Article

Overall View About TGD from Particle Physics Perspective

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Abstract

Topological Geometrodynamics is able to make rather precise and often testable predictions. In this and two other articles I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

In the first article I will concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I will represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.
- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the imbedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces and one can understand the generalization of Equivalence Principle in terms of coset representations for the two super Virasoro algebras associated with lightlike boundaries of so called causal diamonds defined as intersections of future and past directed lightcones (CDs) and with light-like 3-surfaces. Super-conformal symmetries imply generalization of the space-time supersymmetry in TGD framework consistent with the supersymmetries of minimal supersymmetric variant of the standard model. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization of the Yangian symmetry identified originally as symmetry of $\mathcal{N}=4$ SYMs is postulated as basic symmetry of quantum TGD.
- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electro-weak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realized quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.
- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical constructing involving the concept of Dirac operator. As a matter fact, the construction of TGD involves three Dirac operators. The Kähler Dirac equation holds true in the interior of space-time surface and its solutions havea natural interpretation in terms of description of matter, in particular condensed matter. Chern-Simons Dirac action is associated with the light-like 3-surfaces and space-like 3-surfaces at ends of space-time surface at light-like boundaries of CD. One can assign to it a generalized eigenvalue equation and the matrix valued eigenvalues correspond to the the action of Dirac operator on momentum eigenstates. Momenta are however not usual momenta but pseudo-momenta very much analogous to region momenta of twistor approach. The third Dirac operator is associated with super Virasoro generators and super Virasoro conditions define Dirac equation in WCW. These conditions characterize zero energy states as modes of WCW spinor fields and code for the generalization of S-matrix to a collection of what I call M-matrices defining the rows of unitary U-matrix defining unitary process.

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- Twistor approach has inspired several ideas in quantum TGD during the last years and it seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness. Twistor approach also inspires the conjecture that quantum TGD allows also formulation in terms of 6-dimensional holomorphic surfaces in the product $CP_3 \times CP_3$ of two twistor spaces and general arguments allow to identify the partial different equations satisfied by these surfaces.

Contents

1 Introduction

Topological Geometrodynamics is able to make rather precise and often testable predictions. In this and two other articles I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

During these 32 years TGD has become quite an extensive theory involving also applications to quantum biology and quantum consciousness theory. Therefore it is difficult to decide in which order to proceed. Should one represent first the purely mathematical theory as done in the articles in Prespacetime Journal [45, 46, 49, 50, 47, 44, 48, 51]? Or should one start from the TGD inspired heuristic view about space-time and particle physics and represent the vision about construction of quantum TGD briefly after that? In this and other two articlesI have chosen the latter approach since the emphasis is on the applications on particle physics.

Second problem is to decide about how much material one should cover. If the representation is too brief no-one understands and if it is too detailed no-one bothers to read. I do not know whether the outcome was a success or whether there is any way to success but in any case I have been sweating a lot in trying to decide what would be the optimum dose of details.

In the first article I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.

- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the imbedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces and one can understand the generalization of Equivalence Principle in terms of coset representations for the two super Virasoro algebras associated with lightlike boundaries of so called causal diamonds defined as intersections of future and past directed lightcones (CDs) and with light-like 3-surfaces. Super-conformal symmetries imply generalization of the space-time supersymmetry in TGD framework consistent with the supersymmetries of minimal supersymmetric variant of the standard model. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization of the Yangian symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.

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The discussion of this articleis rather sketchy and the reader interesting in details can consult the books about TGD [40, 29, 23, 19, 30, 34, 38].

2 Some aspects of quantum TGD

In the following I summarize very briefly those basic notions of TGD which are especially relevant for the applications to particle physics. The representation will be practically formula free. The article series published in Prespacetime Journal [45, 46, 49, 50, 47, 44, 48, 51] describes the mathematical theory behind TGD. The seven books about TGD [40, 29, 23, 30, 33] provide a detailed summary about the recent state of TGD.

2.1 New space-time concept

The physical motivation for TGD was what I have christened the energy problem of General Relativity. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The way out is based on assumption that space-times are imbeddable as 4-surfaces to certain 8-dimensional space by replacing the points of 4-D empty Minkowski space with 4-D very small internal space. This space -call it S- is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [51] , is the unique choice.

The replacement of the abstract manifold geometry of general relativity with the geometry of surfaces brings the shape of surface as seen from the perspective of 8-D space-time and this means additional degrees of freedom giving excellent hopes of realizng the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any general coordinate invariant variational principle led soon to the realization that the space-time in this framework is much more richer than in general relativity.

1. Space-time decomposes into space-time sheets with finite size: this lead to the identification of physical objects that we perceive around us as space-time sheets. For instance, the outer boundary of the table is where that particular space-time sheet ends. Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

2. Elementary particles are identified as topological inhomogenities glued to these space-time sheets. In this conceptual framework material structures and shapes are not due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in general relativity.

3. Also the view about classical fields changes. One can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [24]. One can speak about field body or magnetic body of the system.

Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [92] and the color magnetic body of u quark whose size is somethat larger than the Bohr radius could explain the anomaly [18].

2.2 Zero energy ontology

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology physical states decompose to pairs of positive and negative energy states such that all net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events. By quantum classical correspondences zero energy states must have space-time and imbedding space correlates.

1. Positive and negative energy parts reside at future and past light-like boundaries of causal diamond (CD) defined as intersection of future and past directed light-cones and visualizable as double cone. The analog of CD in cosmology is big bang followed by big crunch. CDs for a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs can also intersect.

2. p-Adic length scale hypothesis [20] motivates the hypothesis that the temporal distances between the tips of the intersecting light-cones come as octaves $T = 2^nT_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [2]. This means a direct coupling between microscopic and macroscopic scales.

Zero energy ontology conforms with the crossing symmetry of quantum field theories meaning that the final states of the quantum scattering event are effectively negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem which results in any theory assuming symmetries giving rise to to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in general relativity based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must be speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

For thermodynamical states this is indeed the case and this leads to the idea that quantum theory in ZEO can be regarded as a "complex square root" of thermodynamics obtained as a product of positive diagonal square root of density matrix and unitary S-matrix. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and replaces S-matrix as the fundamental observable. In standard quantum measurement theory this time-like entanglement would be reduced in quantum measurement and regenerated in the next quantum jump if one accepts Negentropy Maximization Principle (NMP) [17] as the fundamental variational principle. Various M-matrices define the rows of the unitary U matrix characterizing the unitary process part of quantum jump. From the point of view of consciousness theory the importance of ZEO is that conservation laws in principle pose no restrictions for the new realities created in quantum jumps: free will is maximal.

2.3 The hierarchy of Planck constants

The motivations for the hierarchy of Planck constants come from both astrophysics and biology [28, 8]. In astrophysics the observation of Nottale [101] that planetary orbits in solar system seem to correspond to Bohr orbits with a gigantic gravitational Planck constant motivated the proposal that Planck constant might not be constant after all [31, 25].

This led to the introduction of the quantization of Planck constant as an independent postulate. It has however turned that quantized Planck constant in effective sense could emerge from the basic structure of TGD alone. Canonical momentum densities and time derivatives of the imbedding space coordinates are the field theory analogs of momenta and velocities in classical mechanics. The extreme non-linearity and vacuum degeneracy of Kähler action imply that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many: for vacuum extremals themselves 1-to-infinite.

A convenient technical manner to treat the situation is to replace imbedding space with its n-fold singular covering. Canonical momentum densities to which conserved quantities are proportional would be same at the sheets corresponding to different values of the time derivatives. At each sheet of the covering Planck constant is effectively $\hbar = n\hbar_0$. This splitting to multisheeted structure can be seen as a phase transition reducing the densities of various charges by factor $1/n$ and making it possible to have perturbative phase at each sheet (gauge coupling strengths are proportional to $1/\hbar$ and scaled down by $1/n$). The connection with fractional quantum Hall effect [95] is almost obvious. At the more detailed level one finds that the spectrum of Planck constants would be given by $\hbar = n_a n_b \hbar_0$ [10].

This has many profound implications, which are wellcome from the point of view of quantum biology but the implications would be profound also from particle physics perspective and one could say that living matter represents zoome up version of quantum world at elementary particle length scales.

1. Quantum coherence and quantum superposition become possible in arbitrary long length scales. One can speak about zoomed up variants of elementary particles and zoomed up sizes make it possible to satisfy the overlap condition for quantum length parameters used as a criterion for the presence of macroscopic quantum phases. In the case of quantum gravitation the length scale involved are astrophysical. This would conform with Penrose's intuition that quantum gravity is fundamental for the understanding of consciousness and also with the idea that consciousness cannot be localized to brain.

2. Photons with given frequency can in principle have arbitrarily high energies by $E = hf$ formula, and this would explain the strange anomalies associated with the interaction of ELF em fields with living matter [104]. Quite generally the cyclotron frequencies which correspond to energies much below the thermal energy for ordinary value of Planck constant could correspond to energies above thermal threshold.

3. The value of Planck constant is a natural characterizer of the evolutionary level and biological evolution would mean a gradual increase of the largest Planck constant in the hierarchy characterizing given quantum system. Evolutionary leaps would have interpretation as phase transitions increasing the maximal value of Planck constant for evolving species. The space-time correlate would be the increase of both the number and the size of the sheets of the covering associated with the system so that its complexity would increase.

4. The phase transitions changing Planck constant change also the length of the magnetic flux tubes. The natural conjecture is that biomolecules form a kind of Indra's net connected by the flux tubes and $\hbar$ changing phase transitions are at the core of the quantum bio-dynamics. The contraction of the magnetic flux tube connecting distant biomolecules would force them near to each other making possible for the bio-catalysis to proceed. This mechanism could be central for DNA replication and other basic biological processes. Magnetic Indra's net could be also responsible for the coherence of gel phase and the phase transitions affecting flux tube lengths could induce the contractions and expansions of the intracellular gel phase. The reconnection of flux tubes would allow the restructing of the signal pathways between biomolecules and other subsystems and would be also involved with ADP-ATP transformation inducing a transfer of negentropic entanglement [13]. The braiding of the magnetic flux tubes could make possible topological quantum computation like processes and analog of computer memory realized in terms of braiding patterns [9].

5. p-Adic length scale hypothesis and hierarchy of Planck constants suggest entire hierarchy of zoomed up copies of standard model physics with range of weak interactions and color forces scaling like $\hbar$. This is not conflict with the known physics for the simple reason that we know very little about dark matter (partly because we might be making misleading assumptions about its nature). One implication is that it might be someday to study zoomed up variants particle physics at low energies using dark matter.

   Dark matter would make possible the large parity breaking effects manifested as chiral selection of bio-molecules [93]. What is required is that classical Z^0 and W fields responsible for parity breaking effects are present in cellular length scale. If the value of Planck constant is so large that weak scale is some biological length scale, weak fields are effectively massless below this scale and large parity breaking effects become possible.

   For the solutions of field equations which are almost vacuum extremals Z^0 field is non-vanishing and proportional to electromagnetic field. The hypothesis that cell membrane corresponds to a space-time sheet near a vacuum extremal (this corresponds to criticality very natural if the cell membrane is to serve as an ideal sensory receptor) leads to a rather successful model for cell membrane as sensory receptor with lipids representing the pixels of sensory qualia chart. The surprising prediction is that bio-photons [103] and bundles of EEG photons can be identified as different decay products of dark photons with energies of visible photons. Also the peak frequencies of sensitivity for photoreceptors are predicted correctly [28].

2.4 p-Adic physics and number theoretic universality

p-Adic physics [19, 37] has become gradually a key piece of TGD inspired biophysics. Basic quantitative predictions relate to p-adic length scale hypothesis and to the notion of number theoretic entropy. Basic ontological ideas are that life resides in the intersection of real and p-adic worlds and that p-adic space-time sheets serve as correlates for cognition and intentionality. Number theoretical universality requires the fusion of real physics and various p-adic physics to single coherent whole. On implication is the generalization of the notion of number obtained by fusing real and p-adic numbers to a larger structure.

2.4.1 p-Adic number fields

p-Adic number fields Q_p [69] -one for each prime p- are analogous to reals in the sense that one can speak about p-adic continuum and that also p-adic numbers are obtained as completions of the field of rational numbers. One can say that rational numbers belong to the intersection of real and p-adic numbers. p-Adic number field Q_p allows also an infinite number of its algebraic extensions. Also transcendental extensions are possible. For reals the only extension is complex numbers.

p-Adic topology defining the notions of nearness and continuity differs dramatically from the real topology. An integer which is infinite as a real number can be completely well defined and finite as a p-adic number. In particular, powers p^n of prime p have p-adic norm (magnitude) equal to p^{-n} in Q_p so that at the limit of very large n real magnitude becomes infinite and p-adic magnitude vanishes.

p-Adic topology is rough since p-adic distance $d(x, y) = d(x - y)$ depends on the lowest pinary digit of $x - y$ only and is analogous to the distance between real points when approximated by taking into account only the lowest digit in the decimal expansion of $x - y$. A possible interpretation is in terms of a finite measurement resolution and resolution of sensory perception. p-Adic topology looks somewhat strange. For instance, p-adic spherical surface is not infinitely thin but has a finite thickness and p-adic surfaces possess no boundary in the topological sense. Ultrametricity is the technical term characterizing the basic properties of p-adic topology and is coded by the inequality $d(x-y) \leq Min\{d(x), d(y)\}$. p-Adic topology brings in mind the decomposition of perceptive field to objects.

2.4.2 Motivations for p-adic number fields

The physical motivations for p-adic physics came from the observation that p-adic thermodynamics - not for energy but infinitesimal scaling generator of so called super-conformal algebra [64] acting as symmetries of quantum TGD [29] - predicts elementary particle mass scales and also masses correctly under very general assumptions [19]. The calculations are discussed in more detail in the second article of the series. In particular, the ratio of proton mass to Planck mass, the basic mystery number of physics, is predicted correctly. The basic assumption is that the preferred primes characterizing the p-adic number fields involved are near powers of two: $p \simeq 2^k$, k positive integer. Those nearest to power of two correspond to Mersenne primes $M_n = 2^n - 1$. One can also consider complex primes known as Gaussian primes, in particular Gaussian Mersennes $M_{G,n} = (1 + i)^n - 1$.

It turns out that Mersennes and Gaussian Mersennes are in a preferred position physically in TGD based world order. What is especially interesting that the length scale range 10 nm-2.5 μ assignable to DNA contains as many as 4 Gaussian Mersennes corresponding to $n = 151, 157, 163, 167$ [28]. This number theoretical miracle supports the view that p-adic physics is especially important for the understanding of living matter.

The philosophical for p-adic numbers fields come from the question about the possible physical correlates of cognition and intention [22]. Cognition forms representations of the external world which have finite cognitive resolution and the decomposition of the perceptive field to objects is an essential element of these representations. Therefore p-adic space-time sheets could be seen as candidates of thought bubbles, the mind stuff of Descartes. One can also consider p-adic space-time sheets as correlates of intentions. The quantum jump in which p-adic space-time sheet is replaced with a real one could serve as a quantum correlate of intentional action. This process is forbidden by conservation laws in standard ontology: one cannot even compare real and p-adic variants of the conserved quantities like energy in the general case. In zero energy ontology the net values of conserved quantities for zero energy states vanish so that conservation laws allow these transitions.

Rational numbers belong to the intersection of real and p-adic continua. An obvious generalization of this statement applies to real manifolds and their p-adic variants. When extensions of p-adic numbers are allowed, also some algebraic numbers can belong to the intersection of p-adic and real worlds. The notion of intersection of real and p-adic worlds has actually two meanings.

1. The intersection could consist of the rational and possibly some algebraic points in the intersection of real and p-adic partonic 2-surfaces at the ends of CD. This set is in general discrete. The interpretation could be as discrete cognitive representations.

2. The intersection could also have a more abstract meaning. For instance, the surfaces defined by rational functions with rational coefficients have a well-defined meaning in both real and p-adic context and could be interpreted as belonging to this intersection. There is strong temptation to assume that intentions are transformed to actions only in this intersection. One could say that life resides in the intersection of real and p-adic worlds in this abstract sense.

Additional support for the idea comes from the observation that Shannon entropy $S = -\sum p_n log(p_n)$ allows a p-adic generalization if the probabilities are rational numbers by replacing $log(p_n)$ with $-log(|p_n|_p)$, where $|x|_p$ is p-adic norm. Also algebraic numbers in some extension of p-adic numbers can be allowed. The unexpected property of the number theoretic Shannon entropy is that it can be negative and its unique minimum value as a function of the p-adic prime p it is always negative. Entropy transforms to information!

In the case of number theoretic entanglement entropy there is a natural interpretation for this. Number theoretic entanglement entropy would measure the information carried by the entanglement whereas ordinary entanglement entropy would characterize the uncertainty about the state of either entangled system. For instance, for p maximally entangled states both ordinary entanglement entropy and number theoretic entanglement negentropy are maximal with respect to R_p norm. Entanglement carries maximal information. The information would be about the relationship between the systems, a rule. Schrödinger cat would be dead enough to know that it is better to not open the bottle completely.

Negentropy Maximization Principle [17] coding the basic rules of quantum measurement theory implies that negentropic entanglement can be stable against the effects of quantum jumps unlike entropic entanglement. Therefore living matter could be distinguished from inanimate matter also by negentropic entanglement possible in the intersection of real and p-adic worlds. In consciousness theory negentropic entanglement could be seen as a correlate for the experience of understanding or any other positively colored experience, say love.

Negentropically entangled states are stable but binding energy and effective loss of relative translational degrees of freedom is not responsible for the stability. Therefore bound states are not in question. The distinction between negentropic and bound state entanglement could be compared to the difference between unhappy and happy marriage. The first one is a social jail but in the latter case both parties are free to leave but do not want to. The special characterics of negentropic entanglement raise the question whether the problematic notion of high energy phosphate bond [102] central for metabolism could be understood in terms of negentropic entanglement. This would also allow an information theoretic interpretation of metabolism since the transfer of metabolic energy would mean a transfer of negentropy [13].

3 Symmetries of quantum TGD

Symmetry principles play key role in the construction of WCW geometry have become and deserve a separate explicit treament even at the risk of repetitions.

3.1 General Coordinate Invariance

General coordinate invariance is certainly of the most important guidelines and is much more powerful in TGD framework thanin GRT context.

1. General coordinate transformations as a gauge symmetry so that the diffeomorphic slices of space-time surface equivalent physically. 3-D light-like 3-surfaces defined by wormhole throats define preferred slices and allows to fix the gauge partially apart from the remaining 3-D variant of general coordinate invariance and possible gauge degeneracy related to the choice of the light-like 3-surface due to the Kac-Moody invariance. This would mean that the random light-likeness represents gauge degree of freedome except at the ends of the light-like 3-surfaces.

2. GCI can be strengthed so that the pairs of space-like ends of space-like 3-surfaces at CDs are equivalent with light-like 3-surfaces connecting them. The outcome is effective 2-dimensionality because their intersections at the boundaries of CDs must carry the physically relevant information.

3.2 Generalized conformal symmetries

One can assign Kac-Moody type conformal symmetries to light-like 3-surfaces as isometries of H localized with respect to light-like 3-surfaces. Kac Moody algebra essentially the Lie algebra of gauge group with central extension meaning that projective representation in which representation matrices are defined only modulo a phase factor. Kac-Moody symmetry is not quite a pure gauge symmetry.

One can assign a generalization of Kac-Moody symmetries to the boundaries of CD by replacing Lie-group of Kac Moody algebra with the group of symplectic (contact-) tranformations [70, 67, 66] of H_+ provided with a degenerate Kähler structure made possible by the effective 2-dimensionality of δM^4_+. The light-like radial coordinate of δM^4_+ plays the role of the complex coordinate of conformal transformations or their hyper-complex analogs. These symmetries are also localized with respect to the internal coordinates of the partonic 2-surface so that rather huge symmetry group is in question. The basic hypothesis is that these transformations with possible some restrictions on the depedence on the coordinates of X^2 define the isometries of WCW.

