Article

Nonlinear Theory of Elementary Particles Part XII: The Interaction Description

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Abstract

In the present article it is shown, that all known interactions of elementary particles (except for gravity) have as a basis the electromagnetic interactions. In particular the classical (linear) electrodynamics describes only the 4-vector interaction. In the same time the nonlinear electrodynamics, or more general, the nonlinear theory of elementary particles (NTEP) includes both vector and axial interaction, integrating it into a unified electroweak interaction, and also the strong interaction.

Keywords: non-linear quantum theory, vector interaction, axial-vector interaction.

1. Introduction. The modern state of the interaction description

In previous articles we have shown, that in the nonlinear electrodynamics, i.e., in theory of elementary particles (NTEP), the equations of free particles are mathematically equivalent to the equations of quantum field theory. The purpose of present part is to show that the mathematical description of interaction in NTEP are also equivalent to that in quantum theory.

1.1 The force and energy forms of the interaction description

It is known that interactions define the most important characteristics of the matter motion. They are included in all equations of motion: equations of Newton, Schrödinger, Dirac, equations of weak and strong interactions, etc.

As it is known, interaction can be expressed as force and as energy. The force form of the description of interaction is integral, and the energy form relatively to the last is differential. These forms are interconnected and can be defined one through another. In classical physics, the force is equal to a gradient of potential energy. Generally this dependence is more complex, but is also defined by the operation of differentiation. This implies the particularity of the connection of these two kinds of interaction description: the full unambiguity of transition from force to energy (and on the contrary) does not exist. For example, it is always possible to add to the energy some function (at least, a constant) so that the force value does not change.

In modern physics the most general forms of the interaction description are introduced by Lagrange and Hamilton approaches (Leech, 1958; Landau and Lifshitz, 1977).

1.2 Lagrangian and Hamiltonian aproaches

1.2.1 Mechanical system of a rigid body (particles)

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The Lagrangian mechanics works generaly in n-dimensional configuration space, which includes all parameters, defining the state of mechanical system (coordinates of particles, orientation of rigid body, etc). A point x_ν in this space draws a curve $x_\nu(t)$ in evolution ($\nu = 1,2,...,n$, where n is a number of independent variables). For such curves a functional $S(x(t))$, called action, is introduced. Only those curves, on which the action reaches an extremum, correspond to real evolution (Hamilton principle).

Usually consideration is connected to functional of the form:

$$S = \int_{t_1}^{t_2} \bar{L}\, dt, \tag{12.1.1}$$

with Lagrange function $\bar{L} = \bar{L}(x_\nu, \dot{x}_\nu)$ dependent only on generalized coordinates and velocities $x_\nu, \dot{x}_\nu$. For one material point this expression will be written down as follows

$$\bar{L} = \bar{L}(\vec{r}, \dot{\vec{r}}, t) = \bar{L}(\vec{r}, \vec{\upsilon}, t), \tag{12.1.2}$$

where $\vec{r}, \vec{\upsilon}, t$ are radius-vector, velocity and time correspondingly.

The condition of extremum for the action

$$\delta S = \delta\left(\int_{t_1}^{t_2} \bar{L}\, dt\right) = 0, \tag{12.1.3}$$

leads to Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial \bar{L}}{\partial \dot{x}_i} - \frac{\partial \bar{L}}{\partial x_i} = 0, \tag{12.1.4}$$

This is (normally) a system of second order differential equations, with solutions uniquely defined by initial coordinates and velocities $x_\nu(0), \dot{x}_\nu(0)$.

In Hamiltonian's mechanics approach the state of the system is described by point (x, p) in 2n-dimensional phase space, where *p* is momenta of particle. The dynamics is defined by a function $\bar{\mathrm{H}}(x, p)$, called Hamilton function, via equations:

$$\dot{x} = \frac{\partial \bar{\mathrm{H}}}{\partial p}, \quad \dot{p} = \frac{\partial \bar{\mathrm{H}}}{\partial x}, \tag{12.1.5}$$

Transition from Lagrange's to Hamilton's function mechanics is performed by *Legendre transformation*. It defines the momenta and Hamilton's function as:

$$p(x, \dot{x}) = \frac{\partial \bar{L}}{\partial \dot{x}}, \quad \bar{\mathrm{H}}(x, p) = p\dot{x} - L, \tag{12.1.6}$$

The Hamilton function depends on coordinates and momenta, so one should express the velocities via momenta, inverting the definitions of momenta: $\dot{x} = \dot{x}(x, p)$, and substitute the result into Hamilton's function.

1.2.2 *Continuous systems (fields)*

For fields the Lagrange function is defined by density of Lagrange function L in following way:

$$\overline{L} = \int L d\tau , \qquad (12.1.7)$$

where $d\tau$ is an element of spatial volume. The Lagrange function density or Lagrangian depends generally on field functions and their derivatives, coordinates and time:

$$L = L\left(\psi_\mu , \frac{\partial \psi_\mu}{\partial x_\nu} , x_\nu \right), \qquad (12.1.8)$$

where ψ_μ are the field functions, $\mu = 1,2,...,N$ (N is a number of the functions); $\nu = 1,2,...,n$ (n is a number of independent variables). In this case the action will be written down as follows:

$$S = \int_{t_1}^{t_2} L dx^1 dx^2 ... dx^n , \qquad (12.1.9)$$

and Euler-Lagrange equations in case of the continuous system (field) become:

$$\sum_{i=1}^{n} \frac{\partial}{dx^\nu} \frac{\partial L}{\partial \dot{\psi}_\mu} - \frac{\partial L}{\partial \psi_\mu} = 0 , \qquad (12.1.10)$$

In the present time the Lagrangians are selected on base of some general requirements of symmetry (invariance).

The approach of Hamilton in case of a continuous system performs by following way. Putting the value $\overline{\mathrm{H}}$ named density of Hamilton function, or Hamiltonian of the system:

$$\mathrm{H} = \mathrm{H}\left(\psi_\nu , \frac{\partial \psi_\nu}{\partial x_\nu} , \pi_\nu , x_\nu \right), \qquad (12.1.11)$$

so that

$$\mathrm{H} = \int \overline{\mathrm{H}} d\tau , \qquad (12.1.12)$$

the Legendre transformation can be now writen down as:

$$\pi_\nu = \frac{\partial L}{\partial \dot{\psi}_\nu} , \ \mathrm{H} = \sum_\nu \pi_\nu \dot{\psi}_\nu - L , \qquad (12.1.13)$$

where π_ν is canonical momentum density. Then the dynamics is defined by Hamiltonian via equations:

$$\dot{x} = \frac{\partial \mathrm{H}}{\partial \pi} , \ \ \dot{\pi} = \frac{\partial \mathrm{H}}{\partial x} , \qquad (12.1.14)$$

Hamilton's function defines the full energy of system. When Hamiltonian is known, it is possible to express through it all other characteristics of system. This approach is most frequently used for the description of elementary particles and fields.

1.3 Structure of Lagrangian and Hamiltonians of interaction systems

Numerous testing established that any Lagrangian can be presented in the form:

$$L = L_{free} + L_{\mathrm{int}} , \qquad (12.1.15)$$

where the first term L_{free} answers the sum of the Lagrangian of free particles, and the second term L_{int} answers their interaction by pairs with each other.

Analogously can be recorded the Hamiltonian of the system of the interacting bodies (particles):

$$\mathrm{H} = \mathrm{H}_{free} + \mathrm{H}_{\text{int}}, \qquad (12.1.16)$$

In the classical mechanics free particles are usually the moving solid bodies (material points). As interactions are here considered in essence only elastic and gravitational interactions (note that the development of the relativistic theory of gravity showed that would be more correctly to examine the gravitational field and gravitational interaction as the independent areas of physics).

In the classical electrodynamics the system contains electromagnetic fields and charged particles. The Lagrangian and Hamiltonian of first and second systems are considered as free, without taking into account their interactions. With respect to their interactions there is a special feature: since in the usual cases the EM fields do not interact with each other, Lagrangian of interaction includes only interaction of EM field with the charge particles.

In the quantum field theory (Kaempffer, of 1965; Ryder, 1985), as in the other cases, the Lagrangian is postulated as sum (12.1.15). But the quantum field theory has the following special feature: here fields and particles are considered equally. Therefore the boundary between Lagrangian of free particles and Lagrangian of interaction become relative. Lagrangian of free particles is written as the sum of the Lagrangian of each free particle, but some of these particles can be considered as the interaction fields.

