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FRW Cosmological Solutions with Zero-Mass Scalar Field Attached to Bulk Viscous Fluid in Saez-Ballester Theory of Gravitation

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Abstract

In this paper, we have investigated spatially homogeneous isotropic Friedman cosmological models with zero–mass scalar field attached to bulk viscous fluid in Saez–Ballester theory of Gravitation. The cosmological models are obtained with the help of the special law of variation for Hubble's parameter proposed by Bermann[1] and power law relation. Some physical properties of the models are discussed.

Keywords: Friedman cosmological model, zero-mass scalar field, bulk viscous Fluid, Hubble's parameter, Saez-Ballester theory.

1. Introduction

Einstein's general theory of relativity [2] has been observed very successful in describing gravitational phenomena. It has also served as a basis for models of the universe. However, Einstein first published this theory of the gravitation, there have been many criticisms of general relativity because of the lack of certain 'desirable' features in the theory. For example, Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties, i.e. Mach's principle is not substantiated by general relativity. So, in recent year, there has been considerable interest in studying alternative theories of gravitation, the most important among them being scalar-tensor theories proposed by Lyra [3], Sen [4], Brarns and Dicke [5], Nordtvedt [6], Wagoner [7], Sen and Dunn [8], Barber theory [9].Subsequently Saez-Ballester [10] has developed a new Scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In spite of the dimensionless character of the scalar field, an antigravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies. Saez [11], Singh and Agrawal [12], Shri Ram and Tiwari [13], Reddy and Venkateswara Rao [14], Shri Ram and Singh [15], D. R. K. Reddy [16, 17], Mohanty and Sahu [18, 19, 20]. Adhav et. al. [21, 22], V.U.M. Rao et al [23, 24], Katore et.al. [25, 26, 27] are some of the authors who have studied various aspects of Saez- Ballester [10] of Scalar-tensor theory of gravitation. We hold the view that the investigation is not yet complete and there is a scope of further work which may unravel some of the hidden secrets of the universe.

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The field equations in the scalar-tensor theory proposed by Seaz-Ballester are

$$R_{ij} - \frac{1}{2} R g_{ij} - \omega \phi^n (\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k}) = -T_{ij} \quad , \tag{1.1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi_{,i}^{;i} + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0, \tag{1.2}$$

where n is an arbitrary constant, ω is a dimensionless coupling constant, T_{ij} is energy –momentum tensor and other symbols have their usual meaning.
Also the energy conservation equation

$$T^{ij}_{;j} = 0 \quad , \tag{1.3}$$

is a consequence of the field equations (1.1) and (1.2), here comma (,) and semicolon (;) denote partial and covariant differentiation respectively.

Bulk viscosity is supposed to play very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise. Further there are several aspects which are expected to radiation era and the decoupling of matter and radiation during the recombination era study the effect of bulk viscosity. These are the decoupling of neutrinos during the Bulk viscosity is associated with the GUT phase transition and string creation. It is known that the introduction of bulk viscosity can avoid the 'Big Bang' singularity. The study of Bulk Viscosity has drawn the attention of many workers, Murphy [28] constructed isotropic homogeneous spatially flat cosmological model, in general relativity, with a fluid containing bulk viscosity alone because the shear viscosity cannot exist due to assumption of isotropy. He observed that the 'Big Bang' singularity of bulk viscosity. Santosh et al [29] has obtained exact solution for isotropic homogeneous cosmological model with bulk viscous fluid considering the bulk viscous co-efficient as power function of mass density. Mohanty and pradhan [30] have studied RW cosmological model in bulk viscosity with the help of equation of state $p = (\gamma - 1)\rho$ and the special law of variation for Hubble's deceleration parameter. Mohanty and Pradhan [31] extended the work of Murphy [28] by considering the special law of variation for Hubble's parameter Berman [1] and solved Einstein's field equations when the universe is filled with viscous fluid. Mohanty and pattanaik [32] also constructed the anisotropic cosmological models with constant bulk viscous co–efficient. He observes that all the models discussed are free from initial singularity. D. R. K. Reddy and Rao [14] constructed FRW cosmological models, with bulk viscosity with equation of state $p = (\gamma - 1)\rho,$ in the Scalar-tensor theory proposed by Saez and Ballester [10]. The cosmological model is obtained with the help of special law of variation for Hubble's parameter Bremen[1]. Raj Bali et al [33], has obtained an LRS Bianchi Type V bulk viscous fluid dust distribution string cosmological model in general relativity. Spatially homogeneous and anisotropic LRS Bianchi type I string cosmological models are studied by

