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State-finder Diagnostic for Binary Mixture of Anisotropic Dark Energy and Perfect Fluid in Bianchi Type-III Universe

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Abstract

In the present work, new Bianchi type-III cosmological models with binary mixture of perfect fluid and anisotropic dark energy have been studied. In order to obtain a unique solution, it is assumed that the energy conservation equation of the perfect fluid and dark energy vanishes separately together with a special law for the mean Hubble parameter [Berman ,1983] which generates two types of solutions, one is of power law type and other is of the exponential form. To have a general description of an anisotropic dark energy component, a phenomenological parameterization of dark energy in terms of its equation of state (EoS) $\omega^{(de)}$ and two skewness parameters (γ, δ) have been introduced. The statefinder diagnostic pair [i.e. $\{r, s\}$ parameter] is used to characterize different phases of the universe. The various geometric and kinematic properties of the models and the behavior of the anisotropy of the dark energy have been discussed.

Keywords: Bianchi type-III space-time, Perfect fluid, Anisotropic dark energy, Statefinder parameter.

1. Introduction

Recent cosmological observations obtained by SNe Ia [2-8], WMAP [9-10], SDSS [11-14] and X-ray [15] indicate that the observable universe is experiencing an accelerated expansion. These observations lead to a new type of matter called as dark energy which violates the strong energy condition. i.e. This mysterious dominating negative pressure matter component of the present universe is known as dark energy and it leads to cosmic acceleration. The dark energy occupies about 73% of the energy of our universe, while dark matter occupies about 23% and the usual baryonic matter 4%. The candidates proposed to fit different properties of dark energy are the cosmological constant [16], quintessence [17], K-essence [18], tachyon [19], phantom [20, 21], ghost condensate [22], quintom [23], interacting dark energy models [24], brane world models [25] , Chaplygin gas models [26] , agegraphic dark energy models [27] , holographic dark energy models [28] and Ricci dark energy models [29]etc. Some authors [30, 31] have suggested cosmological models with anisotropic and viscous dark energy in order to explain an anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles. The binary mixture of perfect fluid and dark energy has been studied for

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Bianchi type-I and Bianchi type-V [32, 33]. The anisotropic dark energy has been studied for Bianchi type-III and Bianchi type-VIo [34, 35].

Since there are more and more dark energy models proposed to explain the cosmic acceleration, it is very desirable to find a way to discriminate between the various contenders in a model independent manner. To differentiate among different forms of dark energy, Sahni *et al.*[30] proposed a cosmological diagnostic pair $\{r, s\}$ called as Statefinder which is defined as

$$r = \frac{\ddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} \quad . \tag{1.1}$$

Here H is the Hubble parameter, a is an average scale factor and q is the deceleration parameter.

This pair gives information about dark energy in a model independent way i.e. it categorizes dark energy in the context of back-ground geometry only and it is not dependent on theory of gravity. Hence, these geometrical variables are universal. Therefore, the statefinder is a "geometrical diagnostic" in the sense that it depends upon the expansion factor and hence upon the metric describing space and time. Also, this pair generalizes the well-known geometrical parameters like the Hubble parameter and the deceleration parameter. This pair is algebraically related to the equation of state of dark energy & its first time derivative.

The statefinder pair {1,0} represents a cosmological constant with a fixed equation of state w = -1 and a fixed Newton's gravitational constant. The pair $\{1,1\}$represents the standard cold dark matter model containing no radiation. The Einstein static universe corresponds to pair {∞,-∞}[36]. The statefinder diagnostic pair is analyzed for various dark energy candidates including holographic dark energy [37], agegraphic dark energy [38], quintessence [39], dilation dark energy [40], Yang-Mills dark energy [41], viscous dark energy [42], interacting dark energy [43], tachyon [44], modified Chaplygin gas [45], f(R) gravity [46] and so on.

In the present paper, the spatially homogeneous and anisotropic Bianchi type-III cosmological models with binary bixture of perfect fluid and anisotropic dark energy have been investigated. The statefinder diagnostic pair $\{r, s\}$ is adopted to characterize different phase of the universe. The geometrical and physical behavior of the model is also discussed in both cases.

2. Metric and Field Equations

The homogenous and anisotropic Bianchi-Type-III line element is considered as

$$ds^2 = dt^2 - A(t)^2 dx^2 - B(t)^2 e^{-2\alpha x} dy^2 - C(t)^2 dz^2 \quad , \tag{2.1}$$

where $A(t)$, $B(t)$ and $C(t)$ are functions of cosmic time t only and $\alpha \neq 0$ is a constant.

