Article

Power-Law Cosmic Expansion in *f*(*R*) Gravity for Kaluza-Klein Metric

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Abstract

Taking into account the *f*(*R*) gravity framework in our study, we investigated the Tsallis Holographic Dark Energy (THDE) model utilizing Hubble's Horizon as the infrared (IR) cut-off in Kaluza-Klein metric. We assume a relation $F\alpha a^n$ and volumetric power law expansion to solve the field equations (where n remains unchanged).The resulting model's kinematical and physical characteristics are examined via graphical representation. The corresponding values expected for cosmic expansion are included in the graphical analysis of the equation of state parameter.

Keywords: Tsallis holographic dark energy, *f*(*R*) gravity , volumetric power law expansion.

1. Introduction

According to current cosmological data, the universe is expanding [1]. How to organically integrate the observational predictions into the conventional framework of cosmology is still unclear. Furthermore, it is anticipated that 73% of the matter-energy. Dark Energy (DE), a substance with a negative pressure, is responsible for the universe's content. However, in order to fully answer the observable facts, a small cosmic constant would be helpful. Regretfully, even while this provides a decent solution to the acceleration problem, it also raises other issues, such as the cosmic coincidence problem and the fine tuning problem.

Nonetheless, there are two approaches to solving the accelerating universe problem mentioned above: (i) taking into account a changed theory of gravity, which would alter the gravity sector; or (ii) taking into account an uncommon form of matter or energy that would result in an accelerating phase in the late universe, which would alter the matter sector. Assuming cosmological approaches, the transformation of general relativity (GR) is the most effective method to illustrate the dynamical mechanism of the accelerated expansion of the universe. One

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of the most straightforward changes to general relativity in the last 10 years has been the study of (R) theories, in which a more broad function of the Ricci scalar is used in place of the Hilbert–Einstein action [2]. Using various functional forms of $f(R)$, the researchers analyzed the astrophysical results to characterize the cosmological mechanism of $f(R)$ gravity [3-13].

Higher dimensional space-time research is a particularly active area of study nowadays. There may have been a multidimensional stage before the current four-dimensional space-time. The origin, structure, and development of the cosmos are among the basic problems that the higher dimensional cosmology may provide answers. Moreover, it seeks to integrate gravity and electromagnetics, two basic natural forces. The concept of extra dimension was first introduced by Theodor Kaluza [14] and later developed by Oskar Klein[15].

Kaluza suggested that the geometry of higher-dimension space-time might explain both gravity and electromagnetism in a single framework by introducing a fifth dimension into standard four-dimensional space-time. Klein subsequently shown that the additional dimension is compatible. Nowadays, a lot of researchers are interested in higher dimensions cosmology. Many authors in a variety of scalar tensor theories study the five-dimensional Kaluza–Klein cosmological model [16-18]. As a generalization of the five-dimensional Kaluza–Klein space-time, several scholars have investigated n-dimensional space-time [19-20]. Once more, authors [21–25] study the DE cosmological model in Kaluza–Klein space-time because the hidden effects of higher dimensions could be the source of the DM and DE's invisible impacts.

This study is motivated by the growing interest in higher-dimensional theories, especially in the context of modified theories like $f(R)$ theory. In addition, the Dark Energy and Holographic Dark Energy provide a promising method for comprehending the universe's accelerated expansion.

2. Metric and Field Equations

The $f(R)$ theory of gravity is the most straightforward and widely used variant of GTR. The modified action for the $f(R)$ gravity is as follows:

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R) + L_m\right) d^4x \qquad (1)$$

where L_m is the matter Lagrangian and $f(R)$ is an arbitrary function of the Ricci scalar R. It is important to note that when $f(R) = R$, the Einstein-Hilbert action standard can be recovered. The following are the $f(R)$ gravity field equations that are generated by altering the action in relation to metric g_{uv}:

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \nabla^i \nabla_j F(R) = g_{ij}(T_{ji}^m + T_{ji}^{de}) \tag{2}$$

where $F(R) \equiv \frac{df(R)}{dR}$, ∇_i denotes covariant differentiation and T_{ij}^m , T_{ij}^{de} are the energy momentum tensor corresponding to the matter and HDE respectively.