Tripathy et al [34], in the frame work of general relativity when the source for the energy momentum tensor is a bulk viscous fluid containing one dimensional string

The study of interacting fields, one of the fields being a zero–mass scalar fields, is basically an attempt to look into the yet unsolved problem of the unification of the gravitational and quantum theories . D. R. K. Reddy and R. Venkateswarlu [35] have investigated an exact Bianchi type I cosmological model in the presence of zero-mass scalar field when the source of the gravitation field is a perfect fluid with pressure equal to energy density. V.U.M. Rao et al [36] have investigated an exact Bianchi type VIII and IX models, in the presence of zero-mass scalar fields are presented, when the source of the gravitational field is a perfect fluid with pressure equal to energy density.

Friedman [37] was the first to investigate the most general non-static, homogeneous and isotropic space–time described by the Robertson-Walker metric. The FRW is used as a first approximation for the evolution of the universe because it is simple to calculate and models which calculate the lumpiness in the universe are added onto FRW as Extensions. FRW model studied by G.Mohanty and B.D. Pradhan [38], B. D. Pradhan and G. Mohanty [39], R. K. Tiwari [40], Katore et.al. [41, 42]. The present work is the extension of Reddy and Venkateswara Rao [14] in Saez- Ballester Scalar-tensor theory of gravitation. In section [2], the field equations in the Scalar-tensor theory developed by Saez - Ballester [10] are considered for the FRW space-time in the presence of Zero-mass scalar field attached to Bulk viscous fluid. In section [3], exact solutions of non-static FRW cosmological models are investigated by considering the special law of variation for Hubble's parameter Berman [1] and also the physical behavior of False vacuum model, Zel'dovich model and Radiating model. In section [4], FRW cosmological model with power law relation have investigated using Hubble's parameter when the metric potentials are functions of cosmic time only. We have also discussed the physical properties of the models.

2. Metric and Field Equations

We consider the spatially homogeneous and isotropic Friedmann-Robrtson-Walker (FRW) line element in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{2.1}$$

where *k* is the curvature index which can take the values (-1, 0, +1), *R (t)* represent the radius of the universe and the signature of the metric is (+, -, -, -).
The energy – momentum tensor due to the bulk-viscous fluid and zero – mass scalar fields is written in the form

$$T_{ij} = (\bar{p} + \rho)u_i u^j - \bar{p} g_{ij} + (\psi_{,i}\psi_{,j} - \frac{1}{2} g_{ij}\psi_{,m}\psi^{,m}) \tag{2.2}$$

together with

$$u^i u_i = 1 \qquad \text{and} \qquad \overline{p} = p - \eta u^i_{;i} \quad , \tag{2.2a}$$

where u^i is the four velocity vector of the distribution, ρ is the energy density, p is the pressure, η is the co–efficient of bulk viscosity, ψ is the zero mass scalar field and semi–colon (**;**) denotes covariant differentiation.
The scalar field ψ satisfies the equation

$$\psi_{;j}^{\ i} = 0 . \tag{2.3}$$

Using co-moving co-ordinates, the field equations (1.1) with the help of equation (2.1) and (2.2) can be written as

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} + \frac{k}{R^2} - \frac{\omega}{2}\phi^n \phi_4^2 = -\overline{p} - \frac{\psi_4^2}{2} , \tag{2.4}$$

$$\frac{3R_4^2}{R^2} + \frac{3k}{R^2} - \frac{\omega}{2}\phi^n \phi_4^2 = \rho + \frac{\psi_4^2}{2} , \tag{2.5}$$

$$\frac{\phi_{44}}{\phi_4} + \frac{n}{2} \quad \frac{\phi_4}{\phi} + \frac{3R_4}{R} = 0 , \tag{2.6}$$

$$\psi_{44} + \frac{3R_4}{R}\psi_4 = 0 \quad . \tag{2.7}$$

3. Solution of the field equations

We solve the field equations (2.4) – (2.7) by using the special law of variation for Hubble's parameter proposed by Bermann [1] as

$$H = DR^{-m} , \tag{3.1}$$

where D and $m(\neq 0)$ are constants.