A further physically well-motivated hypothesis inspired by holography and extended GCI is that these symmetries extend so that they apply at the entire space-time sheet. This requires the slicing of space-time surface by partonic 2- surfaces and by stringy world sheets such that each point of stringy world sheet defines a partonic 2-surface and vice versa. This slicing has deep physical motivations since it realizes geometrically standard facts about gauge invariance (partonic 2-surface defines the space of physical polarizations and stringy space-time sheet corresponds to non-physical polarizations) and its existence is a hypothesis about the properties of the preferred extremals of Kähler action. There is a similar decomposition also at the level of CD and so called Hamilton-Jacobi coordinates for M^4_+ [3] define this kind of slicings. This slicing can induced the slicing of the space-time sheet. The number theoretic vision gives a further justification for this hypothesis and also strengthens it by postulating the presence of the preferred time direction having interpretation in terms of real unit of octonions. In ZEO this time direction corresponds to the time-like vector connecting the tips of CD.

Conformal symmetries would provide the realization of WCW as a union of symmetric spaces. Symmetric spaces are coset spaces of form G/H. The natural identification of G and H is as groups of X^2-local symplectic transformations and local Kac-Moody group of X^2-local H isometries. Quantum fluctuating (metrically non-trivial) degrees of freedom would correspond to symplectic transformations of H_+ and induced Kähler form at X^2 would define a local representation for zero modes: not necessarily all of them.

3.3 Equivalence Principle and super-conformal symmetries

Equivalence Principle (EP) is a second corner stone of General Relativity and together with GCI leads to Einstein's equations. What EP states is that inertial and gravitational masses are identical. In this form it is not well-defined even in GRT since the definition of gravitational and inertial four-momenta is highly problematic because Noether theorem is not avaible. The realization is in terms of local equations identifying energy momentum tensor with Einstein tensor.

Whether EP is realized in TGD has been a longstanding open question [39]. The problem has been that at the classical level EP in its GRT form can hold true only in long enough length scales and it took long to time to realize that only the stringy form of this principle is required. The first question is how to identify the gravitational and inertial four-momenta. This is indeed possible. One can assign to the two types super-conformal symmetries assigned with light-like 3-surfaces and space-like 3-surfaces four-momenta to both. EP states that these four momenta are identical and is equivalent with the generalization of GCI and effective 2-dimensionality. The condition generalizes so that it applies to the generators of super-conformal algebras associated with the two super-conformal symmetries. This leads to a generalization of a standard mathematical construction of super-conformal theories known as coset representation [76]. What the construction states is that the differences of super-conformal generators defined by super-symmetric algebra and Kac-Moody algebra annihilate physical states.

3.4 Extension to super-conformal symmetries

The original idea behind the extension of conformal symmetries to super-conformal symmetries was the observation that isometry currents defining infinitesimal isometries of WCW have natural super-counterparts obtained by contracting the Killing vector fields with the complexified gamma matrices of the imbedding space.

This vision has generalized considerably as the construction of WCW spinor structure in terms of modified Dirac action has developed. The basic philosophy behind this idea is that configuration space spinor structure must relate directly to the fermionic sector of quantum physics. In particular, modified gamma matrices should be expressible in terms of the fermionic oscillator operators associated with the second quantized induced spinor fields. The explicit realization of this program leads to an identification of rich spectrum of super-conformal symmetries and generalization of the ordinary notion of space-time supersymmetry. What happens that all fermionic oscillator operator generate broken super-symmetries whereas in SUSYs there is only finite number of them. One can however identify sub-algebra of super-conformal symmetries associated with right handed neutrino and this gives $\mathcal{N} = 1$ super-symmetry [82] of SUSYs [12].

3.5 Space-time supersymmetry in TGD Universe

It has been clear from the beginning that the notion of super-conformal symmetry crucial for the successes of super-string models generalizes in TGD framework. The answer to the question whether space-time SUSY makes sense in TGD framework has not been obvious at all but it seems now that the answer is affirmative. The evolution of the ideas relevant for the formulation of SUSY in TGD framework is summarized in the chapters of [30]. The chapters are devoted to the notion of bosonic emergence [26], to the SUSY QFT limit of TGD [12] , to twistor approach to TGD [41], and to the generalization of Yangian symmetry of $\mathcal{N} = 4$ SYM manifest in the Grassmannian twistor approach [84] to a multi-local variant of super-conformal symmetries [42] represent a gradual development of the ideas about how super-symmetric M-matrix could be constructed in TGD framework. A warning to the reader is in order. In their recent form these chapters do not represent the final outcome but just an evolution of ideas proceeding by trial and error. There are however good reasons to believe that the chapter about Yangian symmetry is nearest to the correct physical interpretation and mathematical formulation.

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the modified Dirac action [4, 11, 7]. The appearance of the momentum and color quantum numbers in the measurement interaction part of the modified Dirac action associated with the light-like wormhole throats [11] couples space-time degrees of freedom to quantum numbers and allows also to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation $\mathcal{N} = 2N$ SUSY algebra (an inherent cutoff on the number of fermionic modes at light-like wormhole throat) or fermionic part of super-conformal algebra with infinite number of oscillator operators results. The addition of fermion in particular mode would define particular super-symmetry. This super-symmetry is badly broken due to the dynamics of the modified Dirac operator which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

Zero energy ontology combined with the analog of the twistor approach to $\mathcal{N} = 4$ SYMs and weak form of electric-magnetic duality has actually led to this kind of formulation [42]. What is new that also virtual particles have massless fermions as their building blocks. This implies manifest finiteness of loop integrals so that the situation simplifies dramatically. What is also new element that physical particles and also string like objects correspond to bound states of massless fermions.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

2. The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in $D = 10$ and $D = 11$ as the only possible candidates for TOE after it turned out that chiral anomalies cancel. It indeed turns out that TGD view about space-time SUSY is internally consistent. Even more, the separate conservation of quark and lepton number is essential for the internal consistency of this view [12].

3. The massivation of particles is the basic problem of both SUSYs and twistor approach. I have discussed several solutions to this problem [41, 42]. The simplest and most convincing solution of the problem is following and inspired by twistor Grassmannian approach to $\mathcal{N} = 4$ SYM and the generalization of the Yangian symmetry of this theory. In zero energy ontology one can construct physical particles as bound states of massless particles associated with the opposite wormhole throats. If the particles have opposite 3-momenta the resulting state is automatically massive. In fact, this forces massivation of also spin one bosons since the fermion and antifermion must move in opposite directions for their spins to be parallel so that the net mass is non-vanishing: note that this means that even photon, gluons, and graviton have small mass. This mechanism makes topologically condensed fermions massive and padic thermodynamics allows to describe the massivation in terms of zero energy states and M-matrix. Bosons receive to their mass besides the small mass coming from thermodynamics also a contribution which is counterpart of the contribution coming from Higgs vacuum expectation value and Higgs gives rise to longitudinal polarizations. No Higgs potential is however needed. The cancellation of infrared divergences necessary for exact Yangian symmetry and the observation that even photon receives small mass suggest that scalar Higgs would disappear completely from the spectrum.

3.5.1 Basic data bits

Let us first summarize the data bits about possible relevance of super-symmetry for TGD before the addition of the 3-D measurement interaction term to the modified Dirac action [4, 11].

1. Right-handed covariantly constant neutrino spinor ν_R defines a super-symmetry in CP_2 degrees of freedom in the sense that Dirac equation is satisfied by covariant constancy and there is no need

for the usual ansatz $\Psi = D\Psi_0$ giving $D^2\Psi = 0$. This super-symmetry allows to construct solutions of Dirac equation in CP_2 [78, 74, 77, 73].

2. In $M^4 \times CP_2$ this means the existence of massless modes $\Psi = \not{p}\Psi_0$, where Ψ_0 is the tensor product of M^4 and CP_2 spinors. For these solutions M^4 chiralities are not mixed unlike for all other modes which are massive and carry color quantum numbers depending on the CP_2 chirality and charge. As matter fact, covariantly constant right-handed neutrino spinor mode is the only color singlet. The mechanism leading to non-colored states for fermions is based on super-conformal representations for which the color is neutralized [16, 21]. The negative conformal weight of the vacuum also cancels the enormous contribution to mass squared coming from mass in CP_2 degrees of freedom.

3. Right-handed covariantly constant neutrino allows to construct the gamma matrices of the world of classical worlds (WCW) as fermionic counterparts of Hamiltonians of WCW. This gives rise super-symplectic symmetry algebra having interpretation also as a conformal algebra. Also more general super-conformal symmetries exist.

4. Space-time (in the sense of Minkowski space M^4) super-symmetry in the conventional sense of the word is impossible in TGD framework since it would require require Majorana spinors. In 8-D space-time with Minkowski signature of metric Majorana spinors are definitely ruled out by the standard argument leading to super string model. Majorana spinors would also break separate conservation of lepton and baryon numbers in TGD framework.

3.5.2 Could one generalize super-symmetry?

Could one then consider a more general space-time super-symmetry with "space-time" identified as space-time surface rather than Minkowski space?

1. The TGD variant of the super-symmetry could correspond quite concretely to the addition to fermion and boson states right-handed neutrinos. Since right-handed neutrinos do not have electro-weak interactions, the addition might not appreciably affect the mass formula although it could affect the p-adic prime defining the mass scale.

2. The problem is to understand what this addition of the right-handed neutrino means. To begin with, notice that in TGD Universe fermions reside at light-like 3-surfaces at which the signature of induced metric changes. Bosons correspond to pairs of light-like wormhole throats with wormhole contact having Euclidian signature of the induced metric. The long standing problem has been that for bosons with parallel light-like four-momenta with same sign of energy the spins of fermion and antifermion are opposite so that one would obtain only scalar bosons!

   I have considered several solutions to the problem but the final solution came from the basic problem of twistor approach to $\mathcal{N} = 4$ SUSY. This theory is believed to be UV finite but has IR divergences spoiling the Yangian SUSY. These infinities cancel if the physical particles are bound states of pairs of wormhole throats with light-like momenta. Just the requirement that spin is equal to one forces massivation. This is true for all spin 1 particles, also those regarded as massless. Massivation of the photon is not a problem if the mass corresponds to the IR cutoff determined by the largest causal diamond (CD) defining the measurement resolution. For electron the size of CD corresponds to the size scale of Earth. The basic prediction is that Higgs disappears completely from the spectrum so that this mechanism is testable at LHC.

   The first proposal to the solution of problem was that either fermion or antifermion in the boson state carries what might be called un-physical polarization in the standard conceptual framework. This means that it has negative energy but three-momentum parallel to that of the second wormhole throat. The assumption that the bosonic wormhole throats correspond to positive and negative energy space-time sheets realizes this constraint in the framework of zero energy ontology. It however turned out that for light-like momenta these states have more natural interpretation in terms of virtual bosons able to have space-like momenta. This means that one can realize virtual particles as pairs of on mass shell wormhole throats with either sign of energy and 3-momentum so that the basic condition of twistorial approach is satisfied. The conservation of 4-momentum at vertices gives extremely powerful kinematical constraints so that there are excellent hopes about cancellation of UV divergences of loop integrals.

3. The super-symmetry as an addition to the fermion state a second wormhole throats carrying right handed neutrino quantum numbers does not make sense since the resulting state cannot be distinguished from gauge boson or Higgs type particle. The light-like 3-surfaces can however carry fermion numbers up to the number of modes of the induced spinor field, which is expected to be infinite inside string like objects having wormhole throats at ends and finite when one has space time sheets containing the throats [11]. In very general sense one could say that each mode defines a very large broken N-super-symmetry with the value of N depending on state and light-like 3-surface. The breaking of this super-symmetry would come from electro-weak - , color - , and gravitational interactions. Right-handed neutrino would by its electro-weak and color inertness define a minimally broken super-symmetry.

4. What this addition of the right handed neutrinos or more general fermion modes could precisely mean? One cannot assign fermionic oscillator operators to right handed neutrinos which are covariantly constant in both M^4 and CP_2 degrees of freedom since the modes with vanishing energy (frequency) cannot correspond to fermionic oscillator operator creating a physical state since one would have $a = a^\dagger$. The intuitive view is that all the spinor modes move in an exactly collinear manner -somewhat like quarks inside hadron do approximately.

3.5.3 Modified Dirac equation briefly

The answer to the question what "collinear motion" means mathematically emerged from the recent progress in the understanding of the modified Dirac equation.

1. The modified Dirac action involves two terms. Besides the original 4-D modified Dirac action there is measurement interaction which can be localized to wormhole throat or to any light-like 3-surfaces "parallel" to it in the slicing of space-time sheet by light-like 3-surfaces. This term correlates space-time geometry with quantum numbers assignable to super-conformal representations and is also necessary to obtain almost-stringy propagator.

2. The modified Dirac equation with measurement action added reads as

$$\begin{aligned} D_K\Psi &= 0 \ , \\ D_3\Psi &= (D_{C-S} + Q\times O)\Psi = 0 \ , \\ [D_3, D_K]\,\Psi &= 0 \ . \end{aligned} \tag{3.1}$$

   (a) D_K corresponds formally to 4-D massless Dirac equation in X^4. D_3 realizes measurement interaction. D_{C-S} is the 3-D modified Dirac action defined by Chern-Simons action.

   (b) Q is linear in Cartan algebra generators of the isometry algebra of imbedding space (color isospin and hypercharge plus four-momentum or two components of four momentum and spin and boost in direction of 3-momentum). Q is expressible as

$$Q = Q_A\partial_\alpha h^k g^{AB} j_{Bk}\hat{\Gamma}^\alpha_{CS} \ . \tag{3.2}$$

   Here Q_A is Cartan algebra generator acting on physical states. Physical states must be eigen states of Q_A since otherwise the equations do not make sense. g^{AB} is the inverse of the matrix defined by the imbedding space inner product of Killing vector fields j^k_A and j^l_B: its existence allows only Cartan algebra charges. $\hat{\Gamma}^\alpha_{CS}$ is the modified gamma matrix associated with the Chern-Simons action.

(c) In general case the modified gamma matrices are defined in terms of action density L as

$$\hat{\Gamma}^{\alpha} = \frac{\partial L}{\partial_{\alpha} h^k}\gamma^k \quad . \tag{3.3}$$

γ^k denotes imbedding space gamma matrices.

(d) The operator O characterizes the conserved fermionic current to which Cartan algebra generators of isometries couple. The simplest conserved currents correspond to quark or lepton currents and corresponding vectorial isospin- and spin currents [11]. Besides this there is an infinite hierarchy of conserved currents relating to quantum criticality and in one-one correspondence with vanishing second variations of Kähler action for preferred extremal. These couplings allow to represent measurement interaction for any observable.

3. The equation $D_3\nu_R = 0$ would reduce for vanishing color charges and covariantly constant spinor to the analog of algebraic fermionic on mass shell condition $p_A\gamma^A\nu_R = 0$ since Q is obtained by projecting the total four-momentum of the parton state interpreted as a vector-field of H to the space-time surface and by replacing ordinary gamma matrices with the modified ones. This equation cannot be exact since Q depends on the point of the light-like 3-surface so that covariant constancy fails and D_{C-S} cannot annihilate the state. This is the space-time correlate for the breaking of super-symmetry. The action of the Cartan algebra generators is purely algebraic and on the state of super-conformal representations rather than that of a differential operator on spinor field. The modified equation implies that all spinor modes represent fermions moving collinearly in the sense an equation with the same total four-momentum and total color quantum numbers is satisfied by all of them. Note that p_A represents the total four-momentum of the state rather than individual four-momenta of fermions.

3.5.4 TGD counterpart of space-time super-symmetry

This picture allows to define more precisely what one means with the approximate super-symmetries in TGD framework.

1. One can in principle construct many-fermion states containing both fermions and anti-fermions at given light-like 3-surface. The four-momenta of states related by super-symmetry need not be same. Super-symmetry breaking is present and has as the space-time correlate the deviation of the modified gamma matrices from the ordinary M^4 gamma matrices. In particular, the fact that $\hat{\Gamma}^{\alpha}$ possesses CP_2 part in general means that different M^4 chiralities are mixed: a space-time correlate for the massivation of the elementary particles.

2. For right-handed neutrino super-symmetry breaking is expected to be smallest but also in the case of the right-handed neutrino mode mixing of M^4 chiralities takes place and breaks the TGD counterpart of super-symmetry.

3. The fact that all helicities in the state are physical for a given light-like 3-surface has important implications. For instance, the addition of a right-handed antineutrino to right-handed (left-handed) electron state gives scalar (spin 1) state. Also states with fermion number two are obtained from fermions. For instance, for e_R one obtains the states $\{e_R, e_R\nu_R\overline{\nu}_R, e_R\overline{\nu}_R, e_R\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,0,1)$. For e_L one obtains the states $\{e_L, e_L\nu_R\overline{\nu}_R, e_L\overline{\nu}_R, e_L\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,1,0)$. In the case of gauge boson and Higgs type particles -allowed by TGD but not required by p-adic mass calculations- gauge boson has 15 super partners with fermion numbers $[2,1,0,-1,-2]$.

The cautious conclusion is that the recent view about quantum TGD allows the analog of super-symmetry which is necessary broken and for which the multiplets are much more general than for the ordinary super-symmetry. Right-handed neutrinos might however define something resembling ordinary super-symmetry to a high extent. The question is how strong prediction one can deduce using quantum TGD and proposed super-symmetry.

1. For a minimal breaking of super-symmetry only the p-adic length scale characterizing the super-partner differs from that for partner but the mass of the state is same. This would allow only a discrete set of masses for various super-partners coming as half octaves of the mass of the particle in question. A highly predictive model results.

2. The quantum field theoretic description should be based on QFT limit of TGD formulated in terms of bosonic emergence [26]. This formulation should allow to calculate the propagators of the super-partners in terms of fermionic loops.

3. This TGD variant of space-time super-symmetry resembles ordinary super-symmetry in the sense that selection rules due to the right-handed neutrino number conservation and analogous to the conservation of R-parity hold true. The states inside super-multiplets have identical electro-weak and color quantum numbers but their p-adic mass scales can be different. It should be possible to estimate reaction reaction rates using rules very similar to those of super-symmetric gauge theories.

4. It might be even possible to find some simple generalization of standard super-symmetric gauge theory to get rough estimates for the reaction rates. There are however problems. The fact that spins $J = 0, 1, 2, 3/2, 2$ are possible for super-partners of gauge bosons forces to ask whether these additional states define an analog of non-stringy strong gravitation. Note that graviton in TGD framework corresponds to a pair of wormhole throats connected by flux tube (counterpart of string) and for gravitons one obtains 2^8-fold degeneracy.

3.6 Twistorial approach, Yangian symmetry, and generalized Feynman diagrams

There has been impressive steps in the understanding of $\mathcal{N} = 4$ maximally sypersymmetric YM theory possessing 4-D super-conformal symmetry. This theory is related by AdS/CFT duality to certain string theory in $AdS_5 \times S^5$ background. Second stringy representation was discovered by Witten and is based on 6-D Calabi-Yau manifold defined by twistors. The unifying proposal is that so called Yangian symmetry is behind the mathematical miracles involved.

The notion of Yangian symmetry would have a generalization in TGD framework obtained by replacing conformal algebra with appropriate super-conformal algebras. Also a possible realization of twistor approach and the construction of scattering amplitudes in terms of Yangian invariants defined by Grassmannian integrals is considered in TGD framework and based on the idea that in zero energy ontology one can represent massive states as bound states of massless particles. There is also a proposal for a physical interpretation of the Cartan algebra of Yangian algebra allowing to understand at the fundamental level how the mass spectrum of n-particle bound states could be understood in terms of the n-local charges of the Yangian algebra.

Twistors were originally introduced by Penrose to characterize the solutions of Maxwell's equations. Kähler action is Maxwell action for the induced Kähler form of CP_2. The preferred extremals allow a very concrete interpretation in terms of modes of massless non-linear field. Both conformally compactified Minkowski space identifiable as so called causal diamond and CP_2 allow a description in terms of twistors. These observations inspire the proposal that a generalization of Witten's twistor string theory relying on the identification of twistor string world sheets with certain holomorphic surfaces assigned with Feynman diagrams could allow a formulation of quantum TGD in terms of 3-dimensional holomorphic surfaces of $CP_3 \times CP_3$ mapped to 6-surfaces dual $CP_3 \times CP_3$, which are sphere bundles so that they are projected in a natural manner to 4-D space-time surfaces. Very general physical and mathematical arguments lead to a highly unique proposal for the holomorphic differential equations defining the complex 3-surfaces conjectured to correspond to the preferred extremals of Kähler action.

3.6.1 Background

I am outsider as far as concrete calculations in $\mathcal{N} = 4$ SUSY are considered and the following discussion of the background probably makes this obvious. My hope is that the reader had patience to not care about this and try to see the big pattern.