For example, in QED we have:

$$L_{free} = L_e + L_\gamma, \qquad (12.1.17)$$

where $\overline{L}_e, \overline{L}_\gamma$ are the Lagrangians of free electron and photon respectively. But photon is simultaneously both particle and interaction carrier.

In this case Lagrangian of free electron is selected to give the Dirac electron equation. Lagrangian of interaction is postulated in the form "current-on-current" interaction (or it is derived on the basis of the gauge invariance principle).

In other divisions of Standard Model (SM) the Lagrangian of free particles and their interaction are the generalization of Lagrangian of QED (Ryder, 1985; Frauenfelder and Henley, 1974).

1.4 Some significant Lagrangians and Hamiltonians of classical physics

1.4.1 *Conservative systems of the material bodies*

As "conservative" we call here the narrow class of the systems, for which the forces are potential gradients. For such systems of material bodies (paticles), it was established that Lagrange's function could be expressed as follows:

$$\overline{L} = \overline{L}(x_\nu, \dot{x}_\nu) = T(x_\nu, \dot{x}_\nu) - V(x_\nu), \qquad (12.1.18)$$

where $T(x_\nu, \dot{x}_\nu) = \sum_{\nu=1}^{n} \frac{m_\nu \vec{\upsilon}_\nu^2}{2}$ is total kinetic energy of the system of n particles; $V(x_\nu) = V(x_1, x_2, ..., x_n)$ is potential energy of system. Here the first term answers energy of free particles and corresponds to L_{free}, and the second term answers interaction energy of particles and corresponds to L_{int}.

Thus the expression (12.1.18) can be rewritten as:

$$\overline{L} = \overline{L}_{free} + \overline{L}_{\text{int}}, \tag{12.1.19}$$

Moreover force is expressed as gradient of potential energy $\vec{F} = -gradV(\vec{r})$.

Note also that in relativistic mechanics the correct equations of motion is obtained only when instead of kinetic energy the value is entered:

$$K = m_0 c^2 \left(1 - \sqrt{1 - \frac{\upsilon^2}{c^2}}\right), \tag{12.1.20}$$

which is named the kinetic potential.

From Euler-Lagrange equations we obtain the equations of motion of material point (which are practically the Newton equations of motion):

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}_i} = \frac{\partial T}{\partial x_i} - \frac{\partial V}{\partial x_i}, \tag{12.1.21}$$

where $\frac{d}{dt}\frac{\partial T}{\partial \dot{x}_i} = Q_i$ are inertial forces; $\frac{\partial T}{\partial x_i} = Q_{c-K}$ are the generalized form of centrifugal and Coriolis forces; $\frac{\partial V}{\partial x_i} = Q_\nu$ are the generalized forces of interaction.

1.4.2 Electrodynamics. Non-conservative forces

In the vacuum Lagrangian of classical electrodynamics (Landau and Lifshitz, 1977; Jackson, 1999) looks like sum (12.1.15). The Lagrangian:

$$L_{free} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right), \tag{12.1.22}$$

answers free electromagnetic field. The Lagrangian of interaction describes only interaction of EM field and charged particles:

$$L_{\text{int}} = j_\mu A^\mu$$

Expression for the Hamiltonian of free EM field has the form:

$$\mathrm{H}_{free} = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right), \tag{12.1.23}$$

The Hamiltonian of interaction (i.e. the energy density of interaction) is written as:

$$\mathrm{H}_{\mathrm{int}} = -\frac{1}{2}\left(\rho\varphi + \vec{j}\cdot\vec{A}\right), \tag{12.1.24}$$

In nature it is not always possible to assign the forces in the form of gradient of potential. In particular, this does not occur in the electrodynamics. But surprisingly (Leech, 1958) in this case the generalized components of force can be assigned so that it keep the form of Euler-Lagrange equations.

It appears that instead of potential $V(x_\nu)=V(x_1,x_2,...,x_n)$, which does not dependent on time, it is often possible to set the function $\overline{M}=\overline{M}(x_\nu,\dot{x}_\nu)$ so that the generalized force, instead of $Q_i=-\frac{\partial V}{\partial x_i}$, can be written as:

$$Q_i^{'}=\frac{d}{dt}\left(\frac{\partial\overline{M}}{\partial\dot{x}_i}\right)-\frac{\partial\overline{M}}{\partial x_i}, \tag{12.1.25}$$

For example in such important case as electrodynamics the Lorentz force can be expressed in the above form if as $\overline{M}$ -function we will choose the following expression:

$$\overline{M}=e\left(\varphi-\frac{1}{c}\vec{\upsilon}\cdot\vec{A}\right), \tag{12.1.26}$$

where φ is a scalar potential, and $\vec{A}$ is a vector potential of an electromagnetic field. Actually, substituting this expression in (12.1.20), we will obtain:

$$F_i=\left[\frac{d}{dt}\left(\frac{\partial}{\partial\dot{x}_{i\,i}}\right)-\frac{\partial}{\partial x_i}\right]\overline{M}=\left[\frac{d}{dt}\left(\frac{\partial}{\partial\dot{x}_i}\right)-\frac{\partial}{\partial x_i}\right]e\left(\varphi-\frac{1}{c}\vec{\upsilon}\cdot\vec{A}\right), \tag{12.1.27}$$

By differentiation of (12.1.27) and taking into account that $\vec{B}=\vec{\nabla}\times\vec{A}$ and $\vec{E}=-\vec{\nabla}\varphi-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$, it is easy to obtain the usual expression of Lorentz force:

$$\vec{F}=e\vec{E}+\frac{e}{c}\vec{\upsilon}\times\vec{B}=e\vec{E}+\frac{1}{c}\vec{j}\times\vec{B}, \tag{12.1.28}$$

Since in this case

$$\overline{L}_{\mathrm{int}}=-\overline{M}(x_\nu,\dot{x}_\nu), \tag{12.1.29}$$

we have that the $\overline{M}$ -function is the energy of the electromagnetic interaction, corresponding to Lorentz force. In fact, using known relationships of 4-current and 4-potential:

$$I_\nu=e\upsilon_\nu=e(ic,\vec{\upsilon}), \tag{12.1.30}$$

$$A_\nu=(i\varphi,\vec{A}), \tag{12.1.31}$$

we obtain the known expression of the current-on-current interaction energy:

$$\overline{M}=-I_\nu A_\nu, \tag{12.1.32}$$

Using 4-current density:

$$j_\nu=\rho\upsilon_\nu=\rho(ic,\vec{\upsilon}), \tag{12.1.33}$$

we can introduce the M -dencity:

$$M = -j_\nu A_\nu , \qquad (12.1.34)$$

1.5 About the interaction description in the quantum field theory

In quantum field theory the general Lagrangian is postulated in form (12.1.15). But there are no enough proved arguments here (Kaempffer, 1965), which allow to deduce theoretically the real interactions.

The rule of replacement in the presence of an electromagnetic field p_μ with $\left(p_\mu + j_\mu A_\mu\right)$ is known for a long time. It is successfully applied to the correct description of experimental situations, when the representation of an electromagnetic field through classical potentials is meaningful. The substantiation of this choice can be made, proceeding from the gauge invariance principle. But the gauge invariance is introduced in this case as postulate, which does not explain the matter.

Let us note something that is important in connection with our theory. It is noted (Kaempffer, 1965) that derivative of phase $\phi(x)$ of ψ -function can be expressed through the electromagnetic potentials as follows:

$$\frac{\partial \phi}{\partial x_\nu} = -eA_\mu , \qquad (12.1.35)$$

This relationship leads to some observable effects, whose sense for the understanding of interaction has been realized from Aharonov and Bohm (Aharonov and Bohm, 1959). In connection with NTEP it is interesting also (Kaempffer, 1965) that it is possible to formulate the QED without the potentials if we recognize that non-locality is inherent to the concept of the phase, which depends on the integration way, as Mandelstam has shown (Mandelstam, 1962). Then it is more reasonable to consider the Bohm-Aharonov experiment as the instruction of essential non-locality of ψ - function in EM field.