Generally, with the usual (four-dimensional) space–time, the consolidation of gravitational forces with other forces of nature is not possible. Thus to improve the possibility of geometrically unifying the fundamental interactions of the universe, study of higher dimensional space–time is essential. In particle physics, various experiments were initiated to develop the higher dimensional cosmological models that resulted into the formulation of the Kaluza–Klein theory. Kaluza–Klein (KK) theories reveal how the gravity and the electromagnetism can be unified from Einstein's field equations generalized to five dimensions. Subsequently, different authors studied physics of the universe in the context of higher dimensional space–time viz. We consider a five dimensional Kaluza-Klein metric in the form

$$ds^2 = -dt^2 + A^2\left(dx^2 + dy^2 + dz^2\right) + B^2 d\phi^2 \tag{3}$$

where A and B are the functions of t only and where the fifth coordinate ϕ is taken to be space-like. Here the Ricci scalar is obtained as $R = 6\frac{\dot{A}^2}{A} + 6\frac{\ddot{A}}{A} + 6\frac{\dot{A}}{A}\frac{\dot{B}}{B} + 2\frac{\ddot{B}}{B}$. The energy-momentum tensor for DE fluid $\left(T_{ij}^{de}\right)$ and pressure-less matter $\left(T_{ij}^{m}\right)$ are given by

$$T_{ij}^{(de)} = \left(\rho_{de} + p_{de}\right)u^i u_j + p_{de} g_{ij} \; ; \; T_{ij}^{(m)} = \rho_m u_i u_j \tag{4}$$

where ρ_{de} is DE density, p_{de} is DE pressure, ρ_m the energy density of matter and

$$u^i u_i = -1 \, , \; u^i u_j = 0 \, . \tag{5}$$

The EoS parameter refers to the relation between pressure and energy density of fluid by $\omega_{de} = \frac{p_{de}}{\rho_{de}}$. To ensure the present acceleration of universe, here, we consider the anisotropic distribution of DE. After parameterize the energy–momentum tensor of DE $\left(T_{ij}^{de}\right)$, it can be expressed as

$$T_{ij}^{(de)} = diag\left[p_{de}, p_{de}, p_{de}, p_{de}, -\rho_{de}\right] = diag\left[w_{de}, w_{de}, w_{de}, w_{de}, -1\right]\rho_{de} \tag{6}$$

In co-moving system of coordinates from equation (4), one finds

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = w_{de}\rho_{de} \quad ; T_5^5 = -\left(\rho_m + \rho_{de}\right) \tag{7}$$

The equation of continuity for the matter and Holographic Dark Energy are

$$\dot{\rho}_m + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\left(\rho_m + p_m\right) = 0 \tag{8}$$

and $$\dot{\rho}_{de} + \frac{4}{3}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\rho_{de} = 0 \tag{9}$$

Using equations (3)-(7), the field equations are expressed as follows

$$F\left\{2\frac{\dot{A}^2}{A^2} + \frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} - \frac{1}{2}f - 2\frac{\dot{A}}{A}\dot{F} - \frac{\dot{B}}{B}\dot{F} - \ddot{F} = -w_{de}\rho_{de} \tag{10}$$

$$F\left\{\frac{\ddot{B}}{A} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} - \frac{1}{2}f - 3\frac{\dot{A}}{A}\dot{F} - \ddot{F} = -w_{de}\rho_{de} \tag{11}$$

$$F\left\{-3\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B}\right\} + \frac{1}{2}f + 3\frac{\dot{A}}{A}\dot{F} + \frac{\dot{B}}{B}\dot{F} = \rho_m + \rho_{de} \tag{12}$$

where the derivative with respect to cosmic time t is denoted by the superscript '.'.