The developments began from the observation of Parke and Taylor [89] that n-gluon tree amplitudes with less than two negative helicities vanish and those with two negative helicities have unexpectedly simple form when expressed in terms of spinor variables used to represent light-like momentum. In fact, in the formalism based on Grassmanian integrals the reduced tree amplitude for two negative helicities is just "1" and defines Yangian invariant. The article *Perturbative Gauge Theory As a String Theory In Twistor Space* [91] by Witten led to so called Britto-Cachazo-Feng-Witten (BCFW) recursion relations for tree level amplitudes [90, 85, 90] allowing to construct tree amplitudes using the analogs of Feynman rules in which vertices correspond to maximally helicity violating tree amplitudes (2 negative helicity gluons) and propagator is massless Feynman propagator for boson. The progress inspired the idea that the theory might be completely integrable meaning the existence of infinite-dimensional un-usual symmetry. This symmetry would be so called Yangian symmetry [42] assigned to the super counterpart of the conformal group of 4-D Minkowski space.

Drumond, Henn, and Plefka represent in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory* [86] an argument suggesting that the Yangian invariance of the scattering amplitudes ins an intrinsic property of planar $\mathcal{N}=4$ super Yang Mills at least at tree level.

The latest step in the progress was taken by Arkani-Hamed, Bourjaily, Cachazo, Carot-Huot, and Trnka and represented in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ super Yang-Mills theory* [84]. At the same day there was also the article of Rutger Boels entitled *On BCFW shifts of integrands and integrals* [83] in the archive. Arkani-Hamed *et al* argue that a full Yangian symmetry of the theory allows to generalize the BCFW recursion relation for tree amplitudes to all loop orders at planar limit (planar means that Feynman diagram allows imbedding to plane without intersecting lines). On mass shell scattering amplitudes are in question.

3.6.2 Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [42]. Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [87]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N}=4$ SUSY). One of the conditions conditions is that the tensor product $R\otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n=0$ and $n=1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n=1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

3.6.3 How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, I have nothing to say. I am just perplexed. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [57] and Virasoro algebras [64] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

2. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context;-)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algedbras ontains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of partonic 2-surface. A coset construction is applied to these two Virasoro algebras so that the differences of the corresponding Super-Virasoro

generators and Kac-Moody generators annihilate physical states. This implies that the corresponding four-momenta are same: this expresses the equivalence of gravitational and inertial masses. A generalization of the Equivalence Principle is in question. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.6.4 Is there any hope about description in terms of Grassmannians?

At technical level the successes of the twistor approach rely on the observation that the amplitudes can be expressed in terms of very simple integrals over sub-manifolds of the space consisting of k-dimensional planes of n-dimensional space defined by delta function appearing in the integrand. These integrals define super-conformal Yangian invariants appearing in twistorial amplitudes and the belief is that by a proper choice of the surfaces of the twistor space one can construct all invariants. One can construct also the counterparts of loop corrections by starting from tree diagrams and annihilating pair of particles by connecting the lines and quantum entangling the states at the ends in the manner dictated by the integration over loop momentum. These operations can be defined as operations for Grassmannian integrals in general changing the values of n and k. This description looks extremely powerful and elegant and nosta importantly involves only the external momenta.

The obvious question is whether one could use similar invariants in TGD framework to construct the momentum dependence of amplitudes.

1. The first thing to notice is that the super algebras in question act on infinite-dimensional representations and basically in the world of classical worlds assigned to the partonic 2-surfaces correlated by the fact that they are associated with the same space-time surface. This does not promise anything very practical. On the other hand, one can hope that everything related to other than M^4 degrees of freedom could be treated like color degrees of freedom in $\mathcal{N} = 4$ SYM and would boil down to indices labeling the quantum states. The Yangian conditions coming from isometry quantum numbers, color quantum numbers, and electroweak quantum numbers are of course expected to be highly non-trivial and could fix the coefficients of various singlets resulting in the tensor product of incoming and outgoing states.

2. The fact that incoming particles can be also massive seems to exclude the use of the twistor space. The following observation however raises hopes. The Dirac propagator for wormhole throat is massless propagator but for what I call pseudo momentum. It is still unclear how this momentum relates to the actual four-momentum. Could it be actually equal to it? The recent view about pseudo-momentum does not support this view but it is better to keep mind open. In any case this finding suggests that twistorial approach could work in in more or less standard form. What would be needed is a representation for massive incoming particles as bound states of massless partons. In particular, the massive states of super-conformal representations should allow this kind of description.

Could zero energy ontology allow to achieve this dream?

1. As far as divergence cancellation is considered, zero energy ontology suggests a totally new approach producing the basic nice aspects of QFT approach, in particular unitarity and coupling constant evolution. The big idea related to zero energy ontology is that all virtual particle particles correspond

to wormhole throats, which are pairs of on mass shell particles. If their momentum directions are different, one obtains time-like continuum of virtual momenta and if the signs of energy are opposite one obtains also space-like virtual momenta. The on mass shell property for virtual partons (massive in general) implies extremely strong constraints on loops and one expect that only very few loops remain and that they are finite since loop integration reduces to integration over much lower-dimensional space than in the QFT approach. There are also excellent hopes about Cutkoski rules.

2. Could zero energy ontology make also possible to construct massive incoming particles from massless ones? Could one construct the representations of the super conformal algebras using only massless states so that at the fundamental level incoming particles would be massless and one could apply twistor formalism and build the momentum dependence of amplitudes using Grassmannian integrals.

   One could indeed construct on mass shell massive states from massless states with momenta along the same line but with three-momenta at opposite directions. Mass squared is given by $M^2 = 4E^2$ in the coordinate frame, where the momenta are opposite and of same magnitude. One could also argue that partonic 2-surfaces carrying quantum numbers of fermions and their superpartners serve as the analogs of point like massless particles and that topologically condensed fermions and gauge bosons plus their superpartners correspond to pairs of wormhole throats. Stringy objects would correspond to pairs of wormhole throats at the same space-time sheet in accordance with the fact that space-time sheet allows a slicing by string worlds sheets with ends at different wormhole throats and definining time like braiding.

The weak form of electric magnetic duality indeed supports this picture. To understand how, one must explain a little bit what the weak form of electric magnetic duality means.

1. Elementary particles correspond to light-like orbits of partonic 2-surfaces identified as 3-D surfaces at which the signature of the induced metric of space-time surface changes from Euclidian to Minkowskian and 4-D metric is therefore degenerate. The analogy with black hole horizon is obvious but only partial. Weak form of electric-magnetic duality states that the Kähler electric field at the wormhole throat and also at space-like 3-surfaces defining the ends of the space-time surface at the upper and lower light-like boundaries of the causal diamond is proportonial to Kähler magnetic field so that Kähler electric flux is proportional Kähler magnetic flux. This implies classical quantization of Kähler electric charge and fixes the value of the proportionality constant.

2. There are also much more profound implications. The vision about TGD as almost topological QFT suggests that Kähler function defining the Kähler geometry of the "world of classical worlds" (WCW) and identified as Kähler action for its preferred extremal reduces to the 3-D Chern-Simons action evaluted at wormhole throats and possible boundary components. Chern-Simons action would be subject to constraints. Wormhole throats and space-like 3-surfaces would represent extremals of Chern-Simons action restricted by the constraint force stating electric-magnetic duality (and realized in terms of Lagrange multipliers as usual).

   If one assumes that Kähler current and other conserved currents are proportional to current defining Beltrami flow whose flow lines by definition define coordinate curves of a globally defined coordinate, the Coulombic term of Kähler action vanishes and it reduces to Chern-Simons action if the weak form of electric-magnetic duality holds true. One obtains almost topological QFT. The absolutely essential attribute "almost" comes from the fact that Chern-Simons action is subject to constraints. As a consequence, one obtains non-vanishing four-momenta and WCW geometry is non-trivial in M^4 degrees of freedom. Otherwise one would have only topological QFT not terribly interesting physically.

Consider now the question how one could understand stringy objects as bound states of massless particles.

1. The observed elementary particles are not Kähler monopoles and there much exist a mechanism neutralizing the monopole charge. The only possibility seems to be that there is opposite Kähler magnetic charge at second wormhole throat. The assumption is that in the case of color neutral

particles this throat is at a distance of order intermediate gauge boson Compton length. This throat would carry weak isospin neutralizing that of the fermion and only electromagnetic charge would be visible at longer length scales. One could speak of electro-weak confinement. Also color confinement could be realized in analogous manner by requiring the cancellation of monopole charge for many-parton states only. What comes out are string like objects defined by Kähler magnetic fluxes and having magnetic monopoles at ends. Also more general objects with three strings branching from the vertex appear in the case of baryons. The natural guess is that the partons at the ends of strings and more general objects are massless for incoming particles but that the 3-momenta are in opposite directions so that stringy mass spectrum and representations of relevant super-conformal algebras are obtained. This description brings in mind the description of hadrons in terms of partons moving in parallel apart from transversal momentum about which only momentum squared is taken as observable.

2. Quite generally, one expects for the preferred extremals of Kähler action the slicing of space-time surface with string world sheets with stringy curves connecting wormhole throats. The ends of the stringy curves can be identified as light-like braid strands. Note that the strings themselves define a space-like braiding and the two braidings are in some sense dual. This has a concrete application in TGD inspired quantum biology, where time-like braiding defines topological quantum computer programs and the space-like braidings induced by it its storage into memory. Stringlike objects defining representations of super-conformal algebras must correspond to states involving at least two wormhole throats. Magnetic flux tubes connecting the ends of magnetically charged throats provide a particular realization of stringy on mass shell states. This would give rise to massless propagation at the parton level. The stringy quantization condition for mass squared would read as $4E^2 = n$ in suitable units for the representations of super-conformal algebra associated with the isometries. For pairs of throats of the same wormhole contact stringy spectrum does not seem plausible since the wormhole contact is in the direction of CP_2. One can however expect generation of small mass as deviation of vacuum conformal weight from half integer in the case of gauge bosons.

If this picture is correct, one might be able to determine the momentum dependence of the scattering amplitudes by replacing free fermions with pairs of monopoles at the ends of string and topologically condensed fermions gauge bosons with pairs of this kind of objects with wormhole throat replaced by a pair of wormhole throats. This would mean suitable number of doublings of the Grassmannian integrations with additional constraints on the incoming momenta posed by the mass shell conditions for massive states.

3.6.5 Could zero energy ontology make possible full Yangian symmetry?

The partons in the loops are on mass shell particles have a discrete mass spectrum but both signs of energy are possible for opposite wormhole throats. This implies that in the rules for constructing loop amplitudes from tree amplitudes, propagator entanglement is restricted to that corresponding to pairs of partonic on mass shell states with both signs of energy. As emphasized in [84] , it is the Grassmannian integrands and leading order singularities of $\mathcal{N} = 4$ SYM, which possess the full Yangian symmetry. The full integral over the loop momenta breaks the Yangian symmetry and brings in IR singularities. Zero energy ontologist finds it natural to ask whether QFT approach shows its inadequacy both via the UV divergences and via the loss of full Yangian symmetry. The restriction of virtual partons to discrete mass shells with positive or negative sign of energy imposes extremely powerful restrictions on loop integrals and resembles the restriction to leading order singularities. Could this restriction guarantee full Yangian symmetry and remove also IR singularities?

3.6.6 Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

4 Weak form electric-magnetic duality and color and weak forces

The notion of electric-magnetic duality [81] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [5]. What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter

associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found. The solution ansatz applies also to the extremals of Chern-Simons action and and to the conserved currents associated with the modified Dirac equation defined as contractions of the modified gamma matrices between the solutions of the modified Dirac equation. The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

6. The general solution ansatz works for induced Kähler Dirac equation and Chern-Simons Dirac equation and reduces them to ordinary differential equations along flow lines. The induced spinor fields are simply constant along flow lines of indued spinor field for Dirac equation in suitable gauge. Also the generalized eigen modes of the modified Chern-Simons Dirac operator can be deduced explicitly if the throats and the ends of space-time surface at the boundaries of CD are extremals of Chern-Simons action. Chern-Simons Dirac equation reduces to ordinary differential equations along flow lines and one can deduce the general form of the spectrum and the explicit representation of the Dirac determinant in terms of geometric quantities characterizing the 3-surface (eigenvalues are inversely proportional to the lengths of strands of the flow lines in the effective metric defined by the modified gamma matrices).

4.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the configuration space metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

4.1.1 Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this

duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} . \tag{4.1}$$

A more general form of this duality is suggested by the considerations of [14] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [79] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} . \tag{4.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the configuration space metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J , \tag{4.3}$$

where J can denotes the Kähler magnetic flux, makes it possible to have a non-trivial configuration space metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

4.1.2 Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [43] ,[43] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{4.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{4.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{4.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{4.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

4.1.3 The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [27] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{4.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and antifermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{4.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole throat. Full self-duality is indeed an un-necessarily strong condition.

4.1.4 Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced K"ahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{4.10}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [43]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [28]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [39]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

4.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

4.2.1 How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

4.2.2 Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2}-X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four Gaussian Mersennes corresponding to $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [98].

4.2.3 Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [12]. The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles

allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [17]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [18].

5 Quantum TGD very briefly

There are two basic approaches to the construction of quantum TGD. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry [58] for the "world of classical worlds" (WCW) identified as the space of 3-surfaces in in certain 8-dimensional space. Essentially a generalization of the Einstein's geometrization of physics program is in question. The second vision is the identification of physics as a generalized number theory involving p-adic number fields and the fusion of real numbers and p-adic numbers to a larger structure, classical number fields, and the notion of infinite prime.

With a better resolution one can distinguish also other visions crucial for quantum TGD. Indeed, the notion of finite measurement resolution realized in terms of hyper-finite factors, TGD as almost topological quantum field theory, twistor approach, zero energy ontology, and weak form of electric-magnetic duality play a decisive role in the actual construction and interpretation of the theory. One can however argue that these visions are not so fundamental for the formulation of the theory than the first two.

5.1 Physics as infinite-dimensional geometry

It is good to start with an attempt to give overall view about what the dream about physics as infinite-dimensional geometry is. The basic vision is generalization of the Einstein's program for the geometrization of classical physics so that entire quantum physics would be geometrized. Finite-dimensional geometry is certainly not enough for this purposed but physics as infinite-dimensional geometry of what might be called world of classical worlds (WCW) -or more neutrally configuration space of 3-surfaces of some higher-dimensional imbeddign space- might make sense. The requirement that the Hermitian conjugation of quantum theories has a geometric realization forces Kähler geometry for WCW. WCW defines the fixed arena of quantum physics and physical states are identified as spinor fields in WCW. These spinor fields are classical and no second quantization is needed at this level. The justification comes from the observation that infinite-dimensional Clifford algebra [54] generated by gamma matrices allows a natural identification as fermionic oscillator algebra.

The basic challenges are following.

1. Identify WCW.
2. Provide WCW with Kähler metric and spinor structure
3. Define what spinors and spinor fields in WCW are.

There is huge variety of finite-dimensional geometries and one might think that in infinite-dimensional case one might be drowned with the multitude of possibilities. The situation is however exactly opposite. The loop spaces associated with groups have a unique Kähler geometry due to the simple condition that Riemann connection exists mathematically [72]. This condition requires that the metric possesses maximal symmetries. Thus raises the vision that infinite-dimensional Kähler geometric existence is unique once one poses the additional condition that the resulting geometry satisfies some basic constraints forced by physical considerations.

The observation about the uniqueness of loop geometries leads also to a concrete vision about what this geometry could be. Perhaps WCW could be refarded as a union of symmetric spaces [65] for which every point is equivalent with any other. This would simplify the construction of the geometry immensely and would mean a generalization of cosmological principle to infinite-D context [14] ,[46].

This still requires an answer to the question why $M^4 \times CP_2$ is so unique. Something in the structure of this space must distinguish it in a unique manner from any other candidate. The uniqueness of M^4 factor can be understood from the miraculous conformal symmetries of the light-cone boundary but in the case of CP_2 there is no obvious mathematical argument of this kind although physically CP_2 is unique [51]. The observation that $M^4 \times CP_2$ has dimension 8, the space-time surfaces have dimension 4, and partonic 2-surfaces, which are the fundamental objects by holography have dimension 2, suggests that classical number fields [60, 55, 62] are involved and one can indeed end up to the choice $M^4 \times CP_2$ from physics as generalized number theory vision by simple arguments [37] ,[47]. In particular, the choices M^8 -a subspace of complexified octonions (for octonions see [60]), which I have used to call hyper-octonions- and $M^4 \times CP_2$ can be regarded as physically equivalent: this "number theoretical compactification" is analogous to spontaneous compactification in M-theory. No dynamical compactification takes place so that $M^8 - H$ duality is a more appropriate term.

5.2 Physics as generalized number theory

Physics as a generalized number theory (for an overview about number theory see [59]) program consists of three separate threads: various p-adic physics and their fusion together with real number based physics to a larger structure [36] ,[50] , the attempt to understand basic physics in terms of classical number fields [37] ,[47] (in particular, identifying associativity condition as the basic dynamical principle), and infinite primes [35] ,[44] , whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory. In this article a summary of the philosophical ideas behind this dream and a summary of the technical challenges and proposed means to meet them are discussed.

The construction of p-adic physics and real physics poses formidable looking technical challenges: p-adic physics should make sense both at the level of the imbedding space, the "world of classical worlds" (WCW), and space-time and these physics should allow a fusion to a larger coherent whole. This forces to generalize the notion of number by fusing reals and p-adics along rationals and common algebraic numbers. The basic problem that one encounters is definition of the definite integrals and harmonic analysis [56] in the p-adic context [20]. It turns out that the representability of WCW as a union of symmetric spaces [65] provides a universal group theoretic solution not only to the construction of the Kähler geometry of WCW but also to this problem. The p-adic counterpart of a symmetric space is obtained from its discrete invariant by replacing discrete points with p-adic variants of the continuous symmetric space. Fourier analysis [56] reduces integration to summation. If one wants to define also integrals at space-time level, one must pose additional strong constraints which effectively reduce the partonic 2-surfaces and perhaps even space-time surfaces to finite geometries and allow assign to a given partonic 2-surface a unique power of a unique p-adic prime characterizing the measurement resolution in angle variables. These integrals might make sense in the intersection of real and p-adic worlds defined by algebraic surfaces.

The dimensions of partonic 2-surface, space-time surface, and imbedding space suggest that classical number fields might be highly relevant for quantum TGD. The recent view about the connection is based on hyper-octonionic representation of the imbedding space gamma matrices, and the notions of associative and co-associative space-time regions defined as regions for which the modified gamma matrices span quaternionic or co-quaternionic plane at each point of the region. A further condition is that the tangent space at each point of space-time surface contains a preferred hyper-complex (and thus commutative) plane identifiable as the plane of non-physical polarizations so that gauge invariance has a purely number theoretic interpretation. WCW can be regarded as the space of sub-algebras of the local octonionic Clifford algebra [54] of the imbedding space defined by space-time surfaces with the property that the local sub-Clifford algebra spanned by Clifford algebra valued functions restricted at them is associative or co-associative in a given region.

The recipe for constructing infinite primes is structurally equivalent with a repeated second quantization of an arithmetic super-symmetric quantum field theory. At the lowest level one has fermionic and bosonic states labeled by finite primes and infinite primes correspond to many particle states of this theory. Also infinite primes analogous to bound states are predicted. This hierarchy of quantizations can be continued indefinitely by taking the many particle states of the previous level as elementary particles at the next level. Construction could make sense also for hyper-quaternionic and hyper-octonionic primes although non-commutativity and non-associativity pose technical challenges. One can also construct infinite number of real units as ratios of infinite integers with a precise number theoretic anatomy. The fascinating finding is that the quantum states labeled by standard model quantum numbers allow a representation as wave fuctions in the discrete space of these units. Space-time point becomes infinitely richly structured in the sense that one can associate to it a wave function in the space of real (or octonionic) units allowing to represent the WCW spinor fields. One can speak about algebraic holography or number theoretic Brahman=Atman identity and one can also say that the points of imbedding space and space-time surface are subject to a number theoretic evolution.

5.3 Questions

The experience has shown repeatedly that a correct question and identification of some weakness of existing vision is what can only lead to a genuine progress. In the following I discuss the basic questions, which have stimulated progress in the challenge of constructing WCW geometry.

5.3.1 What is WCW?

Concerning the identification of WCW I have made several guesses and the progress has been basically due to the gradual realization of various physical constraints and the fact that standard physics ontology is not enough in TGD framework.