In natural units $(8\pi G = 1, c = 1)$, the Einstein's field equations and the components of the energy momentum tensor of a mixture of perfect fluid and anisotropic dark energy are

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -\left({}^{m}T_{ij} + {}^{de}T_{ij} \right) \tag{2.2}$$

with

$$\begin{aligned} {}^{m}T_i^{\,j} &= diag\left[\rho^{(m)}, -p^{(m)}, -p^{(m)}, -p^{(m)}\right] \\ &= diag\left[1, -\omega^{(m)}, -\omega^{(m)}, -\omega^{(m)}\right]\rho^{(m)} \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} {}^{de}T_i^{\,j} &= diag\left[\rho^{(de)}, -p_x^{\,(de)}, -p_y^{\,(de)}, -p_z^{\,(de)}\right] \\ &= diag\left[1, -\omega_x^{\,(de)}, -\omega_y^{\,(de)}, -\omega_z^{\,(de)}\right]\rho^{(de)} \\ &= diag\left[1, -\omega^{(de)}, -\left(\omega^{(de)} + \gamma\right), -\left(\omega^{(de)} + \delta\right)\right]\rho^{(de)} \quad , \end{aligned} \tag{2.4}$$

where g_{ij} is the metric potential with $g_{ij}u^i u^j = 1$; u^i is the flow vector ; R_{ij} is the Ricci tensor ; R is the Ricci scalar ; $\rho^{(m)}$ and $\rho^{(de)}$ are the energy densities of perfect fluid and dark energy components respectively .

Here $\omega^{(m)}$ is the EoS parameter of perfect fluid with $\omega^{(m)} \geq 0$ and $\omega^{(de)}$ is the deviation-free EoS parameter of the dark energy.

One can parameterize the deviation from isotropy by setting $\omega_x^{(de)} = \omega^{(de)}$ and then one can introduce skewness parameters γ and δ which are the deviations from $\omega^{(de)}$ respectively on both the y and z axes respectively.

Here ω and γ are not necessarily constants and can be functions of the cosmic time *t*. The Einstein's field equations (2.2) for metric (2.1) with the help of equations (2.3) and (2.4) reduce to

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = \rho^{(m)} + \rho^{(de)} \ , \tag{2.5}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)}, \tag{2.6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-\omega^{(m)}\rho^{(m)}-\left(\omega^{(de)}+\delta\right)\rho^{(de)}, \tag{2.7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-\omega^{(m)}\rho^{(m)}-\left(\omega^{(de)}+\gamma\right)\rho^{(de)}, \tag{2.8}$$

$$\alpha\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0. \tag{2.9}$$

The Bianchi identity is given by

$$G^{ij}{}_{;j}={}^{m}T^{ij}{}_{;j}+{}^{de}T^{ij}{}_{;j}=0. \tag{2.10}$$

This (2.10) yields

$$\begin{aligned}&\dot{\rho}^{(m)}+\left(1+\omega^{(m)}\right)\rho^{(m)}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\dot{\rho}^{(de)}\\&\quad+\left(1+\omega^{(de)}\right)\rho^{(de)}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\rho^{(de)}\left(\frac{\dot{A}}{A}+\delta\frac{\dot{B}}{B}+\gamma\frac{\dot{C}}{C}\right)=0\end{aligned}. \tag{2.11}$$

Here dot $(\cdot)$ indicates the derivative with respect to *t*.

The directional Hubble parameters in the direction of x, y and z for the Bianchi type-III metric (2.1) are defined as

$$H_x=\frac{\dot{A}}{A},\quad H_y=\frac{\dot{B}}{B}\quad \text{and } H_z=\frac{\dot{C}}{C}. \tag{2.12}$$

The mean Hubble parameter is given by

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{2.13}$$

where $V=ABC$ is the spatial volume of the universe.

The anisotropy parameter of the expansion is defined as

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \tag{2.14}$$

where $H_i(i=1,2,3)$ represents the directional Hubble parameters in the directions of x, y and z respectively.

Solving equation (2.9), we get

$$B = c_1 A \quad , \tag{2.15}$$

where c_1 is the positive constant of integration.