3. Solutions of Field Equations

Equations (10)-(12) are a system of three equations in seven unknowns $A, B, F, f, w_{de}, \rho_{de}, \rho_m$. Therefore, we apply the following physically feasible conditions to get an approximate solution to the system of field equations mentioned above. We consider the power law connection between the scale factor and the scalar field as

$F \propto a^n$, where n is arbitrary constant and a is the mean scale factor of model. (13)

Using equation (13), we get

$F = ba^n$, where b is a proportionality constant. (14)

Using equations (10) and (11), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)+2\dot{A}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{F}}{F}=0 \tag{15}$$

which on integration yields

$\frac{A}{B}=\exp\left\{\int\frac{C}{A^3BF}dt\right\}+C'$, where C' is integration constant.

Hence $$\frac{A}{B}=\exp\left\{\int\frac{C}{V^2}dt\right\}+C' \tag{16}$$

where $F=V$ for $(b=1, n=4)$ and $V=A^3B=a^4$.

4. Physical Parameters

We define the spatial volume and mean Hubble's parameter for the Kaluza-Klein metric (3) may be defined as

Spatial Volume $$V=a^4=A^3B \tag{17}$$

where a is the mean scale factor.

Hubble parameter $$H=\frac{1}{4}\left(3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right) \tag{18}$$

Let us first define the deceleration parameter and the dynamical scalars, such as the

mean anisotropic parameter , expansion scalar , shear scalar as

Deceleration parameter $$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \tag{19}$$

The sign of q predicts the universe's cosmic behavior. The deceleration parameter, q, is positive for a decelerating cosmos and negative for an accelerating universe.

Mean Anisotropic parameter $$\Delta=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2 \tag{20}$$

The mean anisotropic parameter of the expansion, Δ, plays a major role in determining whether the model is isotropic or anisotropic. It calculates the deviation from isotropic expansion; when Δ = 0, the universe expands isotropically.

Scalar expansion $\theta = 4H$ (21)

Shear Scalar $\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^{\,2} - H^2\right) = 2\Delta^2 H$ (22)

5. Power Law Expansion

Here, we consider a power law volumetric expansion

$$V = C_1 t^{4n} \tag{23}$$

where n is a positive constant, encompasses all potential expansion histories throughout the universe's development. The exponent n's positive value is consistent with observational data that points to an expanding universe.

The directional scale factors can be obtained by using (16), (17) and (23) as

$$A = C_1^{\,1/4} t^n \exp\left\{\frac{C}{4C_1^{\,2}} \frac{t^{1-8n}}{(1-8n)}\right\} \tag{24}$$

$$B = C_1^{\,1/4} t^n \exp\left\{\frac{-3C}{4C_1^{\,2}} \frac{t^{1-8n}}{(1-8n)}\right\} \tag{25}$$

Using equation (23) , we get $F = V = c_1 t^{4n}$. (26)

The Ricci scalar and function of the Ricci scalar are found out to be

$$R = \frac{20n^2}{t^2} - \frac{8n}{t^2} + \frac{3}{4}\frac{C^2}{C_1^4} t^{-18n} \tag{27}$$

$$f = -40C_1 n^2 \frac{t^{4n-2}}{4n-2} + 16C_1 n \frac{t^{4n-2}}{4n-2} + \frac{C^2}{C_1^3} t^{-12n} \tag{28}$$

Holographic Dark Energy (HDE) is based on the holographic principle [26, 27], which asserts that a physical system's number of degrees of freedom scales with its boundaries rather than with its loudness. The history of a flat FRW Universe cannot be adequately explained by the HDE model, which was hypothesized with Benkenstein entropy and the Hubble horizon as the