Note that the interaction of the type (10.1.34) is called the vector interaction in quantum field theory. Besides this, there is the axial-vector interaction, which, together with the vector interaction, defines the so-called electro-weak interactions of elementary particles

1.5.1 *Some features of the description of the interactions in quantum field theory*

In the classical field theory coordinate representation of the material particles and their interactions is used. In quantum field theory, in contrast to the classical field theory, there is the possibility of more than one equivalent descriptions of the motion of particles and their interactions. In particular, the use of Laplace or Fourier transformations allows to pass from the coordinate representation to the momentum representation (in other words, we can talk about the transition from coordinate space to momentum space).

Currently, in the QFT the momentum representation is mainly used. As we noted, this creates difficulties in the modern theory (in particular, the renormalization procedure is then needed). In

NTEP we use the coordinate representation of the wave functions and energy (potential) interactions (see previous part of the study).

In this section we will make a brief comparison of the interaction description in both views (Lokhtin, 2009; Valecka, 2008).

The complete theory - quantum field theory - must incorporate both QM and relativity and individual forces are described here by QFT Lagrangians which essentially tell us, which particles interact with which other particles. But in special cases these equations coincide with the classical non-relativistic equations.

For example, the Coulomb force equation is classical (both non-quantum mechanical and nonrelativistic) limit of QM theory. As for gravitation Newton law, that can be derived from general relativity. Strong force is confining, and we can not ever observe individual color charged particles; so we can not really have a macroscopic equation for them. But at the level of quantum hadron dynamics the Yukawa potential, though it is a semi-classical approximation, is good enough for experiment description. Etc.

In the QFT in contrast to classical physics the forces among the elementary particles are considered as the *exchange forces*, i.e., the forces produced by the exchange of force carrier particles: photons, intermediate bosons and gluons.

Electromagnetic forces

In coordinate space r, the interaction energy is:

$$\varepsilon_{em}(\vec{r}) = \frac{e_1 e_2}{4\pi}\frac{1}{r}, \tag{12.1.36}$$

To get the force related to this we would take the derivative in *r*:

$$\vec{F} = grad\varepsilon_{em}(\vec{r}) = \frac{e_1 e_2}{4\pi}\frac{1}{r^2}\frac{\vec{r}}{|\vec{r}|}, \tag{12.1.37}$$

In momentum space q the energy is written as:

$$\varepsilon_{em}(q) = \int \varepsilon_{em}(r) e^{-i\vec{q}\vec{r}} d^3\vec{r} \sim \frac{\alpha}{q^2}, \tag{12.1.38}$$

$\alpha = 1/137$ is electromagnetic constant.

The exchange of energy (or force action) is realized by the virtual photons' exchange.

Strong interactions

In coordinate space

$$\varepsilon_{strong}(\vec{r}) = \frac{g_s^2(\vec{r})}{4\pi}\frac{1}{r}, \tag{12.1.39}$$

In the momentum space it is:

$$\varepsilon_{st}(q) = \int \varepsilon_{st}(r) e^{-i\vec{q}\vec{r}} d^3\vec{r} \sim \frac{\alpha_{st}}{q^2}, \qquad (12.1.40)$$

α_{st} is strong constant.

The exchange of energy (or force action) is realized by the virtual gluons' exchange.

From the study of the spectrum of quarkonium (bound system of quark and antiquark) and the comparison with positronium one finds as potential for the strong force

$$\varepsilon_{st}(\vec{r}) = -\frac{4\hbar c g_s(\vec{r})}{3}\frac{1}{r} + kr, \qquad (12.1.41)$$

where the constant k determines the field energy per unit length and is called string tension. For short distances $r \approx 0{,}4$ fm this resembles the Coulomb law, while for large distances the confinement factor kr dominates.

Yukawa potential of nuclear force

The nuclear force is now understood as a residual effect of the more powerful strong force, or strong interaction. This force is mediated by particles called pions:

$$\varepsilon_{nuc}(\vec{r}) = \frac{g_n^2}{4\pi c^2}\frac{1}{r}e^{-M_{pi}r}, \qquad (12.1.42)$$

where M_{pi} is roughly the pion mass and g_n is an effective coupling constant.

Weak force

In coordinate space

$$\varepsilon_{weak}(\vec{r}) = \frac{g_1 g_2}{4\pi}\frac{1}{r}e^{-M_Z r}, \qquad (12.1.43)$$

In momentum space

$$\varepsilon_w(q) = \int \varepsilon_w(r) e^{-i\vec{q}\vec{r}} d^3\vec{r} \sim \frac{\alpha_w}{q^2 + M_Z^2}, \qquad (12.1.44)$$

$G_F \sim 1/M_Z^2$ is the Fermi constant of weak interaction.

The exchange of energy (or force action) is realized by the virtual Z bosons' exchange.

Gravitational Newton force

For complitness we also add to this list the gravitational interaction.

In coordinate space

$$\varepsilon_{gr}(\vec{r}) = G_N \frac{m_1 m_2}{r}, \qquad (12.1.45)$$

In momentum space

$$\varepsilon_{gr}(q) = \int \varepsilon_{gr}(r) e^{-i\vec{q}\vec{r}}\, d^3\vec{r} \sim G_N \frac{m_1 m_{21}}{q^2}, \qquad (12.1.46)$$

The exchange of energy (or force action) is realized by the virtual gravitons' exchange. G_N is the gravitation constant.

The target of following sections is to show that NTEP allows us to obtain the mathematical description of these interactions.

2. The derivation of the Lagrangian and the Hamiltonian of 4-vector interactions

2.1 The theorem of the Lagrangian and Hamiltonian structure of the vector interaction

As we showed, special feature of NTEP is the unified description of all elementary particles as the nonlinear quantized electromagnetic wave fields. This result gives grounds to assume that the Lagrangian and Hamiltonian of non-linear electromagnetic field must describe all sides of the behavior of particles, including their interaction.

We will prove below the theorem, which confirms this assumption. The proof of theorem is based on the properties of Lagrangian, which follow from the above brief review. A Lagrangian is a very convenient tool for the operating with composite systems: in the case of the joining up of several non-interacting bodies into the system (or even several systems into one), their Lagrange functions are added. With the appearance of interactions between the bodies, the corresponding interaction energy of these particles is added into Lagrange's function. The same can be said of the Hamiltonian.

We will accept as initial the expressions of Lagrangian and Hamiltonian for the free electromagnetic field (particle):

$$L = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right), \qquad (12.2.1)$$

$$\mathrm{H} = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right), \qquad (12.2.2)$$

Let us show that within the framework of NTEP the Lagrangian (12.2.1) and Hamiltonian (12.2.2) allow to obtain the description not only of free fields, but also interactions.

For this purpose we prove the theorem, called by us the **theorem of the interaction structure**:

Due to the principle of superposition of fields the Lagrangian (Hamiltonian) of the system of interacting fields is automatically divided into two parts: the part, which corresponds to free fields, and the part, which corresponds to their interaction; in this case :

1.*The Lagrangian (Hamiltonian) of the system of free particles is defined by the sum of the Lagrangians (Hamiltonians) of the free particles, each of which is determined by the squares of its own fields;*

2. *Lagrangian (Hamiltonian) of interaction of particles is the sum of the Lagrangians (Hamiltonians), each of which describes interaction only of one pair of the particles, does not depend on the presence of other particles and is determined by cross product of their fields.*

Let us first prove the general formula of theorem.

Let the system consists of two parts (particles) 1 and 2, which have both the electric and magnetic fields: $\vec{E}_1$, $\vec{H}_1$ and $\vec{E}_2$, $\vec{H}_2$. According to the superposition principle a total field of system of particles is equal to the sum of the fields, created by each particle separately: $\vec{E} = \vec{E}_1 + \vec{E}_2$, $\vec{H} = \vec{H}_1 + \vec{H}_2$.

Thus, for Lagrangian and Hamiltonian of two interacting particles we obtain:

$$L = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right) = \frac{1}{8\pi}\left(\vec{E}_1^2 - \vec{H}_1^2\right) + \frac{1}{8\pi}\left(\vec{E}_2^2 - \vec{H}_2^2\right) + \frac{1}{8\pi}\left(\vec{E}_1\vec{E}_2 - \vec{H}_1\vec{H}_2\right), \quad (12.2.3)$$

$$\mathrm{H} = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right) = \frac{1}{8\pi}\left(\vec{E}_1^2 + \vec{H}_1^2\right) + \frac{1}{8\pi}\left(\vec{E}_2^2 + \vec{H}_2^2\right) + \frac{1}{8\pi}\left(\vec{E}_1\vec{E}_2 + \vec{H}_1\vec{H}_2\right), \quad (12.2.4)$$

As we see, because of the principle of superposition of fields, Lagrangian (Hamiltonian) of the particles' system is actually divided into two parts, one of which is determined only by own fields of particles, and the second is determined by the fields of the pairs of particles.