Substituting the value of B from equation (2.15) in equation (2.7), and then subtracting it from the equation (2.6), the skewness parameter on the z-axis is obtained as

$$\delta = 0 \quad . \tag{2.16}$$

Now, using equations (2.12), (2.13) and (2.15), the equation (2.14) is reduced to

$$\Delta = \frac{2}{9}\frac{1}{H^2}\left(H_x - H_z\right)^2 . \tag{2.17}$$

Using equations (2.15) and (2.16), the field equations (2.5)-(2.9) reduce to

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = \rho^{(m)} + \rho^{(de)} \ , \tag{2.18}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)}, \tag{2.19}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -\omega^{(m)}\rho^{(m)} - \left(\omega^{(de)} + \gamma\right)\rho^{(de)}. \tag{2.20}$$

Subtracting equation (2.19) from equation (2.20) and solving the resulting equation, we get

$$H_x - H_z = \frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{\lambda}{V} + \frac{1}{V}\int\left(\frac{\alpha^2}{A^2} - \gamma\rho^{(de)}\right)Vdt \ ,$$

where λ is real constant of integration.

Using above equation in equation (2.17), the anisotropy parameter becomes

$$\Delta = \frac{2}{9}\frac{1}{H^2}\left[\lambda + \int\left(\frac{\alpha^2}{A^2} - \gamma\rho^{(de)}\right)Vdt\right]^2 V^{-2} . \tag{2.21}$$

The integral term in above equation (2.21) vanishes for

$$\gamma = \frac{\alpha^2}{\rho^{(de)} A^2} . \tag{2.22}$$

The energy momentum tensor (2.4) for anisotropic dark energy becomes

$$^{de}T_i^{\ j} = diag\left[1, \ -\omega^{(de)}, \ -\omega^{(de)} - \frac{\alpha^2}{\rho^{(de)} A^2}, \ -\omega^{(de)}\right]\rho^{(de)}. \tag{2.23}$$

The anisotropy parameter becomes

$$\Delta = \frac{2}{9}\frac{\lambda^2}{H^2} V^{-2} . \tag{2.24}$$

Also , we get

$$H_x - H_z = \frac{\lambda}{ABC} \tag{2.25}$$

Now, using equations (2.22) and (2.23) in the field equations (2.18)-(2.20), one can obtain the reduced field equations as

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} = \rho^{(m)} + \rho^{(de)} + \frac{\alpha^2}{A^2} = \rho^{(m)} + \rho^{(de)}(1+\gamma) , \tag{2.26}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)}, \tag{2.27}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)}. \tag{2.28}$$

Now, these equations (2.26)-(2.28) are three linearly independent equations with six unknowns $A, \ C, \ \rho^{(m)}, \ \rho^{(de)}, \ \omega^{(m)}, \ \omega^{(de)}$. Hence, we need three extra conditions to solve the field equations completely.

Firstly, we assume that the DE and perfect fluid is minimally interacting, i.e.

$$^{de}T^{ij}{}_{;j} = 0 \qquad \text{and} \qquad {}^{m}T^{ij}{}_{;j} = 0 \qquad .$$

Hence, the Bianchi identity (2.11) can be split into two separately additive conserved components.

Therefore, the conservation of energy momentum tensor of the DE gives

$$^{de}T^{ij}{}_{;j} = \dot{\rho}^{(de)} + \left(1+\omega^{(de)}\right)\rho^{(de)}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \rho^{(de)}\left(\frac{\dot{A}}{A} + \delta\frac{\dot{B}}{B} + \gamma\frac{\dot{C}}{C}\right) = 0 . \tag{2.29}$$

And the conservation of the energy momentum tensor of the perfect fluid gives

$$ {}^{m}T^{ij}{}_{;j} = \dot{\rho}^{(m)} + \left(1+\omega^{(m)}\right)\rho^{(m)}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) = 0 . \tag{2.30} $$

Secondly, we assume that the EoS parameter of the perfect fluid to be constant i.e.

$$ \omega^{(m)} = \frac{p^{(m)}}{\rho^{(m)}} = \text{constant.} \tag{2.31} $$

Lastly, we use respectively the exponential volumetric expansion law as

$$ V = c_2 e^{3kt} \tag{2.32} $$

and power law volumetric expansion as

$$ V = c_2 t^{3m} \quad , \tag{2.33} $$

where c_2, k and m are positive constants.