IR cut-off [28-30]. In their discussion of the THDE model, Tavayef et al. [31] discovered that, in the absence of interaction between two dark sectors of the Universe, the late time accelerated Universe can be obtained by identifying the IR-cutoff with the Hubble radius. Using the generalized Tsallis entropy hypothesis with the Hubble horizon in the background of $f(G,T)$ gravity and the cosmological evolution through cosmic diagnostic parameters and phase planes, Sharif et al. [32] have examined the reconstruction paradigm for the THDE model with dust fluid. The Hubble horizon was used as the IR cutoff while studying Tsallis HDE in BD cosmology, as per Ref. [33]. Tsallis holographic dark energy in the Fractal Universe has been explained by Ghaffari et al. [33]. The stability analysis revealed that both the interacting and non-interacting models are classically unstable. A brief comment on the THD model has been presented by Abdollahi Zadeh et al. [34]. Statefinder diagnostics for interacting THDE models has been covered by Gunjan et al. [35].. Observational restrictions on the interactive THDE model have been highlighted by Sadri [36] . The observational limitation on interacting THD in logarithmic BD theory was examined by Aditya et al. [37] . Using the power law solution of the scale factor, Sharif and Saba [38] have developed a reconstruction scenario for the THDE model against the backdrop of the modified theory of gravity with the Hubble horizon and the generalized Tsallis entropy conjecture. The derivation of original holographic dark energy (HDE), which is presented as

$$\rho_{de} = \frac{3c^2 M_p^2}{L^2} \tag{29}$$

and it depends on the entropy A_1, S area relationship of the black hole i.e $S \propto A$ where A_1 is the area of the event horizon of the black hole. From the relation between the UV and IR cutoffs a new expression for entropy is

$$L^3 \Lambda^3 \leq \rho^{3/4} \tag{30}$$

where L and Λ are IR and UV-cutoffs, respectively. The HDE can be redefined

$$S = \gamma A_1^{\delta}, \tag{31}$$

where γ is an unknown constant and δ denotes the non-additivity parameter. In the limit $\gamma = \frac{1}{G}$ and $\delta = 1$, the Bekenstein-Hawking entropy is recovered. By combining equations (30) and (31), the energy density is given as

$$\rho_{THDE} = DL^{2\delta-4} \tag{32}$$

Where D being an unknown parameter. It is Tsallis Holographic Dark Energy (THDE).By choosing the simplest IR-cutoff as Hubble horizon $\left(L = H^{-1}\right)$, the energy density becomes

$$\rho_{THDE} = DH^{4-2\delta} \tag{33}$$

6. Physical and kinematical properties

Some physical and kinematical properties of the obtained model are as follows:

The spatial volume and Hubble parameter are obtained to be

$$\text{Spatial-Volume} \qquad V = C_1 t^{4n} \tag{34}$$

$$\text{Hubble parameter} = \frac{n}{t} \tag{35}$$

Deceleration parameter and Anisotropy parameter are also obtained

$$\text{Deceleration parameter} = \frac{1}{n} - 1 \tag{36}$$

$$\text{Anisotropy parameter } \Delta = \frac{3C^2 t^{-16n+2}}{16 C_1^4 n^2} \tag{37}$$

Expansion scalar and Shear scalar for the obtained cosmological model are given below,

