

From equations (1.1) and (3.27), we get

$$p_{WDF} = \frac{kC\rho_*}{4kC + 3C_1} \operatorname{cosech}^2 \sqrt{\frac{4kC\rho_*}{3}}t - \frac{\rho_*}{2} \, . \tag{3.28}$$

Using equations (3.7)-(3.10), we can express the physical quantities as

$$\theta = 2\sqrt{k\rho_*} \coth\left(\sqrt{\frac{4k\rho_*}{3}}t\right), \tag{3.29}$$

$$A = \frac{X^2}{8kC + 6C_1} \operatorname{sech}^2\left(\sqrt{\frac{4k\rho_*}{3}}t\right), \tag{3.30}$$

$$\sigma^2 = \frac{X^2}{4(4kC+3C)_1}\operatorname{cosech}^2\left(\sqrt{\frac{4k\rho_*}{3}}t\right), \tag{3.31}$$

$$q = 4\operatorname{sech}^2\sqrt{\frac{4k\rho_*}{3}}t - 1. \tag{3.32}$$

where $X^2 = 3X_1^2 + X_2^2$ is a constant .

4. Models with constant deceleration parameters

Case I : Power law

Here we take $V = at^b$, (4.1)

where a, b are constants.

From equations (2.17),(2.18) and (4.1) , we get

$$R = D_1 a^{1/4} t^{4b} \exp\left\{\frac{X_1}{a(1-b)}t^{1-b}\right\}, \tag{4.2}$$

$$A = D_2 a^{1/4} t^{4b} \exp\left\{\frac{X_2}{a(1-b)}t^{1-b}\right\}, \tag{4.3}$$

where $D_i\,(i=1,2)$ and $X_i\,(i=1,2)$ satisfy the relation $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

From equations (1.3) and (4.1) , we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho_* + \frac{C}{a^{1+\gamma}}\frac{1}{t^{(1+\gamma)b}} \ , \tag{4.4}$$

From equations (1.1) and (4.4), we get

$$p_{WDF} = \gamma\left[\frac{\gamma-1}{\gamma+1}\rho_* + \frac{C}{a^{1+\gamma}}\frac{1}{t^{(1+\gamma)b}}\right]. \tag{4.5}$$

Using equations (3.7)-(3.10), we can express the physical quantities as

$$\theta = \frac{16b}{t}, \tag{4.6}$$

$$A = \frac{1}{64}\frac{X^2}{a^2 b^2}\frac{1}{t^{2(b-1)}}, \tag{4.7}$$

$$\sigma^2 = \frac{1}{2}\frac{X^2}{a^2}\frac{1}{t^{2b}}, \tag{4.8}$$

$$q = \frac{1}{4b} - 1. \tag{4.9}$$

where $X^2 = 3X_1^2 + X_2^2$ is a constant .

For large t , the model tends to be isotropic when b>1.

Case II : Exponential law

Here we take $V = \alpha e^{\beta t}$, (4.10)

where α, β are constants .

From equations (2.17),(2.18) and (4.10) , we get

$$R = D_1 \alpha^{1/4} e^{\beta t/4} \exp\left\{-\frac{X_1}{\alpha\beta} e^{-\beta t}\right\}, \tag{4.11}$$

$$A = D_2 \alpha^{1/4} e^{\beta t/4} \exp\left\{-\frac{X_2}{\alpha\beta} e^{-\beta t}\right\}, \tag{4.12}$$

where $D_i (i = 1,2)$ and $X_i (i = 1,2)$ satisfy the relation $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

From equations (1.3) and (4.10) , we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho_* - \frac{C}{\alpha\beta} e^{-(1+\gamma)\beta t}, \tag{4.13}$$

From equations (1.1) and (4.13), we have

$$p_{WDF} = \gamma\left[\frac{\gamma-1}{\gamma+1}\rho_* - \frac{C}{\alpha\beta}e^{-(1+\gamma)\beta t}\right]. \tag{4.14}$$

Using equations (3.7)-(3.10), we can express the physical quantities as

$$\theta = \beta, \tag{4.15}$$

$$\mathrm{A} = \frac{4X^2}{\alpha^2\beta^2}e^{-2\beta t}, \tag{4.16}$$

$$\sigma^2 = \frac{1}{2}\frac{X^2}{\alpha^2}e^{-2\beta t}, \tag{4.17}$$

$$q = -1. \tag{4.18}$$

5. Conclusion

The Kaluza-Klein Universe has been considered for a new equation of state for the dark energy component of the Universe (known as dark wet fluid). The solution has been obtained in quadrature form. The models with constant deceleration parameter have been discussed in detail. The behaviors of the models for large time have been analyzed. It is interesting to note that our investigations resembles to the result obtained by T. Singh, R. Chaubey [55], R.Chaubey [56].

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