H is the Hubble's parameter defined by

$$H = \frac{R_4}{R} \tag{3.2}$$

and suffix 4 indicates the differentiation with respect to t.

From (3.1) and (3.2), we obtained

$$R(t)=[m(Dt+C)]^{\frac{1}{m}}, \tag{3.3}$$

where C is the constant of integration.

Using equation (3.3) and (2.7) obtained as

$$\psi(t)=\frac{-a_0}{2}[m(Dt+C)]^{\frac{-2}{m}}+a_1, \tag{3.4}$$

where a_0 and a_1 are constants of integration

For metric (2.1), equation (2.2a) leads to

$$\bar{p}=p-3\eta H \tag{3.5}$$

Using equations (3.3) and (3.4), in the field equations (2.4)-(2.7) we get,

$$\bar{p}=-\left\{\frac{D^2(3-2m)}{[m(Dt+C)]^2}+\frac{k}{[m(Dt+C)]^{\frac{2}{m}}}-\frac{(\omega-1)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}}\right\}, \tag{3.6}$$

$$\rho=\left\{\frac{3D^2}{[m(Dt+C)]^2}+\frac{3k}{[m(Dt+C]^{\frac{2}{m}}}-\frac{(\omega+1)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}}\right\}, \tag{3.7}$$

$$\phi^{\left(\frac{n+2}{2}\right)}=\left\{\left(\frac{n+2}{2}\right)\frac{a_0}{2}[m(Dt+C)]^{-2/m}\right\}+a_1. \tag{3.8}$$

where a_0 and a_1 are constants of integration.

Now restricting the distribution with the barotropic equation of state

$$p=(\gamma-1)\rho, 0\le\gamma\le 2, \tag{3.9}$$

where γ the adiabatic parameter is varies continuously with the cosmic time.

In equation (3.9), we obtain the physical quantities p and η as

$$p=(\gamma-1)\left\{\frac{3D^2}{[m(Dt+C)^2}+\frac{3k}{[m(Dt+C]^{\frac{2}{m}}}-\frac{(\omega+1)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}}\right\} \tag{3.10}$$

$$\eta=\frac{1}{3H}\left\{\frac{D^2(3\gamma-2m)}{[m(Dt+C)]^2}+\frac{(3\gamma-2)k}{[m(Dt+C)]^{\frac{2}{m}}}+\frac{(-\omega\gamma-\gamma+2)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}}\right\} \tag{3.11}$$

The deceleration parameter for the model is defined by

$$q=-\frac{R_{44}R}{R_4^2} \tag{3.12}$$

For the special law (3.1), Equation (3.12), yields

$$q=(m-1) \tag{3.13}$$

After a suitable choice of constants and co-ordinates, the geometry of universe for the FRW cosmological model, space-time (2.1) becomes

$$ds^2=dt^2-(mT)^{\frac{2}{m}}\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right] \tag{3.14}$$

The model (3.14) leads to an expanding model of the universe. As the age of the universe increases the radius of the universe increases .At $t=0$, we have a non-zero radius and ρ , p, η are finite. It can be seen from equation (3.8), the scalar field in the model is finite at $t=0$ for $D > 0$ and $m\neq 0$. Thus the models avoid initial singularity at $t=0$ which supports the analysis of Murphy [28] that the introduction of bulk viscous fluid avoids initial singularity. Moreover, we find that the energy density, pressure and bulk viscosity of the fluid decrease with the increase of the age of universe.

Physical Models:

Here we discuss three physical models corresponding to γ = 0, 2, 4/3 of the equation of state given by equation (3.9)

Case (I): False vacuum model $(\gamma=0)$

For $\gamma=0$, we have $p+\rho=0$ which represents false vacuum or degenerate vacuum or ρ vacuum [cho (43)]. The physical significance of this fluid in non–viscous case has been studied by Mohanty and Pattanaik [32] Mohanty and Pradhan [30].