1. The first guess was that WCW corresponds to all possible space-like 3-surfaces in $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and CP_2 denotes complex projective space of two complex dimensions having also representation as coset space $SU(3)/U(2)$ (see the separate article summarizing the basic facts about CP_2 and how it codes for standard model symmetries [43] ,[48, 43]). What led to the this particular choice H was the observation that the geometry of H codes for standard model quantum numbers and that the generalization of particle from point like particle to 3-surface allows to understand also remaining quantum numbers having no obvious explanation in standard model (family replication phenomenon). What is important to notice is that Poincare symmetries act as exact symmetries of M^4 rather than space-time surface itself: this realizes the basic vision about Poincare invariant theory of gravitation. This lifting of symmetries to the level of imbedding space and the new dynamical degrees of freedom brought by the sub-manifold geometry of space-time surface are absolutely essential for entire quantum TGD and distinguish it from

general relativity and string models. There is however a problem: it is not obvious how to get cosmology.

2. The second guess was that WCW consists of space-like 3-surfaces in $H_+ = M^4_+ \times CP2$, where M^4_+ future light-cone having interpretation as Big Bang cosmology at the limit of vanishing mass density with light-cone property time identified as the cosmic time. One obtains cosmology but loses exact Poincare invariance in cosmological scales since translations lead out of future light-cone. This as such has no practical significance but due to the metric 2-dimensionality of light-cone boundary δM^4_+ the conformal symmetries of string model assignable to finite-dimensional Lie group generalize to conformal symmetries assignable to an infinite-dimensional symplectic group of $S^2 \times CP_2$ and also localized with respect to the coordinates of 3-surface. These symmetries are simply too beautiful to be important only at the moment of Big Bang and must be present also in elementary particle length scales. Note that these symmetries are present only for 4-D Minkowski space so that a partial resolution of the old conundrum about why space-time dimension is just four emerges.

3. The third guess was that the light-like 3-surfaces in H or H_+ are more attractive than space-like 3-surfaces. The reason is that the infinite-D conformal symmetries characterize also light-like 3-surfaces because they are metrically 2-dimensional. This leads to a generalization of Kac-Moody symmetries [57] of super string models with finite-dimensional Lie group replaced with the group of isometries of H. The natural identification of light-like 3-surfaces is as 3-D surfaces defining the regions at which the signature of the induced metric changes from Minkowskian $(1, -1, -1, -1)$ to Euclidian $(-1-1-1-1)$- I will refer these surfaces as throats or wormhole throats in the sequel. Light-like 3-surfaces are analogous to blackhole horizons and are static because strong gravity makes them light-like. Therefore also the dimension 4 for the space-time surface is unique.

   This identification leads also to a rather unexpected physical interpretation. Single lightlike wormhole throat carriers elementary particle quantum numbers. Fermions and their superpartners are obtained by glueing Euclidian regions (deformations of so called CP_2 type vacuum extremals of Kähhler action) to the background with Minkowskian signature. Bosons are identified as wormhole contacts with two throats carrying fermion *resp.* antifermionic quantum numbers. These can be identified as deformations of CP_2 vacuum extremals between between two parallel Minkowskian space-time sheets. One can say that bosons and their superpartners emerge. This has dramatic implications for quantum TGD [6] and QFT limit of TGD [26].

   The question is whether one obtains also a generalization of Feynman diagrams. The answer is affirmative. Light-like 3-surfaces or corresponding Euclidian regions of space-time are analogous to the lines of Feynman diagram and vertices are replaced by 2-D surface at which these surfaces glued together. One can speak about Feynman diagrams with lines thicknened to light-like 3-surfaces and vertices to 2-surfaces. The generalized Feynman diagrams are singular as 3-manifolds but the vertices are non-singular as 2-manifolds. Same applies to the corresponding space-time surfaces and space-like 3-surfaces. Therefore one can say that WCW consists of generalized Feynman diagrams-something rather different from the original identification as space-like 3-surfaces and one can wonder whether these identification could be equvalent.

4. The fourth guess was a generalization of the WCW combining the nice aspects of the identifications $H = M^4 \times CP_2$ (exact Poincare invariance) and $H = M^4_+ \times CP_2$ (Big Bang cosmology). The idea was to generalize WCW to a union of basic building bricks -causal diamonds (CDs) - which themselves are analogous to Big Bang-Big Crunch cosmologies breaking Poincare invariance, which is however regained by the allowance of union of Poincare trnsforms of the causal diamonds.

   The starting point is General Coordinate Invariance (GCI). It does not matter, which 3-D slice of the space-time surface one choose to represent physical data as long as slices are related by a diffeomorphism of the space-time surface. This condition implies holography in the sense that 3-D slices define holograms about 4-D reality.

   The question is whether one could generalize GCI in the sense that the descriptions using space-like and light-like 3-surfaces would be equivalent physically. This requires that finite-sized space-like

3-surfaces are somehow equivalent with light-like 3-surfaces. This suggests that the light-like 3-surfaces must have ends. Same must be true for the space-time surfaces and must define preferred space-like 3-surfaces just like wormhole throats do. This makes sense only if the 2-D intersections of these two kinds of 3-surfaces -call them partonic 2-surfaces- and their 4-D tangent spaces carry the information about quantum physics. A strenghening of holopraphy principle would be the outcome. The challenge is to understand, where the intersections defining the partonic 2-surfaces are located.

Zero energy ontology (ZEO) allows to meet this challenge.

(a) Assume that WCW is union of sub-WCWs identified as the space of light-like 3-surfaces assignable to $CD \times CP_2$ with given CD defined as an intersection of future and past directed lightcones of M^4. The tips of CDs have localization in M^4 and one can perform for CD both translations and Lorentz boost for CDs. Space-time surfaces inside CD define the basic building brick of WCW. Also unions of CDs allowed and the CDs belonging to the union can intersect. One can of course consider the possibility of intersections and analogy with the set theoretic realization of topology.

(b) ZEO property means that the light-like boundaries of these objects carry positive and negative energy states, whose quantum numbers are opposite. Everything can be created from vacuum and can be regarded as quantum fluctuations in the standard vocabulary of quantum field theories.

(c) Space-time surfaces inside CDs begin from the lower boundary and end to the upper boundary and in ZEO it is natural to identify space-like 3-surfaces as pairs of space-like 3-surfaces at these boundaries. Light-like 3-surfaces connect these boundaries.

(d) The generalization of GCI states that the descriptions based on space-like 3-surfaces must be equivalent with that based on light-like 3-surfaces. Therefore only the 2-D intersections of light-like and space-like 3-surfaces - partonic 2-surfaces- and their 4-D tangent spaces (4-surface is there!) matter. Effective 2-dimensionality means a strengthened form of holography but does not imply exact 2-dimensionality, which would reduce the theory to a mere string model like theory. Once these data are given, the 4-D space-time surface is fixed and is analogous to a generalization of Bohr orbit to infinite-D context. This is the first guess. The situation is actually more delicate due to the non-determinism of Kähler action motivating the interaction of the hierarchy of CDs within CDs.

In this framework one obtains cosmology: CDs represent a fractal hierarchy of big bang-big crunch cosmologies. One obtains also Poincare invariance. One can also interpret the non-conservation of gravitational energy in cosmology which is an empirical fact but in conflict with exact Poincare invariance as it is realized in positive energy ontology [39, 32]. The reason is that energy and four-momentum in zero energy ontology correspond to those assignable to the positive energy part of the zero energy state of a particular CD. The density of energy as cosmologist defines it is the statistical average for given CD: this includes the contibutions of sub-CDs. This average density is expected to depend on the size scale of CD density is should therefore change as quantum dispersion in the moduli space of CDs takes place and leads to large time scale for any fixed sub-CD.

Even more, one obtains actually quantum cosmology! There is large variety of CDs since they have position in M^4 and Lorentz transformations change their shape. The first question is whether the M^4 positions of both tips of CD can be free so that one could assign to both tips of CD momentum eigenstates with opposite signs of four-momentum. The proposal, which might look somewhat strange, is that this not the case and that the proper time distance between the tips is quantized in octaves of a fundamental time scale $T = R/c$ defined by CP_2 size R. This would explains p-adic length scale hypothesis which is behind most quantitative predictions of TGD. That the time scales assignable to the CD of elementary particles correspond to biologically important time scales [8] forces to take this hypothesis very seriously.

The interpretation for T could be as a cosmic time quantized in powers of two. Even more general quantization is proposed to take place. The relative position of the second tip with respect to the first defines a point of the proper time constant hyperboloid of the future light cone. The hypothesis is that one must replace this hyperboloid with a lattice like structure. This implies very powerful cosmological predictions finding experimental support from the quantization of redshifts for instance [32]. For quite recent further empirical support see [100].

One should not take this argument without a grain of salt. Can one really realize zero energy ontology in this framework? The geometric picture is that translations correspond to translations of CDs. Translations should be done independently for the upper and lower tip of CD if one wants to speak about zero energy states but this is not possible if the proper time distance is quantized. If the relative M^4_+ coordinate is discrete, this pessimistic conclusion is strengthened further.

The manner to get rid of problem is to assume that translations are represented by quantum operators acting on states at the light-like boundaries. This is just what standard quantum theory assumes. An alternative- purely geometric- way out of difficulty is the Kac-Moody symmetry associated with light-like 3-surfaces meaning that local M^4 translations depending on the point of partonic 2-surface are gauge symmetries. For a given translation leading out of CD this gauge symmetry allows to make a compensating transformation which allows to satisfy the constraint.

This picture is roughly the recent view about WCW. What deserves to be emphasized is that a very concrete connection with basic structures of quantum field theory emerges already at the level of basic objects of the theory and GCI implies a strong form of holography and almost stringy picture.

5.3.2 Some Why's

In the following I try to summarize the basic motivations behind quantum TGD in form of various Why's.

1. Why WCW?

   Einstein's program has been extremely successful at the level of classical physics. Fusion of general relativity and quantum theory has however failed. The generalization of Einstein's geometrization program of physics from classical physics to quantum physics gives excellent hopes about the success in this project. Infinite-dimensional geometries are highly unique and this gives hopes about fixing the physics completetely from the uniqueness of the infinite-dimensional Kähler geometric existence.

2. Why spinor structure in WCW?

   Gamma matrices defining the Clifford algebra [54] of WCW are expressible in terms of fermionic oscillator operators. This is obviously something new as compared to the view about gamma matrices as bosonic objects. There is however no deep reason denying this kind of identification. As a consequence, a geometrization of fermionic oscillator operator algebra and fermionic statistics follows as also geometrization of super-conformal symmetries [64, 57] since gamma matrices define super-generators of the algebra of WCW isometries extended to a super-algebra.

3. Why Kähler geometry?

   Geometrization of the bosonic oscillator operators in terms of WCW vector fields and fermionic oscillator operators in terms of gamma matrices spanning Clifford algebra. Gamma matrices span hyper-finite factor of type II_1 and the extremely beautiful properties of these von Neuman algebras [71] (one of the three von Neuman algebras that von Neumann suggests as possible mathematical frameworks behind quantum theory) lead to a direct connection with the basic structures of modern physics (quantum groups, non-commutative geometries,.. [75]).

   A further reason why is the finiteness of the theory.

   (a) In standard QFTs there are two kinds of divergences. Action is a local functional of fields in 4-D sense and one performs path integral over **all** 4-surfaces to construct S-matrix. Mathematically path integration is a poorly defined procedure and one obtains diverging Gaussian determinants and divergences due to the local interaction vertices. Regularization provides the manner to get rid of the infinities but makes the theory very ugly.

(b) Kähler function defining the Kähler geometry is a expected to be non-local functional of the partonic 2-surface (Kähler action for a preferred extremal having as its ends the positive and negative energy 3-surfaces). Path integral is replaced with a functional integral which is mathematically well-defined procedure and one perfoms functional integral only over the partonic 2-surfaces rather than all 4-surfaces (holography). The exponent of Kähler function defines a unique vacuum functional. The local divergences of local quantum field theories of local quantum field theories since there are no local interaction vertices. Also the divergences associated with the Gaussian determinant and metric determinant cancel since these two determinants cancel each other in the integration over WCW. As a matter fact, symmetric space property suggest a much more elegant manner to perform the functional integral by reducing it to harmonic analysis in infinite-dimensional symmetric space [11].

(c) One can imagine also the possibility of divergences in fermionic degrees of freedom but it has turned out that the generalized Feynman diagrams in ZEO are manifestly finite. Even more: it is quite possible that only finite number of these diagrams give non-vanishing contributions to the scattering amplitude. This is essentially due to the new view about virtual particles, which are identified as bound states of on mass shell states assigned with the throats of wormhole contacts so that the integration over loop momenta of virtual particles is extremely restricted [11].

4. Why infinite-dimensional symmetries?

   WCW must be a union of symmetric spaces in order that the Riemann connection exists (this generalizes the finding of Freed for loop groups [72]). Since the points of symmetric spaces are metrically equivalent, the geometrization becomes tractable although the dimension is infinite. A union of symmetric spaces is required because 3-surfaces with a size of galaxy and electron cannot be metrically equivalent. Zero modes distinguish these surfaces and can be regarded as purely classical degrees of freedom whereas the degrees of freedom contributing to the WCW line element are quantum fluctuating degrees of freedom.

   One immediate implication of the symmetric space property is constant curvature space property meaning that the Ricci tensor proportional to metric tensor. Infinite-dimensionality means that Ricci scalar either vanishes or is infinite. This implies vanishing of Ricci tensor and vacuum Einstein equations for WCW.

5. Why $M^4 \times CP_2$?

   This choice provides an explanation for standard model quantum numbers. The conjecture is that infinite-D geometry of 3-surfaces exists only for this choice. As noticed, the dimension of space-time surfaces and M^4 fixed by the requirement of generalized conformal invariance [63] making possible to achieve symmetric space property. If $M^4 \times CP_2$ is so special, there must be a good reason for this. Number theoretical vision [37] ,[47] indeed leads to the identification of this reason. One can assign the hierarchy of dimensions associated with partonic 2-surfaces, space-time surfaces and imbedding space to classical number fields and can assign to imbedding space what might be called hyper-octonionic structure. "Hyper" comes from the fact that the tangent space of H corresponds to the subspaces of complexified octonions with octonionic imaginary units multiplied by a commuting imaginary unit. The space-time reions would be either hyper-quaternionic or co-hyper-quaternionic so that associativity/co-associativity would become the basic dynamical principle at the level of space-time dynamics. Whether this dynamical principle is equivalent with the preferred extremal property of Kähler action remains an open conjecture.

6. Why zero energy ontology and why causal diamonds?

   The consistency between Poincare invariance and GRT requires ZEO. In positive energy ontology only one of the infinite number of classical solutions is realized and partially fixed by the values of conserved quantum numbers so that the theory becomes obsolote. Even in quantum theory conservation laws mean that only those solutions of field equations with the quantum numbers of the initial state of the Universe are interesting and one faces the problem of understanding what the the initial state of the universe was. In ZEO these problems disappear. Everything is creatable from vacuum: if the physical state is mathematically realizable it is in principle reachable by a sequence of quantum jumps. There are no physically non-reachable entities in the theory. Zero energy ontology leads also to a fusion of thermodynamics with quantum theory. Zero energy states ae defined as entangled states of positive and negative energy states and entanglement coefficients define what I call M-matrix identified as "complex square root" of density matrix expressible as a product of diagonal real and positive density matrix and unitary S-matrix [6].

There are several good reasons why for causal diamonds. ZEO requires CDs, the generalized form of GCI and strong form of holography (light-like and space-like 3-surfaces are physically equivalent representations) require CDs, and also the view about light-like 3-surfaces as generalized Feynman diagrams requires CDs. Also the classical non-determinism of Kähler action can be understood using the hierarchy CDs and the addition of CDs inside CDs to obtain a fractal hierarchy of them provides an elegant manner to undersand radiative corrections and coupling constant evolution in TGD framework.

A strong physical argument in favor of CDs is the finding that the quantized proper time distance between the tips of CD fixed to be an octave of a fundamental time scale defined by CP_2 happens to define fundamental biological time scale for electron, u quark and d quark [8] : there would be a deep connection between elementary particle physics and living matter leading to testable predictions.

5.4 Modified Dirac action

The construction of the spinor structure for the world of classical worlds (WCW) leads to the vision that second quantized modified Dirac equation codes for the entire quantum TGD. Among other things this would mean that Dirac determinant would define the vacuum functional of the theory having interpretation as the exponent of Kähler function of WCW and Kähler function would reduce to Kähler action for a preferred extremal of Kähler action. In the following the recent view about the modified Dirac action are explained in more detail.

5.4.1 Identification of the modified Dirac action

The modified Dirac action action involves several terms. The first one is 4-dimensional assignable to Kähler action. Second term is instanton term reducible to an expression restricted to wormhole throats or any light-like 3-surfaces parallel to them in the slicing of space-time surface by light-like 3-surfaces. The third term is assignable to Chern-Simons term and has interpretation as a measurement interaction term linear in Cartan algebra of the isometry group of the imbedding space in order to obtain stringy propagators and also to realize coupling between the quantum numbers associated with super-conformal representations and space-time geometry required by quantum classical correspondence.

This means that 3-D light-like wormhole throats carry induced spinor field which can be regarded as independent degrees of freedom having the spinor fields at partonic 2-surfaces as sources and acting as 3-D sources for the 4-D induced spinor field. The most general measurement interaction would involve the corresponding coupling also for Kähler action but is not physically motivated. There are good arguments in favor of Chern-Simons Dirac action and corresponding measurement interaction.

1. A correlation between 4-D geometry of space-time sheet and quantum numbers is achieved by the identification of exponent of Kähler function as Dirac determinant making possible the entanglement of classical degrees of freedom in the interior of space-time sheet with quantum numbers.

2. Cartan algebra plays a key role not only at quantum level but also at the level of space-time geometry since quantum critical conserved currents vanish for Cartan algebra of isometries and the measurement interaction terms giving rise to conserved currents are possible only for Cartan algebras. Furthermore, modified Dirac equation makes sense only for eigen states of Cartan algebra generators. The hierarchy of Planck constants realized in terms of the book like structure of the generalized imbedding space assigns to each CD (causal diamond) preferred Cartan algebra: in case of Poincare algebra there are two of them corresponding to linear and cylindrical M^4 coordinates.

3. Quantum holography and dimensional reduction hierarchy in which partonic 2-surface defined fermionic sources for 3-D fermionic fields at light-like 3-surfaces Y_l^3 in turn defining fermionic sources for 4-D spinors find an elegant realization. Effective 2-dimensionality is achieved if the replacement of light-like wormhole throat X_l^3 with light-like 3-surface Y_l^3 "parallel" with it in the definition of Dirac determinant corresponds to the $U(1)$ gauge transformation $K \rightarrow K + f + \overline{f}$ for Kähler function of WCW so that WCW Kähler metric is not affected. Here f is holomorphic function of WCW ("world of classical worlds") complex coordinates and arbitrary function of zero mode coordinates.

4. An elegant description of the interaction between super-conformal representations realized at partonic 2-surfaces and dynamics of space-time surfaces is achieved since the values of Cartan charges are feeded to the 3-D Dirac equation which also receives mass term at the same time. Almost topological QFT at wormhole throats results at the limit when four-momenta vanish: this is in accordance with the original vision about TGD as almost topological QFT.

5. A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

6. The inclusion of imaginary instanton term to the definition of the modified gamma matrices is not consistent with the conjugation of the induced spinor fields. Measurement interaction can be however assigned to both Kähler action and its instanton term. CP breaking, irreversibility and the space-time description of dissipation are closely related and the CP and T oddness of the instanton part of the measurement interaction term could provide first level description for dissipative effects. It must be however emphasized that the mere addition of instanton term to Kähler function could be enough.

7. A radically new view about matter antimatter asymmetry based on zero energy ontology emerges and one could understand the experimental absence of antimatter as being due to the fact antimatter corresponds to negative energy states. The identification of bosons as wormhole contacts is the only possible option in this framework.

8. Almost stringy propagators and a consistency with the identification of wormhole throats as lines of generalized Feynman diagrams is achieved. The notion of bosonic emergence leads to a long sought general master formula for the M-matrix elements. The counterpart for fermionic loop defining bosonic inverse propagator at QFT limit is wormhole contact with fermion and cutoffs in mass squared and hyperbolic angle for loop momenta of fermion and antifermion in the rest system of emitting boson have precise geometric counterpart.