3. Model for Exponential Expansion

Using the equations (2.15) ,(2.25), (2.26), (2.27), and (2.28) for exponential volumetric expansion (2.32), we obtain scale factors as follows

$$ A = \left(\frac{c_2}{c_1 c_3}\right)^{\frac{1}{2}} e^{kt - \frac{1}{9}\frac{\lambda}{kc_2}e^{-3kt}} \tag{3.1} $$

$$ B = \left(\frac{c_1 c_2}{c_3}\right)^{\frac{1}{2}} e^{kt - \frac{1}{9}\frac{\lambda}{kc_2}e^{-3kt}} \tag{3.2} $$

$$ C = c_3 e^{kt + \frac{2}{9}\frac{\lambda}{kc_2}e^{-3kt}} \quad , \tag{3.3} $$

where c_3 is positive constant of integration.

The mean Hubble parameter is given by

$$ H = k . \tag{3.4} $$

The directional Hubble parameters on the x, y and z axis are respectively given by

$$H_x = H_y = k + \frac{1}{3}\frac{\lambda}{c_2}e^{-3kt}$$
$$H_z = k - \frac{2}{3}\frac{\lambda}{c_2}e^{-3kt}. \tag{3.5}$$

Using equations (3.4) and (3.5) in equation (2.17), the anisotropy parameter becomes

$$\Delta = \frac{2}{9}\frac{\lambda^2}{c_2^{\,2}k^2}e^{-6kt}. \tag{3.6}$$

Substituting the values of the scale factors in equation (2.20), the energy density of the perfect fluid is found to be

$$\rho^{(m)}(t) = \rho_0 e^{-3k\left(1+\omega^{(m)}\right)t}. \tag{3.7}$$

Using the scale factors and energy density(3.7) of the perfect fluid in equation (2.5), the energy density of DE is calculated as

$$\rho^{(de)} = 3k^2 - \frac{1}{3}\frac{\lambda^2}{c_2^{\,2}}e^{-6kt} - \alpha^2\frac{c_1c_3}{c_2}e^{-2kt+\frac{2}{9}\frac{\lambda}{kc_2}e^{-3kt}} - \rho^{(m)}. \tag{3.8}$$

Now using equations (3.1), (3.7) and (3.8) in equation (2.28), the deviation-free part of the anisotropic EoS parameter may be obtained as

$$\omega^{(de)} = \frac{9c_2^{\,2}k^2 + \lambda^2 e^{-6kt} + 3c_2^{\,2}\omega^{(m)}\rho^{(m)}}{\lambda^2 e^{-6kt} - 9c_2^{\,2}k^2 + 3\alpha^2 c_1c_2c_3 e^{-2kt+\frac{2}{9}\frac{\lambda}{kc_2}e^{-3kt}} + 3c_2^{\,2}\rho^{(m)}}. \tag{3.9}$$

Since $\delta = 0$, the deviation parameter γ can be obtained by using equations (3.1) and (3.8) in equation (2.22) as

$$\gamma = \frac{3\alpha^2 c_1c_2c_3}{9c_2^{\,2}k^2 e^{2kt-\frac{2}{9}\frac{\lambda}{kc_2}e^{-3kt}} - \lambda^2 e^{-4kt-\frac{2}{9}\frac{\lambda}{kc_2}e^{-3kt}} - 3\alpha^2 c_1c_2c_3 e^{-2kt+\frac{2}{9}\frac{\lambda}{kc_2}e^{-3kt}} - 3c_2^{\,2}\rho^{(m)}e^{2kt-\frac{2}{9}\frac{\lambda}{kc_2}e^{-3kt}}} \tag{3.10}$$