It is not difficult to see that the cross terms are determined by the fields, which belong always to two different particles.

Now we need prove that these terms determine the interaction of particles.

2.1 The description of interaction of two charge particles

Taking into account the known results (Landau and Lifshitz, 1977; Jackson, 1999; Brillouin, 1970) we will prove first the above theorem for the case of two charged particles.

2.1.1. *The description of interaction in case of rest particles*

Let us consider first a case when only electrostatic fields are present.

Let we have two charges q_1 and q_2, situated on distance r_a from each other. The values of field in any point of space P from charges, which distances from P are determined with radius-vectors $\vec{r}_1$ and $\vec{r}_2$, are:

$$\vec{E}_1 = \frac{q_1}{r_1^{\,2}}\vec{r}_1^{\,0}, \quad \vec{E}_2 = \frac{q_2}{r_2^{\,2}}\vec{r}_2^{\,0}, \tag{12.2.5}$$

where $\vec{r}_1^{\,0}$ and $\vec{r}_2^{\,0}$ are the unit vectors of corresponding radius-vectors, and r_1 and r_2 are their absolute values. The energy density of an electric field in point P is equal to:

$$\begin{aligned}\frac{1}{8\pi}\vec{E}^2 &= \frac{1}{8\pi}\left(\vec{E}_1+\vec{E}_2\right)^2 = \frac{1}{8\pi}\left[\vec{E}_1^{\,2}+2\vec{E}_1\vec{E}_2+\vec{E}_2^{\,2}\right]= \\ &= \frac{q_1^{\,2}}{8\pi\ r_1^{\,4}}+\frac{q_2^{\,2}}{8\pi\ r_2^{\,4}}+2\frac{q_1q_2}{8\pi\ r_1^{\,2}r_2^{\,2}}\cos\theta\end{aligned}, \tag{12.2.6}$$

where θ is a angle between vectors $\vec{r}_1$ and $\vec{r}_2$. Thus the Lagrangian of total field can be written down as:

$$L = L_1 + L_2 + L_{12}, \tag{12.2.7}$$

where $L_1 = \frac{1}{8\pi}\vec{E}_1^{\,2} = \frac{q_1^{\,2}}{8\pi\ r_1^{\,4}}$, $L_2 = \frac{1}{8\pi}\vec{E}_2^{\,2} = \frac{q_2^{\,2}}{8\pi\ r_2^{\,4}}$, $L_{12} = \frac{1}{4\pi}\vec{E}_1\cdot\vec{E}_2 = 2\frac{q_1q_2}{8\pi\ r_1^{\,2}r_2^{\,2}}\cos\theta$.

Here the first and second terms obviously represent the Lagrangian of free particles (fields). To find out the meaning of third term, we will calculate the Lagrange function, corresponding to this term, using (12.1.7). Since $\vec{E}_2 = -grad\varphi_2 = -\vec{\nabla}\varphi_2$, where $\varphi_2 = \frac{q_2}{r_2}$ is static potential for the second charge, we will obtain:

$$L = -\frac{1}{4\pi}\int\vec{\nabla}\varphi_2\cdot\vec{E}_1 d\tau, \tag{12.2.8}$$

Integrating by parts, we obtain:

$$L = -\frac{1}{4\pi}\int\vec{\nabla}\varphi_2\cdot\vec{E}_1 d\tau = -\frac{1}{4\pi}\varphi_2(E_x+E_y+E_z)\Big|_{-\infty}^{\infty}+\frac{1}{4\pi}\int\varphi_2\left(\vec{\nabla}\vec{E}\right)d\tau, \tag{12.2.9}$$

Here the first term is equal to zero, and in the second term, according to Maxwell we have:

$$\vec{\nabla}\vec{E} = 4\pi\rho_e, \tag{12.2.10}$$

where ρ_e is the density of electric charge q_1. Then, accepting, that $r_o << r_a$, we obtain:

$$L_{12} = \rho_e\varphi\ , \tag{12.2.11}$$

This means that *the Lagrangian, which is adequated to the third term, is Lagrangian of interaction of two charges and has the form of a current - current interaction for the case of a static field.*

2.1.2 *The description of interaction in case of moving particles*

Now we will consider the Lagrangian of two interacting charges, which are in motion. Here together with electric field the magnetic field will also appear. Thus, we should analyze a general view of the Lagrangian in case of any motion of electric charges

First of all, a question arises of whether the electric field varies in case that charges move. This question can be formulated in more general sense: will the Gauss theorem be true in case when the charges move? The experiment answers positively (Purcell, 1975). Hence, the above-stated analysis, concerning static electric field, is true in case of moving charges. Thus, it is enough to analyze further only the term of the general Lagrangian, which contains a magnetic field.

In point P, the magnetic fields from each particle have the form:

$$\vec{H}_1 = \frac{q_1}{r_1^2}\left[\vec{\upsilon}\times\vec{r}_1^{\,0}\right], \vec{H}_2 = \frac{q_2}{r_2^2}\left[\vec{\upsilon}\times\vec{r}_2^{\,0}\right], \tag{12.2.12}$$

where $\vec{\upsilon}$ is the particle velocity. Then the energy density will be

$$\begin{aligned}&\frac{1}{8\pi}\vec{H}^2 = \frac{1}{8\pi}\left(\vec{H}_1+\vec{H}_2\right)^2 = \frac{1}{8\pi}\left[\vec{H}_1^2+\vec{H}_2^2+2\vec{H}_1\vec{H}_2\right]= \\ &= \frac{q_1^{\,2}}{8\pi\ r_1^{\,4}}\left[\vec{\upsilon}\times\vec{r}_1^{\,0}\right]^2 + \frac{q_2^{\,2}}{8\pi\ r_2^{\,4}}\left[\vec{\upsilon}\times\vec{r}_2^{\,0}\right]^2 + 2\frac{q_1q_2}{8\pi\ r_1^{\,2}r_2^{\,2}}\left[\vec{\upsilon}\times\vec{r}_1^{\,0}\right]\cdot\left[\vec{\upsilon}\times\vec{r}_2^{\,0}\right]\end{aligned}, \tag{12.2.13}$$

The result (12.2.13) can be rewritten as:

$$L = L_1 + L_2 + L_{12}, \tag{12.2.14}$$

where $L_1 = \frac{1}{8\pi}\vec{H}_1^2$, $L_2 = \frac{1}{8\pi}\vec{H}_2^2$ are Lagrangian of free particles, and according to our supposition the term $L_{12} = \frac{1}{4\pi}\vec{H}_1\cdot\vec{H}_2$ is the Lagrangian of interaction. This fact follows from direct calculating of the Lagrangian:

$$\bar{L}_{12} = \int L_{12}d\tau, \tag{12.2.15}$$

Since $\vec{H}_2 = \vec{\nabla}\times\vec{A}_2$, where $\vec{A}_2 = \frac{1}{c}\frac{i_2}{r_2}\vec{r}_2^{\,o} = \frac{1}{c}\int\frac{\vec{j}_2}{r_2}d\tau_2$ is the vector potential of current of the second charge, we will obtain:

$$L_{12} = \frac{1}{4\pi}\int\vec{\nabla}\times\vec{A}_2\cdot\vec{H}_1 d\tau, \tag{12.2.16}$$

Integrating by parts in scalar form, we obtain:

$$L_{12} = \frac{1}{4\pi}\sum_{l,n}(\pm)\vec{A}_{2l}\vec{H}_m\Big|_{-\infty}^{+\infty} + \frac{1}{4\pi}\int\vec{A}_2\left(\vec{\nabla}\times\vec{H}\right)d\tau, \tag{12.2.17}$$

where $l=(x,y,z)$, $m=(x,y,z)$, $l\neq m$ and under the sum the signs are alternated. Here the first term is equal to zero, and in the second term according to Maxwell we have:

$$\vec{\nabla}\times\vec{H} = \frac{4\pi}{c}\vec{j}, \tag{12.2.18}$$

Then we obtain:

$$L_{12} = \frac{1}{c}\vec{j}\cdot\vec{A} = L_{\text{int}}, \tag{12.2.19}$$

This means that the *Lagrangian, relatively to magnetic fields of two moving charges, has the form of current-on- current interaction*.