$$\text{Shear Scalar} = \frac{3}{8} \frac{C^2}{C_1^4} t^{-16n} \tag{38}$$

$$\text{Expansion Scalar } \theta = \frac{4n}{t} \tag{39}$$

The Tsallis Holographic Dark Energy is given by

$$\rho_{THDE} = D\left(\frac{n}{t}\right)^{4-2\delta} \tag{40}$$

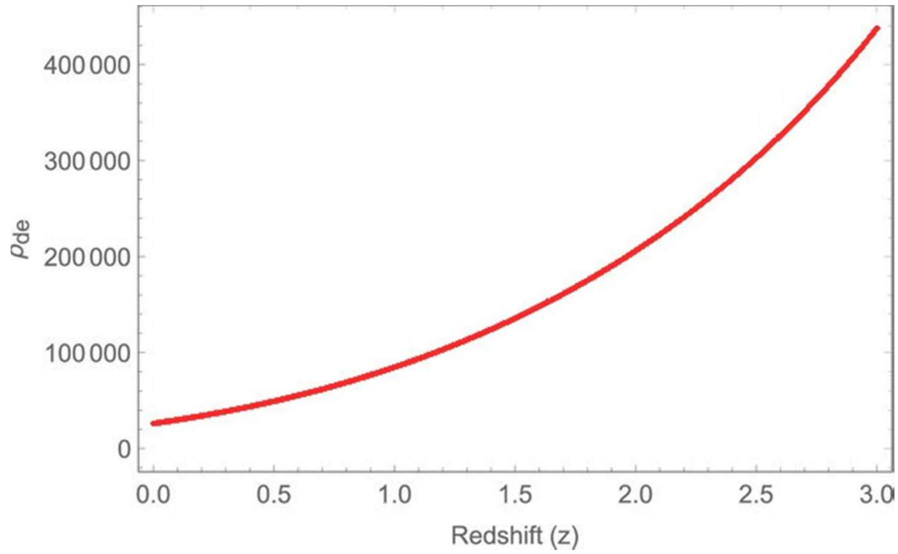

Figure 1. Energy density of Dark Energy vs Redshift (z)

In Fig. 1 , we can see that the variations of energy density of THDE versus redshift z remains positive throughout the evolution of the Universe and is an increasing function of z, or, equivalently, a decreasing function of cosmic time t.

The matter energy density is obtained as

$$\rho_m = 4C_1 n t^{4n-2}\left[-(n-1)+\frac{(-5n+2)}{4n-2}+4n\right]+\frac{9nC}{C_1}t^{-4n-1}+\frac{1}{2}\frac{C^2}{C_1^2}t^{-12n}\left(1+\frac{3}{4C_1}\right)-D\left(\frac{n}{t}\right)^{4-2\delta} \quad (41)$$

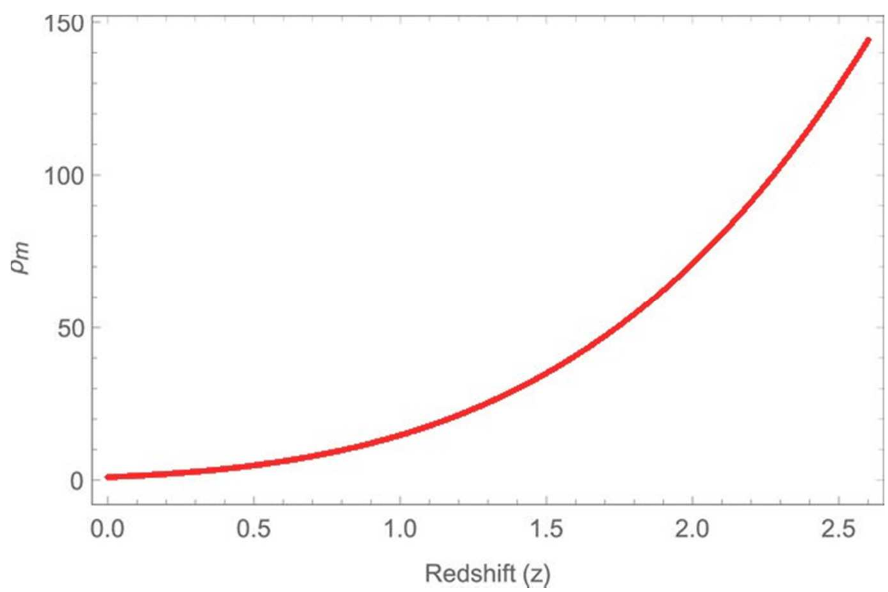

Figure 2. Energy density of matter vs Redshift (z)

Assuming that the constants are chosen appropriately, Fig. 2 shows the resultant model's energy density of matter graphically with respect to redshift. As can be seen from Fig. 2, the energy density of the $f(R)$ gravity model gives an empty cosmos for a long period, confirming the idea that the volume of space rises as the universe expands because the density of matter decreases.