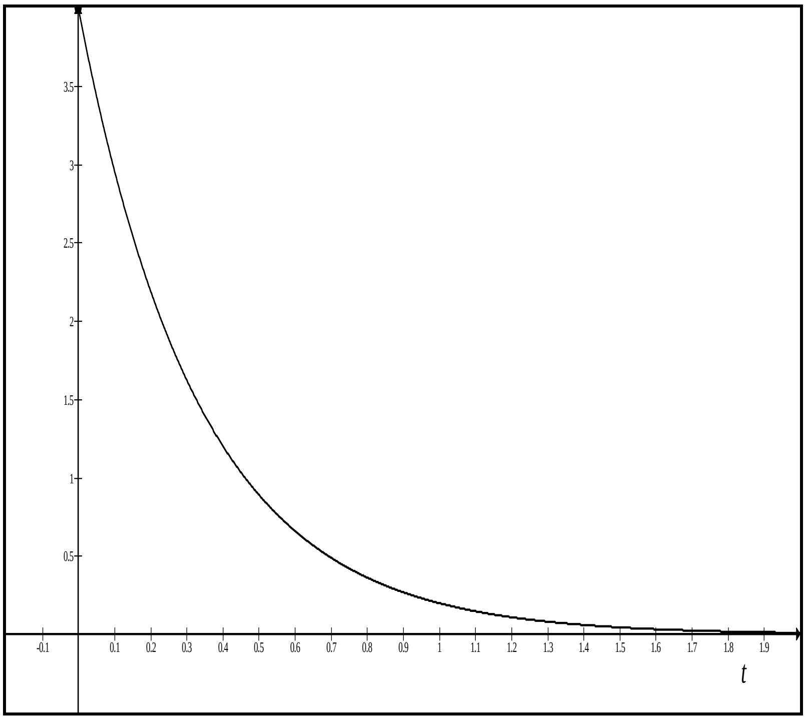

Figure 1: The plot of $\rho^{(m)}$ versus t [with ρ_0 , $k=1$] .

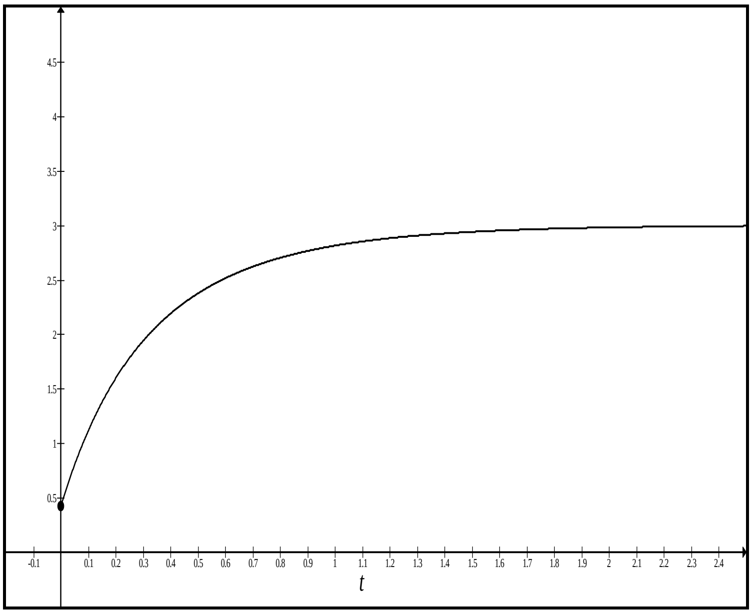

Figure 2 : The plot of $\rho^{(de)}$ with versus *t.*

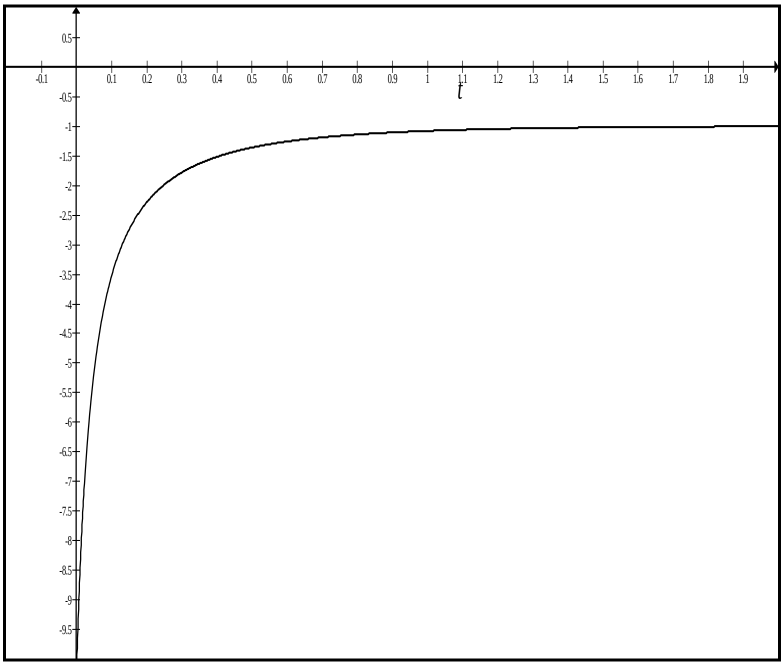

Figure 3 :The plot of $\omega^{(de)}$ with versus t.

Physical behavior of the model:

In this model, $dH/dt = 0 \Rightarrow q = -1$. This shows that the rate of expansion of the universe is the fastest. Thus, this model may represent the inflationary era in the early universe and very late times of the universe.