In this case the physical quantities

$$p = -\rho = \left\{ \frac{3D^2}{[m(Dt+C)]^2} + \frac{3k}{[m(Dt+C)]^{\frac{2}{m}}} - \frac{(\omega+1)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}} \right\} \tag{3.15}$$

$$\eta = \frac{1}{3H} \left\{ \frac{(-2m)D^2}{[m(Dt+C)]^2} - \frac{2k}{[m(Dt+C)]^{\frac{2}{m}}} + \frac{a_0^2}{[m(Dt+C)]^{\frac{6}{m}}} \right\}. \tag{3.16}$$

This model corresponds to a realistic physical situation when $\eta \geq 0$. This is only true when the Hubble parameter $H < 0$ studied by Mohanty and Pradhan [31]

Case (II): Zel' dovich fluid model (γ= 2)

If we take $\gamma = 2$, the distribution reduces to a bulk viscous stiff fluid model [Zel'dovich (44), Barrow (45)] where the density cum pressure and bulk viscous co-efficient take the forms

$$p = \rho = \left\{ \frac{3D^2}{[m(Dt+C)]^2} + \frac{3k}{[m(Dt+C)]^{\frac{2}{m}}} - \frac{(\omega+1)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}} \right\} \tag{3.17}$$

$$\eta = \frac{1}{3H} \left\{ \frac{(6-2m)D^2}{[m(Dt+C)]^2} + \frac{4k}{[m(Dt+C)]^{\frac{2}{m}}} - \frac{\omega a_0^2}{[m(Dt+C)]^{\frac{6}{m}}} \right\} \tag{3.18}$$

Case (III): Radiating model $(\gamma = \frac{4}{3})$

For $(\gamma = \frac{4}{3})$, we have $(p = \frac{1}{3}\rho)$ which represents disordered radiation and the physical quantities in this case take the form,

$$p = \frac{1}{3} \left\{ \frac{3D^2}{[m(Dt+C)]^2} + \frac{3k}{[m(Dt+C)]^{\frac{2}{m}}} - \frac{(\omega+1)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}} \right\}, \tag{3.19}$$

$$\rho = \left\{ \frac{3D^2}{[m(Dt+C)]^2} + \frac{3k}{[m(Dt+C)]^{\frac{2}{m}}} - \frac{(\omega+1)a_0^2}{2[m(Dt+C)]^{\frac{6}{m}}} \right\}, \tag{3.20}$$

$$\eta = \frac{1}{3H}\left\{ \frac{(6-2m)D^2}{[m(Dt+C)]^2} + \frac{2k}{[m(Dt+C)]^{\frac{2}{m}}} - \frac{2\omega a_0^2}{3[m(Dt+C)]^{\frac{6}{m}}} + \frac{a_0^2}{3[m(Dt+C)]^{\frac{6}{m}}} \right\} \quad (3.21)$$

4. Models with power law relation

As a majority of Cosmological models belong to either power law form or exponential form .we presently considering a power law relation between scale factor and time co-ordinate along with well known relation for pressure, density and cosmological 'constant' as

$$R(t) = n_0 t^n \quad , \quad (4.1)$$

where $n_0 (\geq o)$ *is a dimensional constant* and $n(\geq 0)$ is a numerical constant.
Equation (4.1) gives

$$\frac{R_4}{R} = \frac{n}{t} \; and \; \frac{R_{44}}{R} = \frac{n(n-1)}{t^2} . \quad (4.2)$$

With the help of equation (4.2) the field equation (2.7) gives

$$\psi(t) = \frac{b_0 t^{1-3n}}{1-3n} + b , \quad (4.3)$$

where b is a constant of integration.

With the help of equation (4.2), we obtain the equation for the effective pressure energy density ρ and scalar field ϕ from the field equations (2.4sss) - (2.7).as

$$\bar{p} = -\left[\frac{3n^2}{t^2} - \frac{2n}{t^2} + \frac{k}{n_0^2 t^{2n}} + \left(\frac{1-\omega}{2}\right)\frac{b_0^2}{t^{6n}} \right], \quad (4.4)$$

$$\rho = \left[\frac{3n^2}{t^2} + \frac{3k}{n_0^2 t^{2n}} + \left(\frac{1-\omega}{2}\right)\frac{b_0^2}{t^{6n}} \right] \quad \text{and} \quad (4.5)$$

$$\phi^{\left(\frac{n+2}{2}\right)} = \left[\left(\frac{n+2}{2}\right)\left(\frac{b_0 t^{1-3n}}{1-3n}\right) \right] + b_1 \quad (4.6)$$

where b_1 is a constant of integration.