The Equation of state parameter of THDE is given by

$$\omega_{THDE} = \frac{-1}{\rho_{THDE}}\left\{k_1 t^{4n-2} + \frac{3nC}{2C_1}t^{-4n-1} + \frac{C^2}{16C_1^2}t^{-12n} + 3n^2 t^{-2} - \frac{3}{2}\frac{nC}{C_1^2}t^{-8n-1} - \frac{27C^2}{16C_1^4}t^{-16n}\right\} \quad (42)$$

where $k_1 = C_1(n-1)n + \frac{20C_1 n^2}{4n-2} - \frac{8C_1 n}{4n-2} - 12n^2 C_1 - 4n(n-1)C_1$.

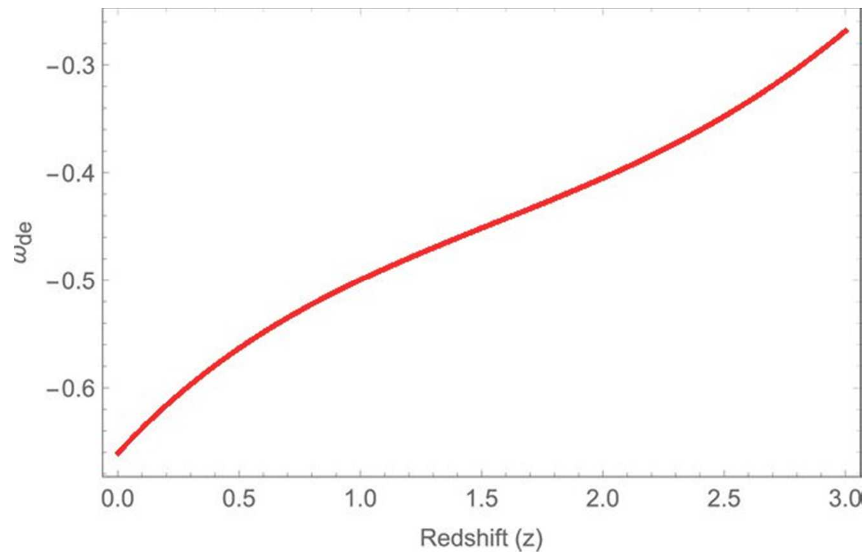

Figure 3. Equation of State parameter vs Redshift (z)

Different EoS parameter values correspond to different epochs of the cosmos, such as the current accelerating expansion phase and the early decelerating phase. For the decelerating phase, it is categorized as matter dominated (dust) $\omega_{de} = 1$, radiation $\omega_{de} = \frac{1}{3}$, and stiff fluid $\omega_{de} = 0$, accordingly. It stands for the phantom phase $\omega_{de} < -1$, cosmological constant $\omega_{de} = -1$, and quintessence $-1 < \omega_{de} < \frac{1}{3}$. The phantom behaviour of the universe is depicted in the Fig. 3, where the EoS parameter varies fully in the region.

7. State-finder diagnostic

Sahni et al. [39] introduced a new geometrical pair called the statefinder pair (r, s) to discern between the various DE models and their effects on the universe's dynamics. It investigates the dynamic expansion of the cosmos since it is based on the higher derivatives of the average scale factor. The definition of the statefinder pair is