So, generally we obtain that the interaction Lagrangian of two moving charge particles is defined by the commutator of the electric and magnetic fields of two particles, and can be written down in the form of a current-on-current interaction:

$$L_{\text{int}} = -j_\nu A_\nu, \tag{12.2.20}$$

Obviously, in this case the general Hamilton function of interactions will be written down as follows:

$$\overline{\text{H}}_{\text{int}} = -\frac{1}{2}\int\left(\rho\varphi + \vec{j}\cdot\vec{A}\right)d\tau = q\varphi - \frac{q}{c}\vec{\upsilon}\cdot\vec{A}, \tag{12.2.21}$$

2.3 Consequences of the theorem

We have shown that the cross product of fields in Lagrangian (Hamiltonian) corresponds to a current-on-current interaction form. From this the next important consequences follow:

1. *The full energy of two interacting objects is bigger than the sum of energies of free objects, and the energy difference corresponds to the energy of cross product of fields.*

An important question arises: how the interaction energy of two objects is divided between them?

Let's consider one concrete case of electric field. The full of energy density of two interacting particles looks like:

$$u = \frac{1}{8\pi}\left(\vec{E}_1 + \vec{E}_2\right)^2 = u_{1o} + u_{\text{int}} + u_{2o}, \tag{12.2.22}$$

where $u_{1o} = \frac{1}{8\pi}\vec{E}_1^{\,2}$ and $u_{2o} = \frac{1}{8\pi}\vec{E}_2^{\,2}$ are the energy densities of first and second particles in a free state respectively, and $u_{\text{int}} = \frac{1}{8\pi}\left[\vec{E}_1\vec{E}_2 + \vec{E}_2\vec{E}_1\right]$ is the density of interaction energy of these particles.

2. Since both components $u_{\text{int}1} = \frac{1}{8\pi}\vec{E}_1\vec{E}_2$ and $u_{\text{int}2} = \frac{1}{8\pi}\vec{E}_2\vec{E}_1$ in the above formula are equal:

$$u_{\text{int}1} = u_{\text{int}2}, \tag{12.2.23}$$

we can accept that *the interaction energy is divided fifty-fifty between two interacting particles.* Then the full energy density of each interacting particles is equal to $u_1 = \frac{1}{8\pi}\left(\vec{E}_1^{\,2} + \vec{E}_1\vec{E}_2\right)$ for first particle, and to $u_2 = \frac{1}{8\pi}\left(\vec{E}_2^{\,2} + \vec{E}_1\vec{E}_2\right)$ for second particle.

Since the energy is defined as $\varepsilon = \frac{1}{c^2}\int_0^\infty u d\tau$, the same conclusion refers also to the energies of rest and interacting particles.

3. From above, in conformity with the known expression $m = \frac{\varepsilon}{c^2}$, it follows also that *the mass of each interacting particle increases in comparison with the mass of free particle on half value of the term of the field cross product.*

In other words, the mass of interaction of two particles divides fifty-fifty between them so that $m_{\text{int}1} = m_{\text{int}2}$, and for the masses of interacting particles we have

$$m_1 = m_{01} + m_{\text{int}1},\ \ m_2 = m_{02} + m_{\text{int}2}, \qquad (12.2.24)$$

where m_{01} and m_{02} the rest masses of particles without interaction.

3. Vector electromagnetic interaction as the basis of the interactions in macro-word

3.1 Interaction description of systems of many charge particles

In case when the system consists of a number of charged particles of both (+) and (-) signs and different sizes, we receive the object, possessing various new electromagnetic properties.

As it is known, the system of moving charges possesses in general the electromagnetic moments. In some cases the total charge of such system (which in this case are also named zero moments of a system) can be equal to zero, while other moments are not. This means that these systems are capable to interact due to other moments. The interaction energy of such systems is much lower than the energy of interaction of the charged systems, but is not equal to zero. For example, the atoms, being the neutral objects, nevertheless are capable to interact between themselves by various forces, which frequently are named Van der Waals forces. In QM these forces depend additionally on spin orientation and other quantum parameters.

Since in framework of NTEP the neutral particles are an electromagnetic fields, their interactions must also described by the formula of a current-on-current interaction.

For description of the charge system the potentials are usually used. As it is known, the use of potentials facilitates the mathematical analysis of electrodynamics problems. Since in the NTEP instead of potentials the strengths of electromagnetic field is considered as wave function, we remember that in electrodynamics the interaction can be written through field strengths. In general case (Bredov, Ruma'ntsev et al, 1985) the electromagnetic fields of a moving charge can be described as following:

$$\vec{E}(\vec{r},t) = \frac{e(1-\vec{\upsilon}^2/c^2)(\vec{n}-\vec{\upsilon}/c)}{R^2(1-\vec{n}\cdot\vec{\upsilon}/c)^3}\Big|_{t'} + \frac{e\vec{n}\times[(\vec{n}-\vec{\upsilon}/c)\times\dot{\upsilon}]}{R^2(1-\vec{n}\cdot\vec{\upsilon}/c)^3}\Big|_{t'}, \qquad (12.3.1)$$

$$\vec{H}(\vec{r},t) = \frac{e(1-\vec{\upsilon}^2/c^2)(\vec{\upsilon}\times\vec{n})}{R^2(1-\vec{n}\cdot\vec{\upsilon}/c)^3}\Big|_{t'} + \frac{e\{c\dot{\upsilon}\vec{n}+\vec{n}\times[(\vec{\upsilon}\times\dot{\upsilon})\times\vec{n}]\}}{R^2(1-\vec{n}\cdot\vec{\upsilon}/c)^3}\Big|_{t'}, \qquad (12.3.2)$$

Each of these expressions consists of two components. The first components of (12.3.1)and (12.3.2) forms quasi-stationary fields, which change in space as R^{-2} and does not contain the acceleration of a charge. Hence, the quasi-stationary field remains all time connected with a particle and does not create energy flax on infinity.

The second components describe a wave field of radiation: it is proportional to acceleration $\dot{\upsilon}$ and decreases as R^{-1}. It is easy to show, that in this case the energy flax decreases as R^{-2}. Thus on large distances from a particle in expressions (12.3.1)-(12.3.2) only the second terms remains, named wave field. This means that the electromagnetic perturbations can propagate from charge particle to the infinity. Due to these fields, i.e. electromagnetic waves, the particle systems interact with each other on a long distance.

The description of charged particles' system by means of potentials is more often, although it have mathematical advantages only. In general case of arbitrarily moving charges we obtain the so-called Lienar - Wiechert potentials:

$$\vec{A}(\vec{r},t)=\frac{e\vec{\upsilon}}{cR(1-\vec{n}\cdot\vec{\upsilon}/c)}\Big|_{t'}, \qquad (12.3.3)$$

$$\varphi(\vec{r},t)=\frac{e}{cR(1-\vec{n}\cdot\vec{\upsilon}/c)}\Big|_{t'}, \qquad (12.3.4)$$

where $\vec{s}(t')$ are the coordinates of the particle, $\vec{r}$ are the coordinates of the observation point, $\vec{\upsilon}(t)=\dot{\vec{s}}(t)$ is its velocity, e is the charge, $\vec{R}(t')=\vec{r}-\vec{s}(t')$ and t' is the retarded moment of time, which is defined by the relation:

$$c(t-t')=\left|\vec{r}-\vec{s}(t')\right|, \qquad (12.3.5)$$

so that difference $t-t'=\left|\vec{r}-\vec{s}(t')\right|/c$ represents the time of distribution of the electromagnetic perturbation from the particle up to an observation point of field.

3.2 The case of stationary system of electric charges

In particular, for the scalar potential in large distances from the system of charges, we have the expansion:

$$\varphi(r)=\frac{q}{r}+\frac{\vec{p}\cdot\vec{r}}{r^3}+\frac{Q_{\alpha\beta}x_\alpha x_\beta}{2r^5}+\ldots, \qquad (12.3.6)$$

where in case of continuous distribution of charges we have: $q=\int\rho(r')dV'$ is the full charge of the system, $\vec{p}=\int r'\rho(r')dV'$ is the dipole moment of the system, $Q_{\alpha\beta}=\int\rho(r')(3x'_\alpha x'_\beta-r'\delta_{\alpha\beta})dV'$ is the tensor of the quadrupole moment of the system of charges, etc. (in case of a discrete system of point charges we have sums instead of integrals).