Using the barotropic equation of state (3.9), the equation (2.2a) and (4.5) yields the expression for the pressure p and bulk viscosity co–efficient η as

$$p=(\gamma-1)\left[\frac{3n^2}{t^2}+\frac{3k}{n_0^2t^{2n}}+\left(\frac{1-\omega}{2}\right)\frac{b_0^2}{t^{6n}}\right], \tag{4.7}$$

$$\eta=\frac{1}{3n}\left[\frac{3\gamma\, n^2}{t}-\frac{2n}{t}+(3\gamma-2)\left(\frac{k}{n_0^2t}\right)+(-\gamma\omega-\gamma+2)\left(\frac{b_0^2}{2t^{5n}}\right)\right], \tag{4.8}$$

After a suitable choice of constant and co-ordinates, the geometry of universe for the FRW cosmological model, space-time becomes

$$ds^2=dt^2-t^{2n}\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right] \tag{4.9}$$

It is interesting to note that the model is isotropic and free from singularity.

Physical Models:

Here we discuss physical quantities for the model $\gamma=0,2,4/3$

Case (I): False vacuum model ($\gamma=0$)

For $\gamma=0$, we have the false vacuum or the degenerate vacuum. In this case physical quantities take the explicit form

$$\rho=-p=\left[\frac{3n^2}{t^2}+\frac{3k}{n_0^2t^{2n}}+\left(\frac{1-\omega}{2}\right)\frac{b_0^2}{t^{6n}}\right] \quad \text{and} \tag{4.10}$$

$$\eta=\frac{2}{3n}\left[\frac{n}{t}-\left(\frac{k}{n_0^2t}\right)+\left(\frac{b_0^2}{2t^{5n}}\right)\right] \tag{4.11}$$

Case (II): Zel'dovich fluid model $(\gamma=2)$

For $\gamma=2$, we have $\rho=p$ which represents stiff – fluid. In this case the phys quantities take the form

$$\rho=p=\left[\frac{3n^2}{t^2}+\frac{3k}{n_0^2t^{2n}}+\left(\frac{1-\omega}{2}\right)\frac{b_0^2}{t^{6n}}\right] \quad \text{and} \tag{4.13}$$

$$\eta = \frac{1}{3n}\left[\frac{6\,n^2}{t} - \frac{2n}{t} + 4\left(\frac{k}{n_0^2 t}\right) - 2\omega\left(\frac{b_0^2}{2t^{5n}}\right)\right]. \tag{4.14}$$

Case (III): Radiating model $(\gamma = \frac{4}{3})$

For $(\gamma = \frac{4}{3})$, the distribution reduces to the special case with equation of state $(p = \frac{1}{3}\rho)$ and the physical quantities in this case take the forms

$$p = \left[\frac{n^2}{t^2} + \frac{k}{n_0^2 t^{2n}} + \left(\frac{1-\omega}{6}\right)\frac{b_0^2}{t^{6n}}\right], \tag{4.15}$$

$$\rho = 3\left[\frac{n^2}{t^2} + \frac{k}{n_0^2 t^{2n}} + \left(\frac{1-\omega}{6}\right)\frac{b_0^2}{t^{6n}}\right] \quad \text{and} \tag{4.16}$$

$$\eta = \frac{1}{3n}\left[\frac{n}{t}(4n-2) + \left(\frac{2k}{n_0^2 t}\right) - \frac{\omega}{3}\left(\frac{b_0^2}{t^{5n}}\right) + \left(\frac{b_0^2}{t^{5n}}\right)\right] \tag{4.17}$$

I5. Conclusion

In this paper we have investigated the spatially homogeneous isotropic FRW cosmological models in presence of zero-mass scalar field attached to Bulk viscous fluid. To obtain determinate solutions of the field equations with the help of the special law of variation for Hubble's parameter proposed by Bermann [1] and power law of metric potentials have been used. We have discussed the physical models corresponding to False vacuum model, Zel'dovich fluid model, and radiating model, respectively, which are physically important cosmological models. It is observed that the models are free from singularities.

The sign of q indicates whether the model accelerates or not. The positive sign of q (i.e. $m > 1$) corresponds to decelerating models whereas the negative sign of $-1 \le q < 0$ for $0 \le m < 1$ indicates acceleration and $q=0$ for $m=1$ corresponds to expansion with constant velocity. It is interesting to note that our investigation resembles to the result obtained by D. R. K. Reddy and N. Venkateswra Rao [14] in absence of zero-mass scalar fields. This study will throw some light on the structure formation of the Universe which has Astrophysical significance.

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