5.4.2 Hyper-quaternionicity and quantum criticality

The conjecture that quantum critical space-time surfaces are hyper-quaternionic in the sense that the modified gamma matrices span a quaternionic subspace of complexified octonions at each point of the space-time surface is consistent with what is known about preferred extremals. The condition that both the modified gamma matrices and spinors are quaternionic at each point of the space-time surface leads to a precise ansatz for the general solution of the modified Dirac equation making sense also in the real context. The octonionic version of the modified Dirac equation is very simple since $SO(7,1)$ as vielbein group is replaced with G_2 acting as automorphisms of octonions so that only the neutral Abelian part of the classical electro-weak gauge fields survives the map.

Octonionic gamma matrices provide also a non-associative representation for the 8-D version of Pauli sigma matrices and encourage the identification of 8-D twistors as pairs of octonionic spinors conjectured to be highly relevant also for quantum TGD. Quaternionicity condition implies that octo-twistors reduce to something closely related to ordinary twistors.

5.4.3 The exponent of Kähler function as Dirac determinant for the modified Dirac action

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

1. The Dirac determinant defined by the product of Dirac determinants associated with the light-like partonic 3-surfaces X_l^3 associated with a given space-time sheet X^4 is the simplest candidate for vacuum functional identifiable as the exponent of the Kähler function. Individual Dirac determinant is defined as the product of eigenvalues of the dimensionally reduced modified Dirac operator $D_{K,3}$ and there are good arguments suggesting that the number of eigenvalues is finite. p-Adicization requires that the eigenvalues belong to a given algebraic extension of rationals. This restriction would imply a hierarchy of physics corresponding to different extensions and could automatically imply the finiteness and algebraic number property of the Dirac determinants if only finite number of eigenvalues would contribute. The regularization would be performed by physics itself if this were the case.

2. It remains to be proven that the product of eigenvalues gives rise to the exponent of Kähler action for the preferred extremal of Kähler action. At this moment the only justification for the conjecture is that this the only thing that one can imagine.

3. A long-standing conjecture has been that the zeros of Riemann Zeta are somehow relevant for quantum TGD. Rieman zeta is however naturally replaced Dirac zeta defined by the eigenvalues of $D_{K,3}$ and closely related to Riemann Zeta since the spectrum consists essentially for the cyclotron energy spectra for localized solutions region of non-vanishing induced Kähler magnetic field and hence is in good approximation integer valued up to some cutoff integer. In zero energy ontology the Dirac zeta function associated with these eigenvalues defines "square root" of thermodynamics assuming that the energy levels of the system in question are expressible as logarithms of the eigenvalues of the modified Dirac operator defining kind of fundamental constants. Critical points correspond to approximate zeros of Dirac zeta and if Kähler function vanishes at criticality as it indeed should, the thermal energies at critical points are in first order approximation proportional to zeros themselves so that a connection between quantum criticality and approximate zeros of Dirac zeta emerges.

4. The discretization induced by the number theoretic braids reduces the world of classical worlds to effectively finite-dimensional space and configuration space Clifford algebra reduces to a finite-dimensional algebra. The interpretation is in terms of finite measurement resolution represented in terms of Jones inclusion $\mathcal{M} \subset \mathcal{N}$ of HFFs with $\mathcal{M}$ taking the role of complex numbers. The finite-D quantum Clifford algebra spanned by fermionic oscillator operators is identified as a representation for the coset space $\mathcal{N}/\mathcal{M}$ describing physical states modulo measurement resolution. In the sectors of generalized imbedding space corresponding to non-standard values of Planck constant quantum version of Clifford algebra is in question.

5.5 Three Dirac operators and their interpretation

The physical interpretation of Kähler Dirac equation is not at all straightforward. The following arguments inspired by effective 2-dimensionality suggest that the modified gamma matrices and corresponding effective metric could allow dual gravitational description of the physics associated with wormhole throats. This applies in particular to condensed matter physics.

5.5.1 Three Dirac equations

To begin with, Dirac equation appears in three forms in TGD.

1. The Dirac equation in world of classical worlds codes for the super Virasoro conditions for the super Kac-Moody and similar representations formed by the states of wormhole contacts forming the

counterpart of string like objects (throats correspond to the ends of the string. This Dirac generalizes the Dirac of 8-D imbedding space by bringing in vibrational degrees of freedom. This Dirac equation should gives as its solutions zero energy states and corresponding M-matrices generalizing S-matrix and their collection defining the unitary U-matrix whose natural application appears in consciousness theory as a coder of what Penrose calls U-process.

2. There is generalized eigenvalue equation for Chern-Simons Dirac operator at light-like wormhole throats. The generalized eigenvalue is $p^k\gamma_k$. The interpretation of pseudo-momentum p^k has been a problem but twistor Grassmannian approach suggests strongly that it can be interpreted as the counterpart of equally mysterious region momentum appearing in momentum twistor Grassmannian approach to $\mathcal{N} = 4$ SYM. The pseudo-/region momentum p is quantized (this does not spoil the basics of Grasssmannian residues integral approach) and $1/p^k\gamma_k$ defines propagator in lines of generalized Feynman diagrams. The Yangian symmetry discovered generalizes in a very straightforward manner and leads alsoto the realization that TGD could allow also a twistorial formulation in terms of product $CP_3 \times CP_3$ of two twistor spaces [42]. General arguments lead to a proposal for explicit form for the solutions of field equation represented identified as holomorphic 6-surfaces in this space subject to additional partial different equations for homogenenous functions of projective twistor coordinates suggesting strongly the quantal interpretation as analogs of partial waves. Therefore quantum-classical correspondence would be realize in beatiful manner.

3. There is Kähler Dirac equation in the interior of space-time. In this equation the gamma matrices are replaced with modified gamma matrices defined by the contractions of canonical momentum currents $T^\alpha_k = \partial L/\partial_\alpha h^k$ with imbedding space gamma matrices Γ_k. This replacement is required by internal consistency and by super-conformal symmetries.

Could Kähler Dirac equation provide a first principle justification for the light-hearted use of effective mass and the analog of Dirac equation in condensed manner physics? This would conform with the holographic philosophy. Partonic 2-surfaces with tangent space data and their light-like orbits would give hologram like representation of physics and the interior of space-time the 4-D representation of physics. Holography would have in the recent situation interpretation also as quantum classical correspondence between representations of physics in terms of quantized spinor fields at the light-like 3-surfaces on one hand and in terms of classical fields on the other hand.

The resulting dispersion relation for the square of the Kähler-Dirac operator assuming that induced like metric, Kähler field, etc. are very slowly varying contains quadratic and linear terms in momentum components plus a term corresponding to magnetic moment coupling. In general massive dispersion relation is obtained as is also clear from the fact that Kähler Dirac gamma matrices are combinations of M^4 and CP_2 gammas so that modified Dirac mixes different M^4 chiralities (basic signal for massivation). If one takes into account the dependence of the induced geometric quantities on space-time point dispersion relations become non-local.

5.5.2 Does energy metric provide the gravitational dual for condensed matter systems?

The modified gamma matrices define an effective metric via their anticommutators which are quadratic in components of energy momentum tensor (canonical momentum densities). This effective metric vanishes for vacuum extremals. Note that the use of modified gamma matrices guarantees among other things internal consistency and super-conformal symmetries of the theory. The physical interpretation has remained obscure hitherto although corresponding effective metric for Chern-Simons Dirac action has now a clear physical interpretation.

If the above argument is on the right track, this effective metric should have applications in condensed matter theory. In fact, energy metric has a natural interpretation in terms of effective light velocities which depend on direction of propagation. One can diagonalize the energy metric $g_e^{\alpha\beta}$ (contravariant form results from the anticommutators) and one can denote its eigenvalues by (v_0, v_i) in the case that the signature of the effective metric is $(1, -1, -1, -1)$. The 3-vector v_i/v_0has interpretation as components of effective light velocity in various directions as becomes clear by thinking the d'Alembert equation for the energy metric. This velocity field could be interpreted as that of hydrodynamic flow. The study of the extremals of Kähler action shows that if this flow is actually Beltrami flow so that the flow parameter associated with the flow lines extends to global coordinate, Kähler action reduces to a 3-D Chern-Simons action and one obtains effective topological QFT. The conserved fermion current $\overline{\Psi}\Gamma_e^\alpha\Psi$ has interpretation as incompressible hydrodynamical flow.

This would give also a nice analogy with AdS/CFT correspondence allowing to describe various kinds of physical systems in terms of higher-dimensional gravitation and black holes are introduced quite routinely to describe condensed matter systems. In TGD framework one would have an analogous situation but with 10-D space-time replaced with the interior of 4-D space-time and the boundary of AdS representing Minkowski space with the light-like 3-surfaces carrying matter. The effective gravitation would correspond to the "energy metric". One can associate with it curvature tensor, Ricci tensor and Einstein tensor using standard formulas and identify effective energy momentum tensor associated as Einstein tensor with effective Newton's constant appearing as constant of proportionality. Note however that the besides ordinary metric and "energy" metric one would have also the induced classical gauge fields having purely geometric interpretation and action would be Kähler action. This 4-D holography would provide a precise, dramatically simpler, and also a very concrete dual description. This cannot be said about model of graphene based on the introduction of 10-dimensional black holes, branes, and strings chosen in more or less ad hoc manner.

This raises questions. Does this give a general dual gravitational description of dissipative effects in terms of the "energy" metric and induced gauge fields? Does one obtain the counterparts of black holes? Do the general theorems of general relativity about the irreversible evolution leading to black holes generalize to describe analogous fate of condensed matter systems caused by dissipation? Can one describe non-equilibrium thermodynamics and self-organization in this manner?

One might argue that the incompressible Beltrami flow defined by the dynamics of the preferred extremals is dissipationless and viscosity must therefore vanish locally. The failure of complete non-determinism of Kähler action however means generation of entropy since the knowledge about the state decreases gradually. This in turn should have a phenomenological local description in terms of viscosity which characterizes the transfer of energy to shorter scales and eventually to radiation. The deeper description should be non-local and basically topological and might lead to quantization rules. For instance, one can imagine the quantization of the ratio η/s of the viscosity to entropy density as multiples of a basic unit defined by its lower bound (note that this would be analogous to Quantum Hall effect). For the first M-theory inspired derivation of the lower bound of η/s [99]. The lower bound for η/s is satisfied in good approximation by what should have been QCD plasma but found to be something different (RHIC and the first evidence for new physics from LHC [18]).

An encouraring sign comes from the observation that for so called massless extremals representing classically arbitrarily shaped pulses of radiation propagating without dissipation and dispersion along single direction the canonical momentum currents are light-like. The effective contravariant metric vanishes identically so that fermions cannot propate in the interior of massless extremals! This is of course the case also for vacuum extremals. Massless extremals are purely bosonic and represent bosonic radiation. Many-sheeted space-time decomposes into matter containing regions and radiation containing regions. Note that when wormhole contact (particle) is glued to a massless extremal, it is deformed so that CP_2 projection becomes 4-D guaranteing that the weak form of electric magnetic duality can be satisfied. Therefore massless extremals can be seen as asymptotic regions. Perhaps one could say that dissipation corresponds to a decoherence process creating space-time sheets consisting of matter and radiation. Those containing matter might be even seen as analogs blackholes as far as energy metric is considered.

5.5.3 Preferred extremals as perfect fluids

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional conform the discovery [94]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [97]. In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [96].

1. The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gardient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3}\partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{5.1}$$

   The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{5.2}$$

   From $dF^i = T^{ij}S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This termm is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

2. The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} \ . \tag{5.3}$$

   The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p) u^\alpha u^\beta + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{5.4}$$

   Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentions in the definition of Minkowski metric $((1,-1,-1,-1))$.

3. If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parametger. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p) g^{tt} \delta^\alpha_t \delta^\beta_t + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{5.5}$$

   In this case the vanishing of the viscous term means that one has perfect fluid in strong sense.

   The existence of a global flow parameter means that one has

$$v_i = \Psi \partial_i \Phi \ . \tag{5.6}$$

   Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} . \tag{5.7}$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in approriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

1. Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the "topological" half of Maxwell's equations (Faraday's induction law and the statement that no non-topological magnetic are possible) is satisfied.

2. Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The Euclidian regions (or wormhole throats, depends on one's tastes) define what I identify as generalized Feynman diagrams.

   Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x. What causes the failure of the exact perfect fluid property?

1. Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At imbedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).

2. The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of super-conductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.

   The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper}

at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is feeded into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.

This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.

3. The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of $\hbar$ can be large in some situations so that the quantal character of dissipation could become visible even macroscopically. Whether this a situation with large $\hbar$ is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

1. The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by $v_{\parallel}$ *resp.* $v_{\perp}$ the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.

2. The flow of momentum component parallel to the main flow due to the absorbed at S is

$$n_{abs} m v_{\parallel} v_{\perp} S \quad .$$

This momentum flow must be equal to the viscous force

$$F_{visc} = \eta \frac{v_{\parallel}}{d} \times S \quad .$$

From this one obtains

$$\eta = n_{abs} m v_{\perp} d \quad .$$

If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

so that one has

$$\frac{\eta}{s} = m v_{\perp} d \quad .$$

This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x, Angular momentum quantization suggests strongly itself as origin of the quantization.

3. Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities $v_\perp$. Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.

   Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv \ D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon}$$

   Quantization condition would give

$$\epsilon = 4\pi \quad .$$

   One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increasess suggesting that also the radius of the vortices scales like d.

4. One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

5.5.4 Is the effective metric one- or two-dimensional?

The following argument suggests that the effective metric defined by the anti-commutators of the modified gamma matrices is effectively one- or two-dimensional. Effective one-dimensionality would conform with the observation that the solutions of the modified Dirac equations can be localized to one-dimensional world lines in accordance with the vision that finite measurement resolution implies discretization reducing partonic many-particle states to quantum superpositions of braids. This localization to 1-D curves occurs always at the 3-D orbits of the partonic 2-surfaces.

The argument is based on the following assumptions.

1. The modified gamma matrices for Kähler action are contractions of the canonical momentum densities T^α_k with the gamma matrices of H.

2. The strongest assumption is that the isometry currents

$$J^{A\alpha} = T^\alpha_k j^{Ak}$$

   for the preferred extremals of Kähler action are of form

$$J^{A\alpha} = \Psi^A (\nabla\Phi)^\alpha \tag{5.8}$$

   with a common function Φ guaranteeing that the flow lines of the currents integrate to coordinate lines of single global coordinate variables (Beltrami property). Index raising is carried out by using the ordinary induced metric.

3. A weaker assumption is that one has two functions Φ_1 and Φ_2 assignable to the isometry currents of M^4 and CP_2 respectively.:

$$\begin{aligned} J_1^{A\alpha} &= \Psi_1^A (\nabla\Phi_1)^\alpha \ , \\ J_2^{A\alpha} &= \Psi_2^A (\nabla\Phi_2)^\alpha \ . \end{aligned} \tag{5.9}$$

The two functions Φ_1 and Φ_2 could define dual light-like curves spanning string world sheet. In this case one would have effective 2-dimensionality and decomposition to string world sheets [15]. Isometry invariance does not allow more that two independent scalar functions Φ_i.

Consider now the argument.

1. One can multiply both sides of this equation with j^{Ak} and sum over the index A labeling isometry currents for translations of M^4 and $SU(3)$ currents for CP_2. The tensor quantity $\sum_A j^{Ak} j^{Al}$ is invariant under isometries and must therefore satisfy

$$\sum_A \eta_{AB} j^{Ak} j^{Al} = h^{kl} \ , \tag{5.10}$$

where η_{AB} denotes the flat tangent space metric of H. In M^4 degrees of freedom this statement becomes obvious by using linear Minkowski coordinates. In the case of CP_2 one can first consider the simpler case $S^2 = CP_1 = SU(2)/U(1)$. The coset space property implies in standard complex coordinate transforming linearly under $U(1)$ that only the the isometry currents belonging to the complement of $U(1)$ in the sum contribute at the origin and the identity holds true at the origin and by the symmetric space property everywhere. Identity can be verified also directly in standard spherical coordinates. The argument generalizes to the case of $CP_2 = SU(3)/U(2)$ in an obvious manner.

2. In the most general case one obtains

$$\begin{aligned} T_1^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_1)^\alpha \equiv f_1^k (\nabla\Phi_1)^\alpha \ , \\ T_2^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_2)^\alpha \equiv f_2^k (\nabla\Phi_2)^\alpha \ . \end{aligned} \tag{5.11}$$

3. The effective metric given by the anti-commutator of the modified gamma matrices is in turn is given by

$$G^{\alpha\beta} = m_{kl} f_1^k f_1^l (\nabla\Phi_1)^\alpha (\nabla\Phi_1)^\beta + s_{kl} f_2^k f_2^l (\nabla\Phi_2)^\alpha (\nabla\Phi_2)^\beta \ . \tag{5.12}$$

The covariant form of the effective metric is effectively 1-dimensional for $\Phi_1 = \Phi_2$ in the sense that the only non-vanishing component of the covariant metric $G_{\alpha\beta}$ is diagonal component along the coordinate line defined by $\Phi \equiv \Phi_1 = \Phi_2$. Also the contravariant metric is effectively 1-dimensional since the index raising does not affect the rank of the tensor but depends on the other space-time coordinates. This would correspond to an effective reduction to a dynamics of point-like particles for given selection of braid points. For $\Phi_1 \neq \Phi_2$ the metric is effectively 2-dimensional and would correspond to stringy dynamics.

6 The role of twistors in quantum TGD

6.1 Could the Grassmannian program be realized in TGD framework?

In the following the TGD based modification of the approach based on zero energy ontology is discussed in some detail. It is found that pseudo-momenta are very much analogous to region momenta and the approach leading to discretization of pseudo-mass squared for virtual particles - and even the discretization of pseudo-momenta - is consistent with the Grassmannian approach in the simple case considered and allow to get rid of IR divergences. Also the possibility that the number of generalized Feynman diagrams contributing to a given scattering amplitude is finite so that the recursion formula for the scattering amplitudes would involve only a finite number of steps (maximum number of loops) is considered. One especially promising feature of the residue integral approach with discretized pseudo-momenta is that it makes sense also in the p-adic context in the simple special case discussed since residue integral reduces to momentum integral (summation) and lower-dimensional residue integral.

6.1.1 What Yangian symmetry could mean in TGD framework?

The loss of the Yangian symmetry in the integrations over the region momenta x^a ($p^a = x^{a+1} - x^a$) assigned to virtual momenta seems to be responsible for many ugly features. It is basically the source of IR divergences regulated by "moving out on the Coulomb branch theory" so that IR singularities remain the problem of the theory. This raises the question whether the loss of Yangian symmetry is the signature for the failure of QFT approach and whether the restriction of loop momentum integrations to avoid both kind of divergences might be a royal road beyond QFT. In TGD framework zero energy ontology indeed leads to to a concrete proposal based on the vision that virtual particles are something genuinely real.

The detailed picture is of course far from clear but to get an idea about what is involved one can look what kind of assumptions are needed if one wants to realize the dream that only a finite number of generalized Feynman diagrams contribute to a scattering amplitude which is Yangian invariant allowing a description using a generalization of the Grassmannian integrals.

1. Assume the bosonic emergence and its super-symmetric generalization holds true. This means that incoming and outgoing states are bound states of massless fermions assignable to wormhole throats but the fermions can opposite directions of three-momenta making them massive. Incoming and outgoing particles would consist of fermions associated with wormhole throats and would be characterized by a pair of twistors in the general situation and in general massive. This allows also string like mass squared spectrum for bound states having fermion and antifermion at the ends of the string as well as more general n-particle bound states. Hence one can speak also about the emergence of string like objects. For virtual particles the fermions would be massive and have discrete mass spectrum. Also super partners containing several collinear fermions and antifermions at a given throat are possible. Collinearity is required by the generalization of SUSY. The construction of these states bring strongly in mind the merge procedure involving the replacement $Z^{n+1} \rightarrow Z^n$.

2. The basic question is how the momentum twistor diagrams and the ordinary Feynman diagrams behind them are related to the generalized Feynman diagrams.

   (a) It is good to start from a common problem. In momentum twistor approach the relationship of region momenta to physical momenta remains somewhat mysterious. In TGD framework in turn the relationship of pseudo-momenta identified as generalized eigenvalues of the Chern-Simons Dirac operator at the lines of Feynman diagram (light-like wormhole throats) to the physical momenta has remained unclear. The identification of the pseudo-momentum as the TGD counterpart of the region momentum x looks therefore like a natural first guess.