From equations (3.1), (3.2) & (3.3), it is observed that the spatial volume is finite at $t = 0$. It expands exponentially as t increases and becomes infinitely large as $t \to \infty$. The directional Hubble parameters are finite at $t = 0$ and $t = \infty$. They deviate from the mean Hubble parameter due to λ. While λ is supporting (opposing) the expansion on x, y axes, it opposes (supports) the expansion on z axis. The expansion scalar $\theta = 3H = 3k$, is constant throughout the evolution of the universe.

From equation (3.7) & (3.8), we get that the energy density of the perfect fluid $\rho^{(m)}$ decreases exponentially and converges to zero since $\omega^{(m)} \geq 0$. The energy density of the DE component changes slightly at early times and converges to a non-zero value as t increases. Thus, the ratio $\rho^{(de)}/\left(\rho^{(m)} + \rho^{de}\right)$ converges to 1 as t increases. This implies that the dark energy dominates the perfect fluid in the inflationary era. The equation of state parameter of the DE $\omega^{(de)}$ begins in phantom region $\omega^{(de)} < -1$ and tends to -1 by exhibiting various patterns as t increases.

The anisotropy of the expansion (Δ) decreases to null exponentially when t increases. Thus the space approaches to isotropy in this model. The deviation parameter $\delta = 0$ throughout and γ is finite at $t = 0$.

From equation (1.2), the statefinder parameters are found as

$$r = 1 \text{ and } s = 0,$$

which is similar to the ΛCDM cosmological model for which the statefinder parameters are $\{r, s\} = \{1, 0\}$.

According to the conditions given by Collins & Hawking [47], the model isotropizes for large value of *t*. Also the anisotropy of the dark energy isotropizes for large value of *t*. This is consistent with the preset day observations that the universe is isotropic.

4. Model for Power Law Expansion

Solving equations (2.26)-(2.28) for the power law volumetric expansion (2.33) by considering (2.15) and (2.25), we obtain scale factors as follows

$$A = \left(\frac{c_2}{c_1 c_3}\right)^{\frac{1}{2}} t^m e^{-\frac{1}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}} \tag{4.1}$$

$$B = \left(\frac{c_1 c_2}{c_3}\right)^{\frac{1}{2}} t^m e^{-\frac{1}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}} \tag{4.2}$$

$$C = c_3 t^m e^{\frac{2}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}} \tag{4.3}$$

where c_3 is positive constant of integration.

The mean Hubble parameter is obtained as

$$H = \frac{m}{t}. \tag{4.4}$$

And the directional Hubble parameters on the x, y and z are respectively given by

$$H_x = H_y = \frac{m}{t} + \frac{1}{3}\frac{\lambda}{c_2} t^{-3m} \text{ and } H_z = \frac{m}{t} - \frac{2}{3}\frac{\lambda}{c_2} t^{-3m}. \tag{4.5}$$

The anisotropy parameter becomes

$$\Delta = \frac{2}{9}\frac{\lambda^2}{c_2{}^2}\frac{t^{2-6m}}{m^2}. \tag{4.6}$$

Using the value of scale factors in equation (2.30), the energy density of the perfect fluid is found as

$$\rho^{(m)}(t) = \rho_0 t^{-3m\left(1+\omega^{(m)}\right)}. \tag{4.7}$$

The energy density of DE is found by using the scale factors and energy density of the perfect fluid (4.7) in equation (2.5) as

$$\rho^{(de)} = 3m^2 t^{-2} - \frac{1}{3}\frac{\lambda^2}{c_2{}^2} t^{-6m} - \alpha^2 \frac{c_1 c_3}{c_2} t^{-2m} e^{\frac{2}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}} - \rho^{(m)}. \tag{4.8}$$

Now, using equations (4.1), (4.7) and (4.8) in equation (2.28), the deviation-free part of the anisotropic EoS parameter may be obtained as

$$\omega^{(de)} = \frac{3m^2 c_2{}^2 t^{-2} + \lambda^2 e^{-6m} + +4m\lambda c_2 t^{-3m-1} + 3c_2{}^2\omega^{(m)}\rho^{(m)}}{\lambda^2 e^{-6m} - 3m^2 c_2{}^2 t^{-2} + 3\alpha^2 c_1 c_2 c_3 t^{-2m} e^{\frac{2}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}} - 3c_2{}^2\rho^{(m)}} \tag{4.9}$$