If $q=0$ then the system is neutral and is described by other electrical moments, which are not zero.

The interactions that occur between the neutral systems of charges play an important role in the existence of real forms of matter: elementary particles, atoms and molecules.

In the case of interactions between two or more molecules are called *intermolecular* interactions (the interactions between the atoms within a molecule are called *intramolecular* interactions). Intermolecular interactions occur between all types of molecules or ions in all states of matter. They have range from the strong, long-distance electrical attractions and repulsions between ions to the relatively weak dispersion forces. The various types of interactions are classified as (in order of decreasing strength of the interactions): ion - ion, ion - dipole, dipole - dipole, ion - induced dipole, dipole - induced dipole, dispersion forces, etc. They can be often explained using a simple classical electrodynamics approach, but for more accuracy a quantum mechanical approach needs.

4. About the Newton dynamics laws

Let's show that in framework of NTEP the consequences of the above-stated theorem are the equations of classical mechanics (Leech, 1958).

4.1 First Newton's law (inertia law)

First Newton's law can be formulated as follows: "An object at rest or moving at constant velocity will continue to do so unless acted upon by an external force". In this form this law can be considered as the consequence of the second Newton's law (although here there are some difficulties, which there is no sense to examine in this case).

Obviously, an inertial motion is accomplished only when there is no energy loss. Motion without energy loss is characteristic of "linear" photon motion. Then, a neutral particle, formed by the rotation of photon fields, must also move without energy loss. It is possible to assume that this motion must be described by the theory, related to the theory of superfluidity. Since nonlinear equations of NTEP reveal relationship with the equations of superconductivity and superfluidity, our assumption has a base.

4.2. *The second law of Newton*

According to the second Newton's law we have

$$\frac{d\vec{p}}{dt} = \sum \vec{F}_i \,, \qquad (12.5.1)$$

where $\vec{p}$ is a linear momentum of the particle motion, $\vec{F}_i$ are all forces acted on the particle.

For particles in electromagnetic field the equation (12.3.1) follows from relativistic Lagrangian of the charge particle in the electromagnetic fields (see e.g. (Bo Thide, 2002; Jackson, 1999)). If the question is only about the electromagnetic field without particles, in this case "The momentum theorem" exists, which shows the validity of the Newton law concerning the electromagnetic field

as a material carrier. It is understood that in framework of NTEP "The momentum theorem" can be easy to prove.

We should also remind that according to Ehrenfest theorem (Schiff, 1955) it is possible from Dirac equation to obtain the Newton equation (12.3.1) (Leech, 1958; Landau and Lifshitz, 1977; Schiff, 1955). Since the Dirac equation coincides precisely with the electron equation of NTEP, it is possible to assert that Ehrenfest theorem proves that the equation of Newton also follows from NTEP.

4.3 *Third Newton's law*

Recall Newton's Third Law: "For every action there is an equal and opposite reaction". Let us show that third Newton's law follows from the features of electromagnetic theory. Actually, taking into account the expression (12.1.29), we can write:

$$\varepsilon_{\text{int}1} = \varepsilon_{\text{int}2} = \frac{1}{2}\overline{M}\,, \tag{12.5.2}$$

On the base (12.1.25) from (12.3.5) it follows that action and counteraction forces are equal and have opposite directions $\vec{F}_1 = -\vec{F}_2$ in accordance with Newton's Third Law .

5. Connection of de Broglie's waves refraction index with Hamiltonian

In frameworks of NTEP the equations of interaction of the electron with other charged particles (or, in other words, the equations of the electron motion in the field of other particle) can be presented in form of the equations of the classical electrodynamics of medium:

$$\frac{1}{c}\frac{\partial \vec{E}}{\partial t} - rot\vec{H} = -\frac{4\pi}{c}\left(\vec{j}^{\,e} + \vec{j}^{\,e}_{ex}\right), \tag{12.5.1}$$

$$\frac{1}{c}\frac{\partial \vec{H}}{\partial t} + rot\vec{E} = \frac{4\pi}{c}\left(\vec{j}^{\,m} + \vec{j}^{\,m}_{ex}\right), \tag{12.5.2}$$

where $\vec{j}^{\,e}, \vec{j}^{\,m}$ are the electric and magnetic current densities of the particle, $\vec{j}^{\,e}_{ex}, \vec{j}^{\,m}_{ex}$ are the external current densities, which caused by the interaction of the given particle with other particles. In case if other particles (including also the virtual particles of the physical vacuum) form a medium, this equations can be presented as the electromagnetic theory of polarized medium (Jackson, 1999; Purcell, 1975; Bo Thide, 2002). In this case the external currents can be represented in the following way:

$$\vec{j}^{\,e}_{ex} = i\omega^{e}_{ex}\vec{E} \tag{12.5.3}$$

$$\vec{j}^{\,m}_{ex} = i\omega^{m}_{ex}\vec{H} \tag{12.5.4}$$

where ω^{e}_{ex} and ω^{m}_{ex} can be conditionally named electroconductivity and magnetoconductivity in the external medium.

The Hamiltonian of Dirac's electron theory is following:

$$\hat{\mathrm{H}} = c\hat{\vec{\alpha}} \cdot \hat{\vec{p}}\psi - \left[\hat{\beta}\, mc^2 + \left(\hat{\alpha}_o \varepsilon_{ex} - c\hat{\vec{\alpha}} \cdot \vec{p}_{ex}\right)\right]\psi, \qquad (12.5.5)$$

Using (12.5.2) we can obtain the NTEP representation of (12.5.5), which we will conditionally write in the form:

$$\hat{\mathrm{H}} = \pm rot\left(\vec{E}, \vec{H}\right) \mp \frac{4\pi}{c}\left(\vec{j}^{\,e,m} + \vec{j}_{ex}^{\,e,m}\right), \qquad (12.5.5')$$

The expression (12.5.5') show that the connection of Hamiltonian with above currents (12.5.3) and (12.5.4) and correspondingly with the features of external medium ε_{ex} and μ_{ex} exists.

Due to above result in non-relativistic case the Schroedinger equation with an external field can be written down through a "quantum" refraction index of medium. Conformity between electrodynamics of optical waves and electrodynamics of de Broglie waves is the most evident look for the stationary Schroedinger equation. Actually, the stationary Schroedinger equation:

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left(\varepsilon - \varepsilon_{\mathrm{int}}\right)\psi = 0, \qquad (12.5.6)$$

(where the energy ε are Hamiltonian eigenvalues, $\varepsilon_{\mathrm{int}} = e\varphi(r)$ is an interaction energy) is similar (Ebert, 1957) to the optical wave equation, which determinates the light propagation in the medium, whose refraction index changes in space from point to point:

$$\nabla^2\psi + \left(\frac{2\pi\, n}{\lambda_0}\right)^2 \psi = 0, \qquad (12.5.7)$$

where $n = n(r) = \sqrt{\varepsilon_{ex}\mu_{ex}}$ is a refraction index, λ_0 is the light wavelength in vacuum; and the optical wavelength $\lambda = \frac{\lambda_0}{n}$ corresponds to the length of the de Broglie wave $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\left(\varepsilon - \varepsilon_{\mathrm{int}}\right)}}$.

Since the elementary particles of NTEP are the nonlinear electromagnetic waves, from the above follows that when particles propagate through the medium, the refraction, diffraction, interference of these waves takes place like as for usual light waves. In this case of particles' interaction the interconversion of particles can be considered as a dispersion of the curvilinear waves. Therefore we can suppose also that the dispersion matrix of the field theory performs here the same role as a dispersion matrix in optics.

To obtain the corresponding relativistic equation is very difficult, since in this case the permittivity and permeability compose a tensor and it is not possible to introduce refractive index as above.

It is more complicated to perform the introduction of the refractive index in the Dirac equation, and in general, probably, impossible (Bredov, Ruma'ntsev et al, 1985). As is known in electromagnetic theory the properties of the medium are characterized by complex magnetic and electrical permeabilities, through which the refractive index can be expressed only in the

particular case. In this case the equations (12.5.1) - (12.5.4) will contain in place of the electric and magnetic fields, the induction vectors of electric $\vec{D} = f_1\left(\vec{E}\right)$ and magnetic $\vec{B} = f_2\left(\vec{H}\right)$ fields.