   (b) The identification $x_{a+1} - x_a = p_a$ with p_a representing light-like physical four-momentum generalizes in obvious manner. Also the identification of the light-like momentum of the external parton as pseudo-momentum looks natural. What is important is that this does not require the identification of the pseudo-momenta propagating along internal lines of generalized Feynman diagram as actual physical momenta since pseudo-momentum just like x is fixed only apart from an overall shift. The identification allows the physical four-momenta associated with the wormhole throats to be always on mass shell and massless: if the sign of the physical energy can be also negative space-like momentum exchanges become possible.

(c) The pseudo-momenta and light-like physical massless momenta at the lines of generalized Feynman diagrams on one hand, and region momenta and the light-like momenta associated with the collinear singularities on the other hand would be in very similar mutual relationship. Partonic 2-surfaces can carry large number of collinear light-like fermions and bosons since super-symmetry is extended. Generalized Feynman diagrams would be analogous to momentum twistor diagrams if this picture is correct and one could hope that the recursion relations of the momentum twistor approach generalize.

3. The discrete mass spectrum for pseudo-momentum would in the momentum twistor approach mean the restriction of x to discrete mass shells, and the obvious reason for worry is that this might spoil the Grassmannian approach relying heavily on residue integrals and making sense also p-adically. It seems however that there is no need to worry. In [84] the $M_{6,4,l=0}(1234AB)$ the integration over twistor variables z_A and z_B using "entangled" integration contour leads to 1-loop MHV amplitude N^pMHV, $p=1$. The parametrization of the integration contour is $z_A=(\lambda_A,x\lambda_A)$, $z_B=(\lambda_B,x\lambda_B)$, where x is the M^4 coordinate representing the loop momentum. This boils down to an integral over $CP_1\times CP_1\times M^4$ [84]. The integrals over spheres CP_1s are contour integrals so that only an ordinary integral over M^4 remains. The reduction to this kind of sums occurs completely generally thanks to the recursion formula.

4. The obvious implication of the restriction of the pseudo-momenta x on massive mass shells is the absence of IR divergences and one might hope that under suitable assumptions one achieves Yangian invariance. The first question is of course whether the required restriction of x to mass shells in z_A and z_B or possibly even algebraic discretization of momenta is consistent with the Yangian invariance. This seems to be the case: the integration contour reduces to entangled integration contour in $CP_1\times CP_1$ not affected by the discretization and the resulting loop integral differs from the standard one by the discretization of masses and possibly also momenta with massless states excluded. Whether Yangian invariance poses also conditions on mass and momentum spectrum is an interesting question.

5. One can consider also the possibility that the incoming and outgoing particles - in general massive and to be distinguished from massless fermions appearing as their building blocks- have actually small masses presumably related to the IR cutoff defined by the size scale of the largest causal diamond involved. p-Adic thermodynamics could be responsible for this mass. Also the binding of the wormhole throats can give rise to a small contribution to vacuum conformal weight possibly responsible for gauge boson masses. This would imply that a given n-particle state can decay to N-particle states for which N is below some limit. The fermions inside loops would be also massive. This allows to circumvent the IR singularities due to integration over the phase space of the final states (say in Coulomb scattering).

6. The representation of the off mass shell particles as pairs of wormhole throats with non-parallel four-momenta (in the simplest case only the three-momenta need be in opposite directions) makes sense and that the particles in question are on mass shell with mass squared being proportional to inverse of a prime number as the number theoretic vision applied to the modified Dirac equation suggests. On mass shell property poses extremely powerful constraints on loops and when the number of the incoming momenta in the loop increases, the number of constraints becomes larger than the number of components of loop momentum for the generic values of the external momenta. Therefore there are excellent hopes of getting rid of UV divergences.

   A stronger assumption encouraged by the classical space-time picture about virtual particles is that the 3-momenta associated with throats of the same wormhole contact are always in same or opposite directions. Even this allows to have virtual momentum spectrum and non-trivial mass spectrum for them assuming that the three momenta are opposite.

7. The best that one can hope is that only a finite number of generalized Feynman diagrams contributes to a given reaction. This would guarantee that amplitudes belong to a finite-dimensional algebraic

extension of rational functions with rational coefficients since finite sums do not lead out from a finite algebraic extension of rationals. The first problem are self energy corrections. The assumption tht the mass non-renormalization theorems of SUSYs generalize to TGD framework would guarantee that the loops contributing to fermionic propagators (and their super-counterparts) do not affect them. Also the iteration of more complex amplitudes as analogs of ladder diagrams representing sequences of reactions $M \rightarrow M_1 \rightarrow M_2 \cdots . \rightarrow N$ such that at each M_n in the sequence can appear as on mass shell state could give a non-vanishing contribution to the scattering amplitude and would mean infinite number of Feynman diagrams unless these amplitudes vanish. If N appears as a virtual state the fermions must be however massive on mass shell fermions by the assumption about on-mass shell states and one can indeed imagine a situation in which the decay $M \rightarrow N$ is possible when N consists of states made of massless fermions is possible but not when the fermions have non-vanishing masses. This situation seems to be consistent with unitarity. The implication would be that the recursion formula for the all loop amplitudes for a given reaction would give vanishing result for some critical value of loops.

Already these assumptions give good hopes about a generalization of the momentum Grassmann approach to TGD framework. Twistors are doubled as are also the Grassmann variables and there are wave functions correlating the momenta of the the fermions associated with the opposite wormhole throats of the virtual particles as well as incoming gauge bosons which have suffered massivation. Also wave functions correlating the massless momenta at the ends of string like objects and more general many parton states are involved but do not affect the basic twistor formalism. The basic question is whether the hypothesis of unbroken Yangian symmetry could in fact imply something resembling this picture. The possibility to discretize integration contours without losing the representation as residue integral quite generally is basic prerequisite for this and should be shown to be true.

6.1.2 How to achieve Yangian invariance without trivial scattering amplitudes?

In $\mathcal{N} = 4$ SYM the Yangian invariance implies that the MHV amplitudes are constant as demonstrated in [84]. This would mean that the loop contributions to the scattering amplitudes are trivial. Therefore the breaking of the dual super-conformal invariance by IR singularities of the integrand is absolutely essential for the non-triviality of the theory. Could the situation be different in TGD framework? Could it be possible to have non-trivial scattering amplitudes which are Yangian invariants. Maybe! The following heuristic argument is formulated in the language of super-twistors.

1. The dual conformal super generators of the super-Lie algebra $U(2,2)$ acting as super vector fields reducing effectively to the general form $J = \eta_a^K \partial/\partial Z_a^J$ and the condition that they annihilate scattering amplitudes implies that they are constant as functions of twistor variables. When particles are replaced with pairs of wormhole throats the super generators are replaced by sums $J_1 + J_2$ of these generators for the two wormhole throats and it might be possible to achieve the condition

$$(J_1 + J_2)M = 0 \tag{6.1}$$

with a non-trivial dependence on the momenta if the super-components of the twistors associated with the wormhole throats are in a linear relationship. This should be the case for bound states.

2. This kind of condition indeed exists. The condition that the sum of the super-momenta expressed in terms of super-spinors λ reduces to the sum of real momenta alone is not usually posed but in the recent case it makes sense as an additional condition to the super-components of the the spinors λ associated with the bound state. This quadratic condition is exactly of the same general form as the one following from the requirement that the sum of all external momenta vanishes for scattering amplitude and reads as

$$X = \lambda_1\eta_1 + \lambda_2\eta_2 = 0 \ . \tag{6.2}$$

The action of the generators $\eta_1\partial_{\lambda_1} + \eta_2\partial_{\lambda_2}$ forming basic building blocks of the super generators on $p_1+p_2 = \lambda_1\tilde{\lambda}_1 + \lambda_2\tilde{\lambda}_2$ appearing as argument in the scattering amplitude in the case of bound states gives just the quantity X, which vanishes so that one has super-symmetry. The generalization of this condition to n-parton bound state is obvious.

3. The argument does not apply to free fermions which have not suffered topological condensation and are therefore represented by CP_2 type vacuum extremal with single wormhole throat. If one accepts the weak form of electric-magnetic duality, one can circumvent this difficulty. The free fermions carry Kähler magnetic charge whereas physical fermions are accompanied by a bosonic wormhole throat carrying opposite Kähler magnetic charge and opposite electroweak isospin so that a ground state of string like object with size of order electroweak length scale is in question. In the case of quarks the Kähler magnetic charges need not be opposite since color confinement could involve Kähler magnetic confinement: electro-weak confinement holds however true also now. The above argument generalizes as such to the pairs formed by wormhole throats at the ends of string like object. One can of course imagine also more complex hybrids of these basic options but the general idea remains the same.

Note that the argument involves in an essential manner non-locality , which is indeed the defining property of the Yangian algebra and also the fact that physical particles are bound states. The massivation of the physical particles brings in the IR cutoff.

6.1.3 Number theoretical constraints on the pseudo-momenta

One can consider also further assumptions motivated by the recent view about the generalized eigenvalues of Chern-Simons Dirac operator having interpretation as pseudo-momentum. The details of this view need not of course be final.

1. Assume that the pseudo-momentum assigned to fermion lines by the modified Dirac equation [11] is the counterpart of region momentum as already explained and therefore does not directly correspond to the actual light-like four-momentum associated with partonic line of the generalized Feynman diagram. This assumption conforms with the assumption that incoming particles are built out of massless partonic fermions. It also implies that the propagators are massless propagators as required by twistorialization and Yangian generalization of super-conformal invariance.

2. Since (pseudo)-mass squared is number theoretically quantized as the length of a hyper-complex prime in preferred plane M^2 of pseudo-momentum space fermionic propagators are massless propagators with pseudo-masses restricted on discrete mass shells. Lorentz invariance suggests that M^2 cannot be common to all particles but corresponds to preferred reference frame for the virtual particle having interpretation as plane spanned by the quantization axes of energy and spin.

3. Hyper-complex primeness means also the quantization of pseudo-momentum components so that one has hyper-complex primes of form $\pm((p+1)/2, \pm(p-1)/1)$ corresponding to pseudo-mass squared $M^2 = p$ and hypercomplex primes $\pm(p, 0)$ with pseudo-mass squared $M2 = p^2$. Space-like fermionic momenta are not needed since for opposite signs of energy wormhole throats can have space-like net momenta. If space-like pseudo-momenta are allowed/needed for some reason, they could correspond to space-like hyper-complex primes $\pm((p-1)/2, \pm(p+1)/1)$ and $\pm(0, p)$ so that one would obtain also discretization of space-like mass shells also. The number theoretical mass squared is proportional to p, whereas p-adic mass squared is proportional to $1/p$. For p-adic mass calculations canonical identification $\sum x_n p^n$ maps p-adic mass squared to its real counterpart. The simplest mapping consistent with this would be $(p_0, p_1) \to (p_0, p_1)/p$. This could be assumed from the beginning in real context and would mean that the mass squared scale is proportional to $1/p$.

4. Lorentz invariance requires that the preferred coordinate system in which this holds must be analogous to the rest system of the virtual fermion and thus depends on the virtual particle. In accordance with the general vision discussed in [11] Lorentz invariance could correspond to a discrete algebraic subgroup of Lorentz group spanned by transformation matrices expressible in terms of roots of

unity. This would give a discrete version of mass shell and the preferred coordinate system would have a precise meaning also in the real context. Unless one allows algebraic extension of p-adic numbers p-adic mass shell reduces to the set of above number-theoretic momenta. For algebraic extensions of p-adic numbers the same algebraic mass shell is obtained as in real correspondence and is essential for the number theoretic universality. The interpretation for the algebraic discretization would be in terms of a finite measurement resolution. In real context this would mean discretization inducing a decomposition of the mass shell to cells. In the p-adic context each discrete point would be replaced with a p-adic continuum. As far as loop integrals are considered, this vision means that they make sense in both real and p-adic context and reduce to summations in p-adic context. This picture is discussed in detail in [11].

5. Concerning p-adicization the beautiful aspect of residue integral is that it makes sense also in p-adic context provided one can circumvent the problems related to the identification of p-adic counterpart of π requiring infinite-dimensional transcendental extension coming in powers of π. Together with the discretization of both real and virtual four-momenta this would allow to define also p-adic variants of the scattering amplitudes.

6.1.4 Could recursion formula allow interpretation in terms of zero energy ontology?

The identification of pseudo-momentum as a counterpart of region momentum suggests that generalized Feynman diagrams could be seen as a generalization of momentum twistor diagrams. Of course, the generalization from $\mathcal{N} = 4$ SYM to TGD is an enormous step in complexity and one must take all proposals in the following with a big grain of salt. For instance, the replacement of point-like particles with wormhole throats and the decomposition of gauge bosons to pairs of wormhole throats means that naive generalizations are dangerous.

With this in firmly in mind one can ask whether the recursion formula could allow interpretation in terms of zero energy states assigned to causal diamonds (CDs) containing CDs containing $\cdots$. In this framework loops could be assigned with sub-CDs.

The interpretation of the leading order singularities forming the basic building blocks of the twistor approach in zero ontology is the basic source of questions. Before posing these questions recall the basic proposal that partonic fermions are massless but opposite signs of energy are posssible for the opposite throats of wormhole contacts. Partons would be on mass shell but besides physical states identified as bound states formed from partons also more general intermediate states would be possible but restricted by momentum conservation and mass shell conditions for partons at vertices. Consider now the questions.

1. Suppose that the massivation of virtual fermions and their super partners allows only ladder diagrams in which the intermediate states contain on mass shell massless states. Should one allow this kind of ladder diagrams? Can one identify them in terms of leading order singularities? Could one construct the generalized Feynman diagrams from Yangian invariant tree diagrams associated with the hierarchy of sub-CDs and using BCFW bridges and entangled pairs of massless states having interpretation as box diagrams with on mass shell momenta at microscopic level? Could it make sense to say that scattering amplitudes are represented by tree diagrams inside CDs in various scales and that the fermionic momenta associated with throats and emerging from sub-CDs are always massless?

2. Could BCFW bridge generalizes as such and could the interpretation of BCFW bridge be in terms of a scattering in which the four on mass shell massless partonic states (partonic throats have arbitrary fermion number) are exchanged between four sub-CDs. This admittedly looks somewhat artificial.

3. Could the addition of 2-particle zero energy state responsible for addition of loop in the recursion relations and having interpretation in terms of the cutting of line carrying loop momentum correspond to an addition of sub-CD such that the 2-particle zero energy state has its positive and negative energy part on its past and future boundaries? Could this mean that one cuts a propagator line by adding CD and leaves only the portion of the line within CD. Could the reverse operation mean to the addition of zero energy "thermally entangled" states in shorter time and length scales

and assignable as a zero energy state to a sub-CD. Could one interpret the Cutkosky rule for propagator line in terms of this cutting or its reversal. Why only pairs would be needed in the recursion formula? Why not more general states? Does the recursion formula imply that they are included? Does this relate to the fact that these zero energy states have interpretation as single particle states in the positive energy ontology and that the basic building block of Feynman diagrams is single particle state? Could one regard the unitarity as an identity which states that the discontinuity of T-matrix characterizing zero energy state over cut is expressible in terms of $TT^\dagger$ and T matrix is the relevant quantity?

Maybe it is again dangerous to try to draw too detailed correspondences: after all, point like particles are replaced by partonic two-surfaces in TGD framework.

4. If I have understood correctly the genuine l-loop term results from $l-1$-loop term by the addition of the zero energy pair and integration over GL(2) as a representative of loop integral reducing $n+2$ to n and calculating the added loop at the same time [84]. The integrations over the two momentum twistor variables associated with a line in twistor space defining off mass shell four-momentum and integration over the lines represent the integration over loop momentum. The reduction to $GL(2)$ integration should result from the delta functions relating the additional momenta to $GL(2)$ variables (note that GL(2) performs linear transformations in the space spanned by the twistors Z_A and Z_B and means integral over the positions of Z_A an Z_B). The resulting object is formally Yangian invariant but IR divergences along some contours of integration breaks Yangian symmetry.

   The question is what happens in TGD framework. The previous arguments suggests that the reduction of the the loop momentum integral to integrals over discrete mass shells and possibly to a sum over their discrete subsets does not spoil the reduction to contour integrals for loop integrals in the example considered in [84]. Furthermore, the replacement of mass continuum with a discrete set of mass shells should eliminate IR divergences and might allow to preserve Yangian symmetry. One can however wonder whether the loop corrections with on mass shell massless fermions are needed. If so, one would have at most finite number of loop diagrams with on mass shell fermionic momenta and one of the TGD inspired dreams already forgotten would be realized.

6.1.5 What about unitarity?

The approach of Arkani-Hamed and collaborators means that loop integral over four-momenta are replaced with residue integrals around a small sphere $p^2 = \epsilon$. This is very much reminiscent of my own proposal for a few years ago based on the idea that the condition of twistorialization forces to accept only massless virtual states [41, 26]. I of course soon gave up this proposal as too childish.

This idea seems to however make a comeback in a modified form. At this time one would have only massive and quantized pseudo-momenta located at discrete mass shells. Can this picture be consistent with unitarity?

Before trying to answer this question one must make clear what one could assume in TGD framework.

1. Physical particles are in the general case massive and consist of collinear fermions at wormhole throats. External partons at wormhole throats must be massless to allow twistorial interpretation. Therefore massive states emerge. This applies also to stringy states.

2. The simplest assumption generalizing the childish idea is that on mass shell massless states for partons appear as both virtual particles and external particles. Space-like virtual momentum exchanges are possible if the virtual particles can consist of pairs of positive and negative energy fermions at opposite wormhole throats. Hence also partons at internal lines should be massless and this raises the question about the identification of propagators.

3. Generalized eigenvalue equation for Chern-Simons Dirac operator implies that virtual elementary fermions have massive and quantized pseudo-momenta whereas external elementary fermions are massless. The massive pseudo-momentum assigned with the Dirac propagator of a parton line cannot be identified with the massless real momentum assigned with the fermionic propagator line. The region momenta introduced in Grassmannian approach are something analogous.

As already explained, this brings in mind is the identification of this pseudo momentum as the counterpart of the region momentum of momentum twistor diagrams so that the external massless fermionic momenta would be differences of the pseudo-momenta. Indeed, since region momenta are determined apart from a common shift, they need not correspond to real momenta. Same applies to pseudo-momenta and one could assume that both internal and external fermion lines carry light-like pseudo-momenta and that external pseudo-momenta are equal to real momenta.

4. This picture has natural correspondence with twistor diagrams. For instance, the region momentum appearing in BCFW bridge defining effective propagator is in general massive although the underlying Feynman diagram would contain online massless momenta. In TGD framework massless lines of Feynman graphs associated with singularities would correspond to real momenta of massless fermions at wormhole throats. Also other canonical operations for Yangian invariants involve light-like momenta at the level of Feynman diagrams and would in TGD framework have a natural identification in terms of partonic momenta. Hence partonic picture would provide a microscopic description for the lines of twistor diagrams.

Let us assume being virtual particle means only that the discretized pseudo-momentum is on shell but massive whereas all real momenta of partons are light-like, and that negative partonic energies are possible. Can one formulate Cutkosky rules for unitarity in this framework? What could the unitarity condition

$$iDisc(T - T^{\dagger}) = -TT^{\dagger}$$

mean now? In particular, are the cuts associated with mass shells of physical particles or with mass shells of pseudo-momenta? Could these two assignments be equivalent?

1. The restriction of the partons to be massless but having both signs of energy means that the spectrum of intermediate states contains more states than the external states identified as bound states of partons with the same sign of energy. Therefore the summation over intermediate states does not reduce to a mere summation over physical states but involves a summation over states formed from massless partons with both signs of energy so that also space-like momentum exchanges become possible.

2. The understanding of the unitarity conditions in terms of Cutkosky rules would require that the cuts of the loop integrands correspond to mass shells for the virtual states which are also physical states. Therefore real momenta have a definite sign and should be massless. Besides this bound state conditions guaranteeing that the mass spectrum for physical states is discrete must be assumed. With these assumptions the unitary cuts would not be assigned with the partonic light-cones but with the mass shells associated of physical particles.

3. There is however a problem. The pseudo-momenta of partons associated with the external partons are assumed to be light-like and equal to the physical momenta.

   (a) If this holds true also for the intermediate physical states appearing in the unitarity conditions, the pseudo-momenta at the cuts are light-like and cuts must be assigned with pseudo-momentum light-cones. This could bring in IR singularities and spoil Yangian symmetry. The formation of bound states could eliminate them and the size scale of the largest CD involved would bring in a natural IR cutoff as the mass scale of the lightest particle. This assumption would however force to give up the assumption that only massive pseudo-momenta appear at the lines of the generalized Feynman diagrams.