Since $\delta = 0$, γ can be obtained by using equations (4.1) and (4.8) in equation (2.22) as

$$\gamma = \frac{3\alpha^2 c_1 c_2 c_3}{9m^2 c_2{}^2 t^{2m-2} e^{\frac{2}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}} - \lambda^2 t^{-4m} e^{\frac{2}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}} - 3\alpha^2 c_1 c_2 c_3 - 3c_2{}^2\rho^{(m)} t^{2m} e^{\frac{2}{3}\frac{\lambda}{c_2}\frac{t^{1-3m}}{3m-1}}}. \tag{4.10}$$

Physical behavior of the model

From the (4.4), it is observed that the mean Hubble parameter *H* is infinitely large at $t = 0$ and null at $t = \infty$. For $0 \le m < 1$ or $q < 0$ indicates that the universe is accelerating. For $m > 1$ the universe is decelerating. In particular for $m = 1$, we get $q = 0$ indicating that the universe is expanding with constant velocity.

The volume of the universe expands indefinitely for all values of *m*. The anisotropy of the expansion (Δ) behaves monotonically, decays to zero for $m > 1/3$ and diverges for $m < 1/3$ as $t \to \infty$ and is constant for $m = 1/3$.

The terms λ, α and $\rho^{(m)}$ contribute the energy density of the DE component negatively.

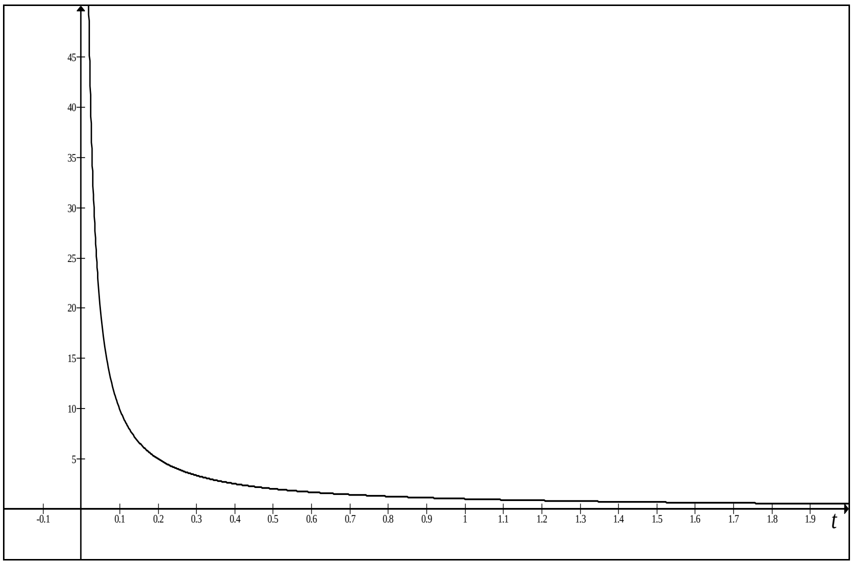

Figure 4: The plot of $\rho^{(m)}$ versus t [with $\rho_0, k = 1$].

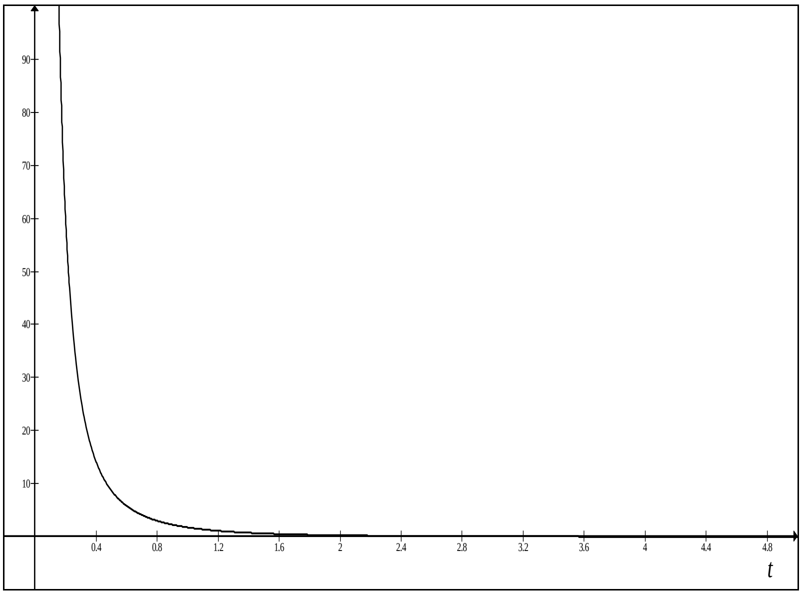

Figure 5 : The plot of $\rho^{(de)}$ with versus *t*.