Thus, the introduction of complex permittivity in the equation of NTEP (or in the corresponding quantum equations) should be associated with the introduction of additional complex functions, reflecting the effect of external or internal fields.

6. The axial-vector interaction

In classical electrodynamics there is only the vector interaction (a vector in this case means a 4-vector-potential, consisting of a scalar and vector parts). In quantum field theory was observed the existence of some different interactions: the axial-vector or the weak interaction, which plays an important role in the interactions of elementary particles. In quantum field theory, this interaction, as well as a vector is in fact postulated. In the nonlinear theory (Kiryakos. 2010a) an axial-vector interaction occurs on the basis of the initial axioms and does not require any additional assumptions. We briefly recall this conclusion here.

6.1 The Lagrangian of the nonlinear lepton equation

The Lagrangian of nonlinear theory (see in details (Kyriakos. 2010a)) can be obtained from the Lagrangian of Dirac's linear equation. Using the linear equivalent of the energy-momentum conservation law for the internal (*in)* field of particle:

$$\hat{\beta}\ m_e c^2 = -\varepsilon_{in} - c\hat{\vec{\alpha}}\ \vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\ \vec{A}_{in}, \qquad \text{(A)}$$

where the inner (self) energy ε_{in} and momentum p_{in} of lepton can be expressed using the inner energy density u and the inner momentum density $\vec{g}$ (or Poynting vector $\vec{S}$) of an EM wave:

$$\varepsilon_{in} = \int_0^\tau u d\tau = \frac{1}{8\pi}\int_0^\tau \left(\vec{E}^2 + \vec{H}^2\right) d\tau, \qquad (12.6.1)$$

$$\vec{p}_{in} = \int_0^\tau \vec{g} d\tau = \frac{1}{c^2}\int_0^\tau \vec{S} d\tau = \frac{1}{4\pi}\int_0^\tau \left[\vec{E}\times\vec{H}\right] d\tau, \qquad (12.6.2)$$

Substituting relationship (A) into this equation, we obtain:

$$L_N = \psi^+\left(\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}}\right)\psi + \psi^+\left(\varepsilon_{in} - c\hat{\vec{\alpha}}\cdot\vec{p}_{in}\right)\psi, \qquad (12.6.3)$$

We will assume that (12.6.3), taking into account (12.6.1) and (12.6.2), *represents the general form of the Lagrangian of nonlinear electron theory.*

Therefore this Lagrangian, together with the vector interaction must also takes into account the weak interaction. In this sense, this Lagrangian is a Lagrangian of electro-weak theory. The correctness of this assumption proves the effective Lagrangian form, derived from it below.

6.2 The effective Lagrangian of the nonlinear lepton equation (the Heisenberg equation)

We can rewrite the equations (12.6.1) and (12.6.2) in the following form:

$$\varepsilon_{in} = \int_0^{\tau_0} u d\tau + \int_{\tau_0}^{\infty} u d\tau , \qquad (12.6.4)$$

$$\vec{p}_{in} = \int_0^{\tau_0} \vec{g} d\tau + \int_{\tau_0}^{\infty} \vec{g} d\tau , \qquad (12.6.5)$$

where the first terms of equations (12.6.4) and (12.6.5) contain the bulk of own energy and momentum of the particle and the second terms are the residual parts of these quantities. Obviously, these residual pieces give a small contribution to the total value of the energy and momentum.

Taking into account that the solution of Dirac's equation for a free electron is the plane wave

$$\psi = \psi_0 \exp\left[i(\omega t - ky)\right], \qquad (12.6.6)$$

we can approximately write (12.6.1) and (12.6.2) as follows:

$$\varepsilon_p = \int_0^{\tau_0} u d\tau = u\tau_0 = \frac{\tau_0}{8\pi} \psi^+ \hat{\alpha}_0 \psi , \qquad (12.6.7)$$

$$\vec{p}_p = \int_0^{\tau_0} \vec{g} d\tau = \vec{g}\tau_0 = \frac{1}{c^2} \vec{S}\tau_0 = -\frac{\tau_0}{8\pi\, c} \psi^+ \hat{\vec{\alpha}} \psi , \qquad (12.6.8)$$

where τ_0 is the volume that contains the main part of the semi-photon's energy.

Using (12.6.7) and (12.6.8), we can represent (12.6.3) in the following quantum form:

$$L_N = i\hbar \left[\frac{\partial}{\partial t} \left[\frac{1}{2} \left(\psi^+ \psi \right) \right] - c\,div \left(\psi^+ \hat{\vec{\alpha}} \psi \right) \right] + \frac{\tau_0}{8\pi} \left[\left(\psi^+ \psi \right)^2 - \left(\psi^+ \hat{\vec{\alpha}} \psi \right)^2 \right], \qquad (12.6.9)$$

or in the electromagnetic forme:

$$L_N = i \frac{\hbar}{2m_e} \left(\frac{1}{c^2} \frac{\partial u}{\partial t} + div\, \vec{g} \right) + \frac{\tau_0}{m_e c^2} \left(u^2 - c^2 \vec{g}^{\,2} \right), \qquad (12.6.10)$$

We can transform here the second term using the known electrodynamics identity (Lightman Alan R. et al.,1975):

$$(8\pi)^2 \left(u^2 - c^2 \vec{g}^{\,2} \right) = \left(\vec{E}^2 + \vec{H}^2 \right)^2 - 4 \left(\vec{E} \times \vec{H} \right)^2 = \left(\vec{E}^2 - \vec{H}^2 \right)^2 + 4 \left(\vec{E} \cdot \vec{H} \right)^2 , \qquad (12.6.11)$$

What is the physical meaning of this transformation? In (12.6.11), the expression $\left(\vec{E}^2 + \vec{H}^2 \right)^2 - 4 \left(\vec{E} \times \vec{H} \right)^2$ is scalar, i.e., is an EM invariant, but the expressions $\left(\vec{E}^2 + \vec{H}^2 \right)^2$ and $4 \left(\vec{E} \times \vec{H} \right)^2$ are separately not invariants. At the same time the expressions $\left(\vec{E}^2 - \vec{H}^2 \right)^2$ and $\left(\vec{E} \cdot \vec{H} \right)^2$ are individually the invariants of the EM theory.

Taking into account electromagnetic representation of the wave function (Kyriakos, 2010b) we can represent the nonlinear part of (12.6.10) in the following form:

$$L'_N = \frac{\tau_0}{(8\pi)^2 mc^2}\left[\left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\left(\vec{E}\cdot\vec{H}\right)^2\right], \qquad (12.6.12)$$

As we can see, it contains only the Maxwell theory invariants.

According to the equation (12.6.9) the quantum form of the Lagrangian density (12.6.12) is:

$$L'_N = \frac{\tau_0}{8\pi}\left[\left(\psi^+\hat{\alpha}_0\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\psi\right)^2\right], \qquad (12.6.13)$$

We can see that in quantum form, the electrodynamics correlation (12.6.11) takes the form of the known Fierz identity (Cheng and Li, 1984; 2000):

$$\left(\psi^+\hat{\alpha}_0\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\psi\right)^2 = \left(\psi^+\hat{\alpha}_4\psi\right)^2 + \left(\psi^+\hat{\alpha}_5\psi\right)^2, \qquad (12.6.14)$$

Using (12.6.14), we obtain from (12.6.9):

$$L_Q = \psi^+\hat{\alpha}_\mu\partial_\mu\psi + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+\hat{\alpha}_4\psi\right)^2 - \left(\psi^+\hat{\alpha}_5\psi\right)^2\right], \qquad (12.6.15)$$

As we see *the Lagrangian* (12.6.15) *coincides with the Lagrangian of Nambu – Jona-Lazinio* (Nambu and Jona-Lazinio, 1961; 1961a). The second term of this Lagrangian describes a vector-axial-vector (VA) interaction.

What is the physical meaning of expression $\left(\psi^+\hat{\alpha}_5\psi\right)$?

6.3. Helicity in fluid mechanics and electrodynamics

The physical meaning of the term $\left(\psi^+\hat{\alpha}_5\psi\right)^2 = 4\left(\vec{E}\cdot\vec{H}\right)^2$ has been studied in both classical and quantum physics. In connection with the geometric (topological) properties, this term is called chirality.