   (b) On the other hand, if pseudo-momenta are not regarded as a property of physical state and are thus allowed to be massive for the real intermediate states in Cutkosky rules, the cuts at parton level correspond to on mass shell hyperboloids and IR divergences are absent.

6.2 Could TGD alllow formulation in terms of twistors

There are many questions to be asked. There would be in-numerable questions upwelling from my very incomplete understanding of the technical issues. In the following I restrict only to the questions which relate to the relationship of TGD approach to Witten's twistor string approach [91] and M-theory like frameworks. The arguments lead to an explicit proposal how the preferred extremals of Kähler action could correspond to holomorphic 4-surfaces in $CP_3 \times CP_3$. The basic motivation for this proposal comes from the observation that Kähler action is Maxwell action for the induced Kähler form and metric. Hence Penrose's original twistorial representation for the solutions of linear Maxwell's equations could have a generalization to TGD framework.

6.2.1 $M^4 \times CP_2$ from twistor approach

The first question which comes to mind relates to the origin of the Grassmannians. Do they have some deeper interpretation in TGD context. In twistor string theory Grassmannians relate to the moduli spaces of holomorphic surfaces defined by string world sheets in twistor space. Could partonic 2-surfaces have analogous interpretation and could one assign Grassmannians to their moduli spaces? If so, one could have rather direct connection with topological QFT defining twistor strings [91] and the almost topological QFT defining TGD. There are some hints to this direction which could be of course seen as figments of a too wild imagination.

1. The geometry of CD brings strongly in mind Penrose diagram for the conformally compactified Minkowski space [61], which indeed becomes CD when its points are replaced with spheres. This would suggest the information theoretic idea about interaction between observer and externals as a map in which M^4 is mapped to its conformal compactification represented by CD. Compactification means that the light-like points at the light-like boundaries of CD are identified and the physical counterpart for this in TGD framework is conformal invariance along light-rays along the boundaries of CD. The world of conscious observer for which CD is identified as a geometric correlate would be conformally compactified M^4 (plus CP_2 or course).

2. Since the points of the conformally compactified M^4 correspond to twistor pairs [88], which are unique only apart from opposite complex scalings, it would be natural to assign twistor space to CD and represent its points as pairs of twistors. This suggest an interpretation for the basic formulas of Grassmannian approach involving integration over twistors. The incoming and outgoing massless particles could be assigned at point-like limit light-like points at the lower and upper boundaries of CD and the lifting of the points of the light-cone boundary at partonic surfaces would give rise to the description in terms of ordinary twistors. The assumption that massless collinear fermions at partonic 2-surfaces are the basic building blocks of physical particles at partonic 2-surfaces defined as many particles states involving several partonic 2-surfaces would lead naturally to momentum twistor description in which massless momenta and described by twistors and virtual momenta in terms of twistor pairs. It is important to notice that in TGD framework string like objects would emerge from these massless fermions.

3. Partonic 2-surfaces are located at the upper and lower light-like boundaries of the causal diamond (CD) and carry energies of opposite sign in zero energy ontology. Quite generally, one can assign to the point of the conformally compactified Minkowski space a twistor pair using the standard description. The pair of twistors is determined apart from $Gl(2)$ rotation. At the light-cone boundary M^4 points are are light-like so that the two spinors of the two twistors differ from each other only by a complex scaling and single twistor is enough to characterize the space-time point this degenerate situation. The components of the twistor are related by the well known twistor equation $\mu^{a'} = -ix^{aa'}\lambda_a$. One can therefore lift each point of the partonic 2-surface to single twistor determined apart from opposite complex scalings of μ and λ so that the lift of the point would be 2-sphere. In the general case one must lift the point of CD to a twistor pair. The degeneracy of the points is given by $Gl(2)$ and each point corresponds to a 2-sphere in projective twistor space.

4. The new observation is that one can understand also CP_2 factor in twistor framework. The basic observation about which I learned in [88] (giving also a nice description of basics of twistor geometry)

is that a pair (X, Y) of twistors defines a point of CD on one hand and complex 2-planes of the dual twistor space -which is nothing but CP_2- by the equations

$$X_\alpha W^\alpha = 0 \ , \quad Y_\alpha W^\alpha = 0 \ .$$

The intersection of these planes is the complex line $CP_1 = S^2$. The action of $G(2)$ on the twistor pair affects the pair of surfaces CP_2 determined by these equations since it transforms the equations to their linear combination but not the the point of conformal CD resulting as projection of the sphere. Therefore twistor pair defines both a point of M^4 and assigns with it pair of CP_2:s represented as holomorphic surfaces of the projective dual twistor space. Hence the union over twistor pairs defines $M^4 \times CP_2$ via this assignment if it is possible to choose "the other" CP_2 in a unique manner for all points of M^4. The situation is similar to the assignment of a twistor to a point in the Grassmannian diagrams forming closed polygons with light-like edges. In this case one assigns to the the "region momenta" associated with the edge the twistor at the either end of the edge. One possible interpretation is that the two CP_2:s correspond to the opposite ends of the CD. My humble hunch is that this observation might be something very deep.

Recall that the assignment of CP_2 to M^4 point works also in another direction. $M^8 - H$ duality associates with so called hyper-quaternionic 4-surface of M^8 allowing preferred hyper-complex plane at each point 4-surfaces of $M^4 \times CP_2$. The basic observation behind this duality is that the hyper-quaternionic planes (copies of M^4) with preferred choices of hyper-complex plane M^2 are parameterized by points of CP_2. One can therefore assign to a point of CP_2 a copy of M^4. Maybe these both assignments indeed belong to the core of quantum TGD. There is also an interesting analogy with Uncertainty Principle: complete localization in M^4 implies maximal uncertainty of the point in CP_2 and vice versa.

6.2.2 Does twistor string theory generalize to TGD?

With this background the key speculative questions seem to be the following ones.

1. Could one relate twistor string theory to TGD framework? Partonic 2-surfaces at the boundaries of CD are lifted to 4-D sphere bundles in twistor space. Could they serve as a 4-D counterpart for Witten's holomorphic twistor strings assigned to point like particles? Could these surfaces be actually lifts of the holomorphic curves of twistor space replaced with the product $CP_3 \times CP_2$ to 4-D sphere bundles? If I have understood correctly, the Grassmannians $G(n, k)$ can be assigned to the moduli spaces of these holomorphic curves characterized by the degree of the polynomial expressible in terms of genus, number of negative helicity gluons, and the number of loops for twistor diagram.

   Could one interpret $G(n, k)$ as a moduli space for the δCD projections of n partonic 2-surfaces to which k negative helicity gluons and $n - k$ positive helicity gluons are assigned (or something more complex when one considers more general particle states)? Could quantum numbers be mapped to integer valued algebraic invariants? IF so, there would be a correlation between the geometry of the partonic 2-surface and quantum numbers in accordance with quantum classical correspondence.

2. Could one understand light-like orbits of partonic 2-surfaces and space-time surfaces in terms of twistors? To each point of the 2-surface one can assign a 2-sphere in twistor space CP_3 and CP_2 in its dual. These CP_2s can be identified. One should be able to assign to each sphere S^2 at least one point of corresponding CP_2s associated with its points in the dual twistor space and identified as single CP_2 union of CP_2:s in the dual twistor space a point of CP_2 or even several of them. One should be also able to continue this correspondence so that it applies to the light-like orbit of the partonic 2-surface and to the space-time surface defining a preferred extremal of Kähler action. For space-time sheets representable as graph of a map $M^4 \rightarrow CP_2$ locally one should select from a CP_2 assigned with a particular point of the space-time sheet a unique point of corresponding CP_2 in a manner consistent with field equations. For surfaces with lower dimensional M^4 projection one must assign a continuum of points of CP_2 to a given point of M^4. What kind equations-could allow to realize this assignment? Holomorphy is strongly favored also by the number theoretic considerations since in this case one has hopes of performing integrals using residue calculus.

(a) Could two holomorphic equations in $CP_3 \times CP_2$ defining 6-D surfaces as sphere bundles over $M^4 \times CP_2$ characterize the preferred extremals of Kähler action? Could partonic 2-surfaces be obtained by posing an additional holomorphic equation reducing twistors to null twistors and thus projecting to the boundaries of CD? A philosophical justification for this conjecture comes from effective 2-dimensionality stating that partonic 2-surfaces plus their 4-D tangent space data code for physics. That the dynamics would reduce to holomorphy would be an extremely beautiful result. Of course this is only an additional item in the list of general conjectures about the classical dynamics for the preferred extremals of Kähler action.

(b) One could also work in $CP_3 \times CP_3$. The first CP_3 would represent twistors endowed with a metric conformally equivalent to that of $M^{2,4}$ and having the covering of $SU(2,2)$ of $SO(2,4)$ as isometries. The second CP_3 defining its dual would have a metric consistent with the Calabi-Yau structure (having holonomy group $SU(3)$). Also the induced metric for canonically imbedded CP_2s should be the standard metric of CP_2 having $SU(3)$ as its isometries. In this situation the linear equations assigning to M^4 points twistor pairs and $CP_2 \subset CP_3$ as a complex plane would hold always true. Besides this two holomorphic equations coding for the dynamics would be needed.

(c) The issues related to the induced metric are important. The conformal equivalence class of M^4 metric emerges from the 5-D light-cone of $M^{2,4}$ under projective identification. The choice of a proper projective gauge would select M^4 metric locally. Twistors inherit the conformal metric with signature $(2,4)$ form the metric of 4+4 component spinors with metric having $(4,4)$ signature. One should be able to assign a conformal equivalence class of Minkowski metric with the orbits of pairs of twistors modulo $GL(2)$. The metric of conformally compactified M^4 would be obtained from this metric by dropping from the line element the contribution to the S^2 fiber associated with M^4 point.

(d) Witten related [91] the degree d of the algebraic curve describing twistor string, its genus g, the number k of negative helicity gluons, and the number l of loops by the following formula

$$d = k - 1 + l \ , \ g \leq l \ . \tag{6.3}$$

One should generalize the definition of the genus so that it applies to 6-D surfaces. For projective complex varieties of complex dimension n this definition indeed makes sense. Algebraic genus [52] is expressible in terms of the dimensions of the spaces of closed holomorphic forms known as Hodge numbers $h^{p,q}$ as

$$g = \sum (-1)^{n-k} h^{k,0} \ . \tag{6.4}$$

The first guess is that the formula of Witten generalizes by replacing genus with its algebraic counterpart . This requires that the allowed holomorphic surfaces are projective curves of twistori space, that is described in terms of homogenous polynomials of the 4+4 projective coordinates of $CP_3 \times CP_3$.

6.2.3 What is the relationship of TGD to M-theory and F-theory?

There are also questions relating to the possible relationship to M-theory and F-theory.

1. Calabi-Yau-manifolds [53, 68] are central for the compactification in super string theory and emerge from the condition that the super-symmetry breaks down to $\mathcal{N} = 1$ SUSY. The dual twistor space CP_3 with Euclidian signature of metric is a Calabi-Yau manifold [91]. Could one have in some sense two Calabi-Yaus! Twistorial CP_3 can be interpreted as a four-fold covering and conformal compactification of $M^{2,4}$. I do not know whether Calabi-Yau property has a generalization to the situation when Euclidian metric is replaced with a conformal equivalence class of flat metrics with Minkowskian signature and thus having a vanishing Ricci tensor. As far as differential forms (no

dependence on metric) are considered there should be no problems. Whether the replacement of the maximal holonomy group $SU(3)$ with its non-compact version $SU(1,2)$ makes sense is not clear to me.

2. The lift of the CD to projective twistor space would replace $CD \times CP_2$ with 10-dimensional space which inspires the familiar questions about connection between TGD and M-theory. If Calabi-Yau with a Minkowskian signature of metric makes sense then the Calabi-Yau of the standard M-theory would be replaced with its Minkowskian counterpart! Could it really be that M-theory like theory based on $CP_3 \times CP_2$ reduces to TGD in $CD \times CP_2$ if an additional symmetry mapping 2-spheres of CP_3 to points of CD is assumed? Could the formulation based on 12-D $CP_3 \times CP_3$ correspond to F-theory which also has two time-like dimensions. Of course, the additional conditions defined by the maps to M^4 and CP_2 would remove the second time-like dimension which is very difficult to justify on purely physical grounds.

3. One can actually challenge the assumption that the first CP_3 should have a conformal metric with signature $(2,4)$. Metric appears nowhere in the definition holomorphic functions and once the projections to M^4 and CP_2 are known, the metric of the space-time surface is obtained from the metric of $M^4 \times CP_2$. The previous argument for the necessity of the presence of the information about metric in the second order differential equation however suggests that the metric is needed.

4. The beginner might ask whether the 6-D 2-sphere bundles representing space-time sheets could have interpretation as Calabi-Yau manifolds. In fact, the Calabi-Yau manifolds defined as complete intersections in $CP_3 \times CP_3$ discovered by Tian and Yau are defined by three polynomials [68]. Two of them have degree 3 and depend on the coordinates of single CP_3 only whereas the third is bilinear in the coordinates of the CP_3:s. Obviously the number of these manifolds is quite too small (taking into account scaling the space defined by the coefficients is 6-dimensional). All these manifolds are deformation equivalent. These manifolds have Euler characteristic $\chi = \pm 18$ and a non-trivial fundamental group. By dividing this manifold by Z_3 one obtains $\chi = \pm 6$, which guarantees that the number of fermion generations is three in heterotic string theory. This manifold was the first one proposed to give rise to three generations and $\mathcal{N} = 1$ SUSY.

6.2.4 What could the field equations be in twistorial formulation?

The fascinating question is whether one can identify the equations determining the 3-D complex surfaces of $CP_3 \times CP_3$ in turn determining the space-time surfaces.

The first thing is to clarify in detail how space-time $M^4 \times CP_2$ results from $CP_3 \times CP_3$. Each point $CP_3 \times CP_3$ define a line in third CP_3 having interpretation as a point of conformally compactified M^4 obtained by sphere bundle projection. Each point of either CP_3 in turn defines CP_2 in in fourth CP_3 as a 2-plane. Therefore one has $(CP_3 \times CP_3) \times (CP_3 \times CP_3)$ but one can reduce the consideration to $CP_3 \times CP_3$ fixing $M^4 \times CP_2$. In the generic situation 6-D surface in 12-D $CP_3 \times CP_3$ defines 4-D surface in the dual $CP_3 \times CP_3$ and its sphere bundle projection defines a 4-D surface in $M^4 \times CP_2$.

1. The vanishing of three holomorphic functions f^i would characterize 3-D holomorphic surfaces of 6-D $CP_3 \times CP_3$. These are determined by three real functions of three real arguments just like a holomorphic function of single variable is dictated by its values on a one-dimensional curve of complex plane. This conforms with the idea that initial data are given at 3-D surface. Note that either the first or second CP_3 can determine the CP_2 image of the holomorphic 3-surface unless one assumes that the holomorphic functions are symmetric under the exchange of the coordinates of the two CP_3s. If symmetry is not assumed one has some kind of duality.

2. Effective 2-dimensionality means that 2-D partonic surfaces plus 4-D tangent space data are enough. This suggests that the 2 holomorphic functions determining the dynamics satisfy some second order differential equation with respect to their three complex arguments: the value of the function and its derivative would correspond to the initial values of the imbedding space coordinates and their normal derivatives at partonic 2-surface. Since the effective 2-dimensionality brings in dependence on the induced metric of the space-time surface, this equation should contain information about the induced metric.

3. The no-where vanishing holomorphic 3-form Ω, which can be regarded as a "complex square root" of volume form characterizes 6-D Calabi-Yau manifold [53, 68] , indeed contains this information albeit in a rather implicit manner but in spirit with TGD as almost topological QFT philosophy. Both CP_3:s are characterized by this kind of 3-form if Calabi-Yau with $(2,4)$ signature makes sense.

4. The simplest second order- and one might hope holomorphic- differential equation that one can imagine with these ingredients is of the form

$$\Omega_1^{i_1j_1k_1}\Omega_2^{i_2j_2k_2}\partial_{i_1i_2}f^1\partial_{j_1j_2}f^2\partial_{k_1k_2}f^3 = 0 \ , \ \partial_{ij} \equiv \partial_i\partial_j \ . \quad (6.5)$$

   Since Ω_i is by its antisymmetry equal to $\Omega_i^{123}\epsilon^{ijk}$, one can divide Ω^{123}:s away from the equation so that one indeed obtains holomorphic solutions. Note also that one can replace ordinary derivatives in the equation with covariant derivatives without any effect so that the equations are general coordinate invariant.

   One can consider more complex equations obtained by taking instead of (f^1,f^2,f^3) arbitrary combinations (f^i,f^j,f^k) which results uniquely if one assumes anti-symmetrization in the labels $(1,2,3)$. In the sequel only this equation is considered.

5. The metric disappears completely from the equations and skeptic could argue that this is inconsistent with the fact that it appears in the equations defining the weak form of electric-magnetic duality as a Lagrange multiplier term in Chern-Simons action. Optimist would respond that the representation of the 6-surfaces as intersections of three hyper-surfaces is different from the representation as imbedding maps $X^4 \rightarrow H$ used in the usual formulation so that the argument does not bite, and continue by saying that the metric emerges in any case when one endows space-time with the induced metric given by projection to M^4.

6. These equations allow infinite families of obvious solutions. For instance, when some f^i depends on the coordinates of either CP_3 only, the equations are identically satisfied. As a special case one obtains solutions for which $f^1 = Z\cdot W$ and $(f^2,f^3) = (f^2(Z),f^3(W))$ This family contains also the Calabi-Yau manifold found by Yau and Tian, whose factor space was proposed as the first candidate for a compactification consistent with three fermion families.

7. One might hope that an infinite non-obvious solution family could be obtained from the ansatz expressible as products of exponential functions of Z and W. Exponentials are not consistent with the assumption that the functions f_i are homogenous polynomials of finite degree in projective coordinates so that the following argument is only for the purpose for learning something about the basic character of the equations.

$$\begin{aligned} & f^1 = E_{a_1,a_2,a_3}(Z)E_{\hat{a}_1,\hat{a}_2,\hat{a}_3}(W) \ , \qquad f^2 = E_{b_1,b_2,b_3}(Z)E_{\hat{b}_1,\hat{b}_2,\hat{b}_3}(W) \ , \\ & f^3 = E_{c_1,c_2,c_3}(Z)E_{\hat{c}_1,\hat{c}_2,\hat{c}_3}(W) \ , \\ & E_{a,b,c}(Z) = exp(az_1)exp(bz_2)exp(cz_3) \ . \end{aligned} \quad (6.6)$$

   The parameters a,b,c, and $\hat{a},\hat{b},\hat{c}$ can be arbitrary real numbers in real context. By the basic properties of exponential functions the field equations are algebraic. The conditions reduce to the vanishing of the products of determinants $det(a,b,c)$ and $det(\hat{a},\hat{b},\hat{c})$ so that the vanishing of either determinant is enough. Therefore the dependence can be arbitrary either in Z coordinates or in W coordinates. Linear superposition holds for the modes for which determinant vanishes which means that the vectors (a,b,c) or $(\hat{a},\hat{b},\hat{c})$ are in the same plane.

   Unfortunately, the vanishing conditions reduce to the conditions $f^i(W) = 0$ for case a) and to $f^i(Z) = 0$ for case b) so that the conditions are equivalent with those obtained by putting the "wave vector" to zero and the solutions reduce to obvious ones. The lesson is that the equations do not commute with the multiplication of the functions f^i with nowhere vanishing functions of W and Z. The equation selects a particular representation of the surfaces and one might argue that this should not be the case unless the hyper-surfaces defined by f^i contain some physically relevant information. One could consider the possibility that the vanishing conditions are replaced with conditions $f^i = c_i$ with $f^i(0) = 0$ in which case the information would be coded by a family of space-time surfaces obtained by varying c_i.