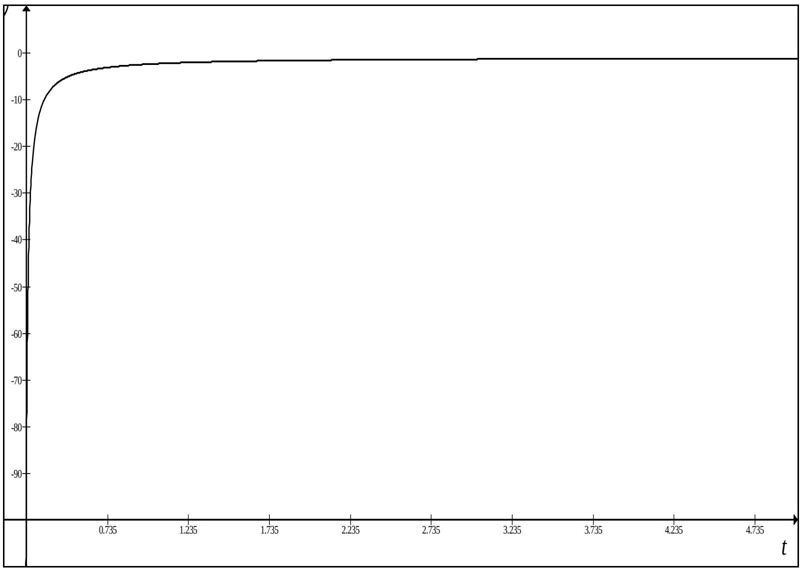

Figure 6 :The plot of $\omega^{(de)}$ with versus t.

From equation (1.2), the statefinder parameters are found as

$$r(t) = \frac{\ddot{a}}{aH^3} = \frac{(m-1)(m-2)}{m^2}$$

and $$s(t) = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = \frac{2(m-1)(m-2)-2m^2}{3m(2-3m)} = \frac{2m(r-1)}{3(2-3m)}$$

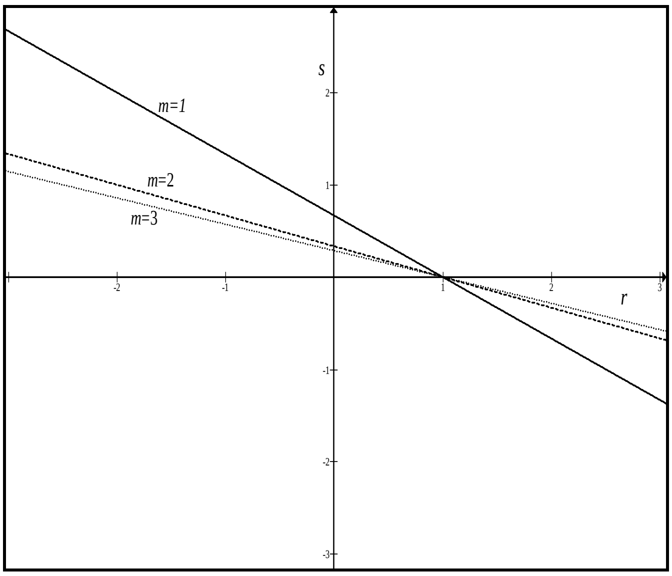

***Figure7** : variation r against s for different values of* $m\,(=1,2,3)$.

5. Conclusion

The Bianchi type-III cosmological models with perfect fluid and anisotropic dark energy have been studied on the basis of exponential expansion and power law expansion . In both the models the anisotropic dark energy isotropizes for large value of *t*. The anisotropy of the space isotropizes for exponential expansion. Also the anisotropy of the space isotropizes for power law expansion model provided $m > 1/3$. The statefinder diagnostic pair i.e. $\{r, s\}$ parameter is adopted to characterize different phases of the universe. It is observed that the exponential expansion model is similar to the ΛCDM cosmological model for which the statefinder parameters are $\{r, s\} = \{1, 0\}$. In case of power law expansion model, the evolution trajectories in the statefinder *r-s* plane are plotted in fig.7. This power law expansion model has the contours in the *r-s* plane which show the different phases of the universe. The most of the observations are similar to the results obtained by Akarsu & Kilinc [34].

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