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}. \tag{43}$$

For the obtained model the statefinder parameters are calculated to be

$$r = \frac{(n-1)(n-2)}{n^2}, \quad s = \frac{2n}{3}. \tag{44}$$

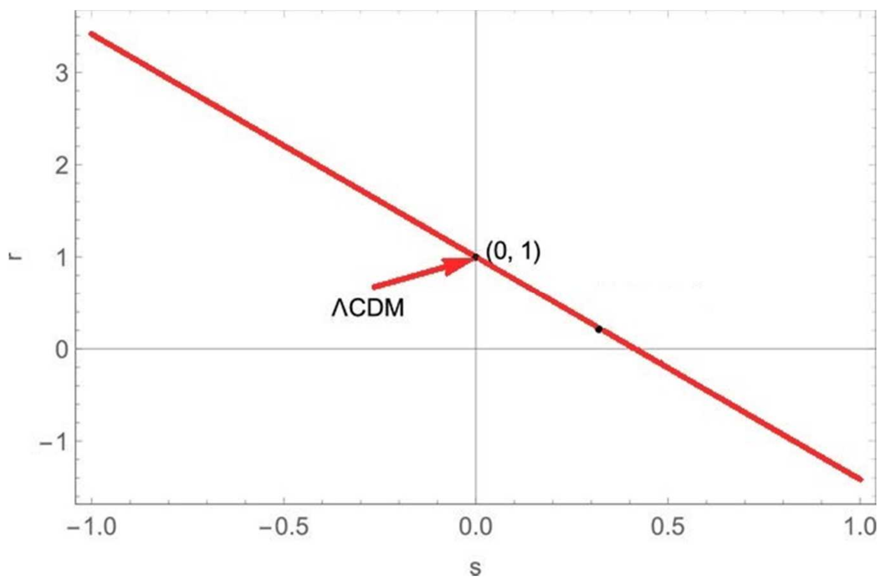

Figure 4. *r* vs *s*

The statefinder pair is constant based on n, as demonstrated by equation (44). The statefinder parameters for the Lambda Cold Dark Matter (or ΛCDM) model and Standard Cold Dark Matter (or SCDM) have fixed values of $\{r, s\} = \{1, 0\}$ and $\{r, s\} = \{1, 1\}$ respectively, according to Sahini et al. [39]. Plotting the evolution trajectories of our model at the $\{r, s\}$-plane, we observe that the evolution trajectory of our model ends at $\{r, s\} = \{1, 0\}$. Additionally, we observe that r and s behave as $r \to 1$ and $s \to 0$, indicating that our model is consistent with the ΛCDM model at late times as depicted in Fig.4.

8. Conclusion

This paper's goal is to examine the recently created $f(R)$ gravity. In order to achieve this, the five-dimensional Kaluza-Klein metric is taken into consideration from a holographic perspective in relation to the Hubble Horizon cut-off when Tsallis Holographic Dark Energy (THDE) is present. The solutions of the field equations have been examined using volumetric power law expansion. The scale factors A and B tend to infinity at vast times $t \to \infty$, which is in perfect agreement with the Big-Bang model of the universe, and the metric potentials disappear at an initial epoch that has initial singularity.

As time passes, the mean Hubble parameter decreases. In the early period, it is very huge, and later on, it approaches to zero. The expansion rate of the cosmos consequently slows down. As a result, the universe's rate of expansion decreases. Recent observations of supernovae Ia have shown that the Universe is expanding and accelerating, and the deceleration parameter lies in the range $-1 < q < 0$.

The universe shows a decelerated model if $n < 1$ and $q > 1$, but an accelerated expansion if $n > 1$ then $q < 0$ which is in agreement with current observations of SNe Ia and CMBR. It is observed that $\theta \to \infty$ for $t \to 0$ and tend to zero as time becomes infinitely large. The shear scalar diverge at $t = 0$. As $t \to \infty$, shear scalar tend to zero.

Therefore, the models would essentially give an empty universe for large time's t, i.e., at late time matter has no shear. It is observed that mean anisotropic parameter is very large at $t = 0$ and tends to zero as $t \to \infty$. It is evident that the universe's energy density increases with cosmic redshift z and stays positive at all redshift values [40-43]. It begins with high positive numbers and eventually gets closer to zero. The EoS parameter's current value in relation to the observational Hubble datasets.

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