One might criticize the above equations since they are formulated directly in the product $CP_3 \times CP_3$ of projective twistor by choosing a specific projective gauge by putting$z^4 = 1, w^4 = 1$. The manifestly projectively invariant formulation for the equations is in full twistor space so that 12-D space would be replaced with 16-D space. In this case one would have 4-D complex permutation symbol giving for these spaces Calabi-Yau structure with flat metric. The product of functions $f = z^4 = constant$ and $g = w^4 = constant$ would define the fourth function $f_4 = fg$ fixing the projective gauge

$$\epsilon^{i_1j_1k_1l_1}\epsilon^{i_2j_2k_2l_2}\partial_{i_1i_2}f^1\partial_{j_1j_2}f^2\partial_{k_1k_2}f^3\partial_{l_1l_2}f^4 = 0 \ , \ \partial_{ij} \equiv \partial_i\partial_j \ . \tag{6.7}$$

The functions f^i are homogenous polynomials of their twistor arguments to guarantee projective invariance. These equations are projectively invariant and reduce to the above form which means also loss of homogenous polynomial property. The undesirable feature is the loss of manifest projective invariance by the fixing of the projective gauge.

A more attractive ansatz is based on the idea that one must have one equation for each f^i to minimize the non-determinism of the equations obvious from the fact that there is single equation in 3-D lattice for three dynamical variables. The quartets (f^1, f^2, f^3, f^i), $i = 1, 2, 3$ would define a possible minimally non-linear generalization of the equation

$$\epsilon^{i_1j_1k_1l_1}\epsilon^{i_2j_2k_2l_2}\partial_{i_1i_2}f^1\partial_{j_1j_2}f^2\partial_{k_1k_2}f^m\partial_{l_1l_2}f^4 = 0 \ , \ \partial_{ij} \equiv \partial_i\partial_j \ , \ m = 1, 2, 3 \ . \tag{6.8}$$

Note that the functions are homogenous polynomials of their arguments and analogous to spherical harmonics suggesting that they can allow a nice interpretation in terms of quantum classical correspondence.

The minimal non-linearity of the equations also conforms with the non-linearity of the field equations associated with Kähler action. Note that also in this case one can solve the equations by diagonalizing the dynamical coefficient matrix associated with the quadratic term and by identifying the eigen-vectors of zero eigen values. One could consider also more complicated strongly non-linear ansätze such as (f^i, f^i, f^i, f^i), $i = 1, 2, 3$, but these do not seem plausible.

1. The explicit form of the equations using Taylor series expansion for multi-linear case

In this section the equations associated with (f_1, f_2, f_3) ansatz are discussed in order to obtain a perspective about the general structure of the equations by using simpler (multilinearity) albeit probably non-realistic case as starting point. This experience can be applied directly to the (f^1, f^2, f^3, f^i) ansatz, which is quadratic in f^i.

The explicit form of the equations is obtained as infinite number of conditions relating the coefficients of the Taylor series of f^1 and f^2. The treatment of the two variants for the equations is essentially identical and in the following only the manifestly projectively invariant form will be considered.

1. One can express the Taylor series as

$$\begin{aligned} f^1(Z,W) &= \sum_{m,n} C_{m,n}M_m(Z)M_n(W) \ , \\ f^2(Z,W) &= \sum_{m,n} D_{m,n}M_m(Z)M_n(W) \ , \\ f^3(Z,W) &= \sum_{m,n} E_{m,n}M_m(Z)M_n(W) \ , \\ M_{m\equiv(m_1,m_2,m_3)}(Z) &= z_1^{m_1}z_2^{m_2}z_3^{m_3} \ . \end{aligned} \tag{6.9}$$

2. The application of derivatives to the functions reduces to a simple algebraic operation

$$\partial_{ij}(M_m(Z)M_n(W)) = m_i n_j M_{m_1-e_i}(Z)M_{n-e_j}(W) \ . \tag{6.10}$$

Here e_i denotes i:th unit vector.

3. Using the product rule $M_m M_n = M_{m+n}$ one obtains

$$\begin{aligned} & \partial_{ij}(M_m(Z)M_n(W))\partial_{rs}(M_k(Z)M_l(W)) \\ = & \ m_i n_j k_r l_s \times M_{m-e_i}(Z)M_{n-e_j}(W) \times M_{k-e_r}(Z)M_{k-e_s}(W) \\ = & \ m_i n_j k_r l_s \times M_{m+k-e_i-e_r}(Z) \times M_{n+l-e_j-e_l}(W) \ . \end{aligned} \tag{6.11}$$

4. The equations reduce to the trilinear form

$$\begin{aligned} & \sum_{m,n,k,l,r,s} C_{m,n}D_{k,l}E_{r,s}(m,k,r)(n,l,s)M_{m+k+r-E}(Z)M_{n+l+s-E}(W) = 0 \ , \\ E = & \ e_1+e_2+e_3 \ , \ (a,b,c) = \epsilon^{ijk}a_i b_j c_c \ . \end{aligned} \tag{6.12}$$

Here (a,b,c) denotes the determinant defined by the three index vectors involved. By introducing the summation indices

$$(M = m+k+r-E, k, r) \ , \quad (N = n+l+s-E, l, s)$$

one obtains an infinite number of conditions, one for each pair (M,N). The condition for a given pair (M,N) reads as

$$\sum_{k,l,r,s} C_{M-k-r+E,N-l-s+E}D_{k,l}E_{r,s} \times (M-k-r+E,k,r)(N-l-s+E,l,s) = 0 \ . \tag{6.13}$$

These equations can be regarded as linear equations by regarding any matrix selected from $\{C,D,E\}$ as a vector of linear space. The existence solutions requires that the determinant associated with the tensor product of other two matrices vanishes. This matrix is dynamical. Same applies to the tensor product of any of the matrices.

5. Hyper-determinant [80] is the generalization of the notion of determinant whose vanishing tells that multilinear equations have solutions. Now the vanishing of the hyper-determinant defined for the tensor product of the three-fold tensor power of the vector space defined by the coefficients of the Taylor expansion should provide the appropriate manner to characterize the conditions for the existence of the solutions. As already seen, solutions indeed exist so that the hyper-determinant must vanish. The elements of the hyper matrix are now products of determinants for the exponents of the monomials involved. The non-locality of the Kähler function as a functional of the partonic surface leads to the argument that the field equations of TGD for vanishing n:th variations of Kähler action are multilinear and that a vanishing of a generalized hyper-determinant characterizes this [11].

6. Since the differential operators are homogenous polynomials of partial derivatives, the total degrees of $M_m(Z)$ and $M_m(W)$ defined as a sum $D = \sum m_i$ is reduced by one unit by the action of both operators ∂_{ij}. For given value of M and N only the products

$$M_m(Z)M_n(W)M_k(Z)M_r(W)M_s(Z)M_l(W)$$

for which the vector valued degrees $D_1 = m + k + r$ and $D_2 = n + l + s$ have the same value are coupled. Since the degree is reduced by the operators appearing in the equation, polynomial solutions for which f^i contain monomials labelled by vectors m_i, n_i, r_i for which the components vary in a finite range $(0, n_{max})$ look like a natural solution ansatz. All the degrees $D_i \leq D_{i,max}$ appear in the solution ansatz so that quite a large number of conditions is obtained.

What is nice is that the equation can be interpreted as a difference equation in 3-D lattice with "time direction" defined by the direction of the diagonal.

1. The counterparts of time=constant slices are the planes $n_1 + n_2 + n_3 = n$ defining outer surfaces of simplices having E as a normal vector. The difference equation does not seem to say nothing about the behavior in the transversal directions. M and N vary in the simplex planes satisfying $\sum M_i = T_1$, $\sum N_i = T_2$. It seems natural to choose $T_1 = T_2 = T$ so that Z and W dynamics corresponds to the same "time". The number of points in the $T = constant$ simplex plane increases with T which is analogous to cosmic expansion.

2. The "time evolution" with respect to T can be solved iteratively by increasing the value of $\sum M_i = N_i = T$ by one unit at each step. Suppose that the values of coefficients are known and satisfy the conditions for (m, k, r) and (n, l, s) up to the maximum value T for the sum of the components of each of these six vectors. The region of known coefficients -"past"- obviously corresponds to the interior of the simplex bounded by the plane $\sum M_i = \sum N_i = T$ having E as a normal. Let (m_{min}, n_{min}), (k_{min}, l_{min}) and (r_{min}, s_{min}) correspond to the smallest values of 3-indices for which the coefficients are non-vanishing- this could be called the moment of "Big Bang". The simplest but not necessary assumption is that these indices correspond zero vectors $(0, 0, 0)$ analogous to the tip of light-cone.

3. For given values of M and N corresponding to same value of "cosmic time" T one can separate from the formula the terms which correspond to the un-known coefficients as the sum $C_{M+E,N+E}D_{0,0}E_{0,0}+ D_{M+E,N+E}D_{0,0}C_{0,0}+E_{M+E,N+E}C_{0,0}D_{0,0}$. The remaining terms are by assumption already known. One can fix the normalization by choosing $C_{0,0} = D_{0,0} = E_{0,0} = 1$. With these assumptions the equation reduces at each point of the outer boundary of the simplex to the form

$$C_{M+E,N+E} + D_{M+E,N+E} + E_{M+E,N+E} = X$$

where X is something already known and contain only data about points in the plane $m+k+r = M$ and $n+r+s = N$. Note that these planes have one "time like direction" unlike the simplex plane so that one could speak about a discrete analog of string world sheet in 3+3+3-D lattice space defined by a 2-plane with one time-like direction.

4. For each point of the simplex plane one has equation of the above form. The equation is non-deterministic since only constrain only the sum $C_{M+E,N+E} + D_{M+E,N+E} + E_{M+E,N+E}$ at each point of the simplex plane to a plane in the complex 3-D space defined by them. Hence the number of solutions is very large. The condition that the solutions reduce to polynomials poses conditions on the coefficients since the quantities X associated with the plane $T = T_{max}$ must vanish for each point of the simplex plane in this case. In fact, projective invariance means that the functions involved are homogenous functions in projective coordinates and thus polynomials and therefore reduce to polynomials of finite degree in 3-D treatment. This obviously gives additional condition to the equations.

2. The minimally non-linear option

The simple equation just discussed should be taken with a caution since the non-determinism seems to be too large if one takes seriously the analogy with classical dynamics. By the vacuum degeneracy also the time evolution associated with Kähler action breaks determinism in the standard sense of the word. The non-determinism is however not so strong and removed completely in local sense for non-vacuum extremals. One could also try to see the non-determinism as the analog for non-deterministic time evolution by quantum jumps.

One can however consider the already mentioned possibility of increasing the number of equations so that one would have three equations corresponding to the three unknown functions f^i so that the determinism associated with each step would be reduced. The equations in question would be of the same general form but with (f^1, f^2, f^3) replaced with some some other combination.

1. In the genuinely projective situation where one can consider the (f^1, f^2, f^3, f^i), $i = 1, 2, 3$ as a unique generalization of the equation. This would make the equations quadratic in f_i and reduce the non-determinism at given step of the time evolution. The new element is that now only monomials $M_m(z)$ associated with the f^i with same degree of homegenity defined by $d = \sum m_i$ are consistent with projective invariance. Therefore the solutions are characterized by six integers $(d_{i,1}, d_{i,2})$ having interpretation as analogs of conformal weights since they correspond to eigenvalues of scaling operators. That homogenous polynomials are in question gives hopes that a generalization of Witten's approach might make sense. The indices m vary at the outer surfaces of the six 3-simplices defined by $(d_{i,1}, d_{i,2})$ and looking like tedrahedrons in 3-D space. The functions f^i are highly analogous to the homogenous functions appearing in group representations and quantum classical correspondence could be realized through the representation of the space-time surfaces in this manner.

2. The 3-determinants (a, b, c) appearing in the equations would be replaced by 4-determinants and the equations would have the same general form. One has

$$\begin{aligned} &\sum_{k,l,r,s,t,u} C_{M-k-r-t+E,N-l-s-u+E} D_{k,l} E_{r,s} C_{t,u} \times \\ &\times (M-k-r-t+E,k,r,t)(N-l-s-u+E,l,s,u) = 0 \ , \\ &E = e_1+e_2+e_3+e_4 \ , \ \ (a,b,c,d) = \epsilon^{ijkl} a_i b_j c_k d_l \ . \end{aligned} \tag{6.14}$$

   and its variants in which D and E appear quadratically. The values of M and N are restricted to the tedrahedrons $\sum M_i = \sum d_{k,1} + d_{1,i}$ and $\sum N_i = \sum d_{i,2} + d_{i,2}$, $i = 1, 2, 3$. Therefore the dynamics in the index space is 3-dimensional. Since the index space is in a well-defined sense dual to CP_3 as is also the CP_3 in which the solutions are represented as counterparts of 3-surfaces, one could say that the 3-dimensionality of the dynamics corresponds to the dynamics of Chern-Simons action at space-like at the ends of CD and at light-like 3-surfaces.

3. The view based on 4-D time evolution is not useful since the solutions are restricted to time=constant plane in 4-D sense. The elimination of one of the projective coordinates would lead however to the analog of the above describe time evolution. In four-D context a more appropriate form of the equations is

$$\sum_{m,n,k,l,r,s} C_{m,n} D_{k,l} E_{r,s} C_{t,u} (m,k,r,t)(n,l,s,u) M_{m+k+r-E}(Z) M_{n+l+s-E}(W) = 0 \tag{6.15}$$

   with similar equations for f^2 and f^3. If one assumes that the CP_2 image of the holomorphic 3-surface is unique (it can correspond to either CP_3) the homogenous polynomials f^i must be symmetric under the exchange of Z and W so that the matrices C, D, and E are symmetric. This is equivalent to a replacement of the product of determinants with a sum of 16 products of determinants obtained by permuting the indices of each index pair (m, n), (k, l),(r, s) and (t, u).

4. The number N_{cond} of conditions is given by the product $N_{cond} = N(d_M)N(d_N)$ of numbers of points in the two tedrahedrons defined by the total conformal weights

$$\sum M_r = d_M = \sum_k d_{k,1} + d_{i,1} \text{ and } \sum N_r = d_N = \sum_k d_{k,2} + d_{i,2} \ , \ i = 1, 2, 3.$$

The number N_{coeff} of coefficients is

$$N_{coeff} = \sum_k n(d_{k,1}) + \sum_k n(d_{k,2}) \ ,$$

where $n(d_{k,i})$ is the number points associated with the tedrahedron with conformal weight $d_{k,i}$. Since one has $n(d) \propto d^3$, N_{cond} scales as

$$N_{cond} \propto d_M^3 d_N^3 = (\sum_k d_{k,1} + d_{1,i})^3 \times (\sum_k d_{k,2} + d_{i,2})^3$$

whereas the number N_{coeff} of coefficients scales as

$$N_{coeff} \propto \sum_k (d_{k,1}^3 + d_{k,2}^3) \ .$$

N_{cond} is clearly much larger than N_{coeff} so the solutions are analogous to partial waves and that the reduction of the rank for the matrices involved is an essential aspect of being a solution. The reduction of the rank for the coefficient matrices should reduce the effective number of coefficients so that solutions can be found. An interesting question is whether the coefficients are rationals with a suitable normalization allowed by independent conformal scalings. An analogy for the dynamics is quantum entanglement for 3+3 systems respecting the conservation of conformal weights and quantum classical correspondence taken to extreme suggests something like this.

5. One can interpret these equations as linear equations for the coefficients of the either linear term or as quadratic equations for the non-linear term. Also in the case of quadratic term one can apply general linear methods to identify the vanishing eigen values of the matrix of the quadratic form involved and to find the zero modes as solutions. The rank of the dynamically determined multiplier matrix must be non-maximal for the solutions to exist. One can imagine that the rank changes at critical surfaces in the space of Taylor coefficients meaning a multi-furcation in the space determined by the coefficients of the polynomials. Also the degree of the polynomial can change at the critical point.

   Solutions for which either determinant vanishes for all terms present in the solution exist. This is is achieved if either the index vectors (m, l, r, t) or (n, l, s, u) in their respective parallel 3-planes are also in a 3-plane going through the origin. These solutions might seen as the analogs of vacuum extremals of Chern-Simons action for which the CP_2 projection is at most 2-D Lagrangian manifold.

   Quantum classical correspondence requires that the space-time surface carries also information about the momenta of partons. This information is quasi-continuous. Also information about zero modes should have representation in terms of the coefficients of the polynomials. Is this really possible if only products of polynomials of fixed conformal weights with strong restrictions on coefficients can be used? The counterpart for the vacuum degeneracy of Kähler action might resolve the problem. The analog for the construction of space-time surfaces as deformations of vacuum extremals would be starting from a trivial solution and adding to the building blocks of f^i some terms of same degree for which the wave vectors are not in the intersection of a 3-plane and simplex planes. The still existing "vacuum part" of the solution could carry the needed information.

6. One can take "obvious solutions" characterized by different common 3-planes for the "wave vectors" characterizing the 8 monomials $M_a(Z)$ and $M_b(W)$, $a \in \{m, k, r, t\}$ and $b \in \{n, l, s, u\}$. The coefficient matrices C, D, E, F are completely free. For the sum of these solutions the equations contain

interaction terms for which at least two "wave vectors" belong to different 3-planes so that the corresponding 4-determinants are non-vanishing. The coefficients are not anymore free. Could the "obvious solutions" have interpretation in terms of different space-time sheets interacting via wormhole contacts? Or can one equate "obvious" with "vacuum" so that interaction between different vacuum space-time sheets via wormhole contact with 3-D CP_2 projection would deform vacuum extremals to non-vacuum extremals? Quantum classical correspondence inspires the question whether the products for functions f_i associated with an obvious solution associated with a particular plane correspond to a tensor products for quantum states associated with a particular partonic 2-surface or space-time sheet.

7. Effective 2-dimensionality realized in terms of the extremals of Chern-Simons actions with Lagrange multiplier term coming from the weak form of electric magnetic duality should also have a concrete counterpart if one takes the analogy with the extremals of Kähler action seriously. The equations can be transformed to 3-D ones by the elimination of the fourth coordinate but the interpretation in terms of discrete time evolution seems to be impossible since all points are coupled. The total conformal weights of the monomials vary in the range $[0, d_{1,i}]$ and $[0, d_{2,i}]$ so that the non-vanishing coefficients are in the interior of 3-simplex. The information about the fourth coordinate is preserved being visible via the four-determinants.

8. It should be possible to relate the hierarchy with respect to conformal weights would to the geometrization of loop integrals if a generalization of twistor strings is in question. One could hope that there exists a hierarchy of solutions with levels characterized by the rank of the matrices appearing in the linear representation. There is a temptation to associate this hierarchy with the hierarchy of deformations of vacuum extremals of Kähler action forming also a hierarchy. If this is the case the obvious solutions would correspond to vacuum exremals. At each step when the rank of the matrices involved decreases the solution becomes nearer to vacuum extremal and there should exist vanishing second variation of Kähler action. This structural similarity gives hopes that the proposed ansatz might work. Also the fact that a generalization of the Penrose's twistorial description for the solutions of Maxwell's equations to the situation when Maxwell field is induced from the Kähler form of CP_2 raises hopes. One must however remember that the consistency with other proposed solution ansätze and with what is believed to be known about the preferred extremals is an enormously powerful constraint and a mathematical miracle would be required.

7 Finiteness of generalized Feynman diagrams zero energy ontology

By effective 2-dimensionality partonic 2-surfaces plus the 4-D tangent space data at them code for quantum physics. The light-like orbits of partonic 2-surfaces in turn have interpretation as analogs of Feynman diagrams which correspond to 3-surfaces defining the regions at which the signature of the induced metric changes and 4-metric becomes degenerate. One could also identify the space-like regions of the space-time surfaces (deformed CP_2 type vacuum extremals, in particular wormhole throats) as the counterparts of generalized Feynman diagrams. The regions with Minkowskian signature of the induced metric would in turn correspond to the many-sheeted version of external space-time in which the particles move. A very concrete connection between particle and space-time geometry and topology is clearly in question.

Zero energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynmann diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. Zero energy ontology encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

7.1 Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_\pm$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermiona and $X_\pm$ migh allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

7.2 Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the modified Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^\alpha p_\alpha + \hat{\Gamma}^\alpha D_\alpha \ , \\ p_\alpha &= p_k \partial_\alpha h^k \ . \end{aligned} \tag{7.1}$$

   The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is modified gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D modified Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

4. Irrrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [12] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N-4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

7.3 Taking into account magnetic confinement

What has been said above is not quite enough. As shown in the accompanying article and in [11] the weak form of electric-magnetic duality [81] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_\pm$ pairs ($X_\pm$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion-$X_\pm$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_\pm$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and modified Dirac operator.

2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [12].

3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \rightarrow F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-antifermion pair).

4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [8].

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

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