In classical physics, the analysis of the physical meaning of this term is based on the mathematical similarity between electromagnetic vectors with hydrodynamic vectors.

Kelvin's vision (Moffatt, 2008) of the role of knotted or linked vortex tubes in a hypothetical ether was largely qualitative in character. He correctly perceived that knots and linkages would be conserved by virtue of the frozen-in property of vortex lines, but he had no quantitative measure of such knottedness or linkage. The simplest such quantitative measure for any localised vorticity distribution is in fact provided by its helicity, the integrated scalar product of the vorticity field $\vec{\omega}$ and the velocity $\vec{\upsilon}$ to which it gives rise: $h = \int \vec{\upsilon}\cdot\vec{\omega}\, d\tau$.

A similar analysis shown that in the case of free electromagnetic fields, the value $\left(\vec{\mathrm{E}}\cdot\vec{\mathrm{H}}\right)$ corresponds to the above definition (Trueba, Jose L. and Ranada, Antonio F. (2000). In this regard, it is called an electromagnetic helicity.

In quantum physics (Ternov, 2000) the study of electromagnetic properties of massive neutrinos have shown that the electric dipole moment of Dirac neutrinos, as well as the magnetic, has a dynamic nature: it depends by complex nonlinear way on the strength of field and particle energy. In weak electric and magnetic fields $\vec{\mathrm{E}},\vec{\mathrm{H}} << \vec{\mathrm{B}}_0\lambda$ is equal to $d_\nu = \frac{2}{9}\frac{(\vec{\mathrm{E}}\cdot\vec{\mathrm{H}})}{\lambda \mathrm{B}_0^2}$.

The results of the analysis of neutrino equation in the NTEP showed that the helicity emerges as one of the distinguishing characteristics of the neutrino (see in details (Kyriakos, 2011)).

7. The general case of the interaction Lagrangian and Hamiltonian of NTEP

The Hamiltonian and Lagrangian of the NTEP as the non-linear theory must contain all possible invariants of non-linear electromagnetic field theory. Thus we can suppose thate Lagrangian must be some function of the field invariants:

$$L = f_L(I_1, I_2), \qquad (12.7.1)$$

where $I_1 = \left(\vec{E}^2 - \vec{H}^2\right), I_2 = \left(\vec{E}\cdot\vec{H}\right)$.

Hamiltonian is fully defined through the Lagrangian. Thus, if the function (12.7.1) is known, using the formulas (12.1.13), it is easy to calculate the Hamiltonian, which will be now the function of the various powers of electromagnetic field vectors:

$$\mathrm{H} = f_{\mathrm{H}}(\vec{E},\vec{H}), \qquad (12.7.2)$$

Apparently, for each problem the functions f_L and f_{H} must have its special form, which is unknown before the problem solution. As is known the approximate form of the function f_{H} can be found on the basis of Schroedinger's or Dirac's wave equation, using the so-called perturbation method. It is supposed here that there is an expansion of the function f_{H} in Taylor–MacLaurent power series with unknown expansion coefficient. Then the problem is reduced to the calculation of these coefficients. The solution is searched for each term of expansion separately, starting from first. Usually this is the problem for a free particle, whose solution is already known. Then using the equation with the two first terms, we find the coefficient of the second term. Further using the equation for the three first terms, we find the coefficient for the third term of expansion, etc. In many cases by this method it is possible to obtain the solution with any desirable accuracy.

In case of function of two variables $\xi = f(x,y)$ the Taylor – MacLaurent power series nearly to a point (x_0, y_0) is:

$$f(x,y) = f(x_0,y_0) + \sum_{k=1}^{n}\frac{1}{k!}\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right)^k f(x_0,y_0) + O\left(\rho^n\right), \qquad (12.7.3)$$

where $\rho = \sqrt{(x-x_0)^2 + (y-y_0)^2}$,

$$\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right) f(x_0,y_0) \equiv (x-x_0)\frac{\partial f(x_0,y_0)}{\partial x} + (y-y_0)\frac{\partial f(x_0,y_0)}{\partial y}, \qquad (12.7.4)$$

$$\left((x-x_0)\frac{\partial}{\partial x}+(y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0)\equiv (x-x_0)^2\frac{\partial f^2(x_0,y_0)}{\partial x^2}+ \\ +2(x-x_0)(y-y_0)\frac{\partial^2 f(x_0,y_0)}{\partial x\partial y}+(y-y_0)^2\frac{\partial f^2(x_0,y_0)}{\partial y^2}, \qquad (12.7.5)$$

Etc. (In case when $x_0=0,\ y_0=0$ we obtain the MacLaurent series).

Obviously, for the most types of the functions $f_L(I_1,I_2)$ the expansion contains approximately the same set of the terms, which distinguish only by the constant coefficients, any of which can be equal to zero (as examples see the expansions of the quantum electrodynamics Lagrangian for particle at the present of physical vacuum (Akhiezer and Berestetskii,. 1965; Schwinger, 1951; Weisskopf, 1936). Generally the expansion will look like:

$$L_M=\frac{1}{8\pi}\left(\vec{E}^2-\vec{B}^2\right)+L', \qquad (12.7.6)$$

where

$$L'=\alpha\ \left(\vec{E}^2-\vec{B}^2\right)^2+\beta\ \left(\vec{E}\cdot\vec{B}\right)^2+\gamma\ \left(\vec{E}^2-\vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)+ \\ +\xi\ \left(\vec{E}^2-\vec{B}^2\right)^3+\zeta\ \left(\vec{E}^2-\vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2+..., \qquad (12.7.7)$$

is the part, which is responsible for the non-linear interaction (here $\alpha,\beta,\gamma,\xi,\zeta,...$ are constants)

Corresponding Hamiltonian will be defined as follows:

$$H=\sum_i E_i\frac{\partial L}{\partial E_i}-L=\frac{1}{8\pi}\left(\vec{E}^2+\vec{B}^2\right)+\overline{H}', \qquad (12.7.8)$$

where the Hamiltonian part responsible for non-linear interaction is:

$$\hat{H}'=\alpha\ \left(\vec{E}^2-\vec{B}^2\right)\left(3\vec{E}^2-\vec{B}^2\right)+\beta\ \left(\vec{E}\cdot\vec{B}\right)^2+ \\ \xi\ \left(\vec{E}^2-\vec{B}^2\right)\left(5\vec{E}^2+\vec{B}^2\right)+\zeta\ \left(3\vec{E}^2-\vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2+..., \qquad (12.7.9)$$

It is not difficult to obtain the quantum representation of Hamiltonian (12.7.9) of non-linear theory. Replacing the electromagnetic wave field vectors by quantum wave function, we will obtain a series of type:

$$\hat{\mathrm{H}}=\left(\psi^+\hat{\alpha}_0\psi\right)+\sum c_{1i}\left(\psi^+\hat{\alpha}_i\psi\right)\left(\psi^+\hat{\alpha}_j\psi\right)+\sum c_{2i}\left(\psi^+\hat{\alpha}_i\psi\right)\left(\psi^+\hat{\alpha}_j\psi\right)\left(\psi^+\hat{\alpha}_k\psi\right)+..., \qquad (12.7.10)$$

where $\hat{\alpha}_i,\hat{\alpha}_j,\hat{\alpha}_k$ are Dirac matrixes, c_i are the coefficients of expansion.

As we see, the terms of Lagrangian and Hamiltonian series contain the same elements, such as $\left(\vec{E}^2+\vec{B}^2\right)$, $\left(\vec{E}\cdot\vec{B}\right)^2$, $\left(\vec{E}^2-\vec{B}^2\right)$ and some others. It is possible to assume that each element of series has some particular physical sense. In this case it is possible to see the analogy with expansion of fields on the electromagnetic moments, and also with decomposition of a S-matrix on the elements (Akhiezer and Berestetskii,, 1965), each of which corresponds to the particularities of interaction of separate particles.

Conclusions

The above results show that in framework of NTEP the descriptions of the interactions, used in different areas of physics (mechanics, classical electrodynamics, quantum electrodynamics, etc.), are electromagnetic. This allows us to speak about NTEP as about the unified field theory of elementary particles. We have not considered the gravitational interaction. There are serious grounds for believing that it is also a consequence of electromagnetic interaction, but the proof of this assertion requires a separate study.

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