Article

Friedmann Universe with Decaying Vacuum Energy

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Abstract

In this paper, we investigate the flat FRW universe with cosmological terms and bulk viscous fluid. We have solved the field equations analytically by assuming Hubble parameter H in the model to be a suitable function of time t which yields time dependent deceleration parameter q. The model of universe represents earlier decelerating and the current accelerating universe passing through a transition phase. The stability of the outcome has also been discussed, along with the physical, geometrical, and kinematic behaviour of the model.

Keywords: Energy density, deceleration parameter, cosmological term, Hubble parameter, bulk viscosity.

1. Introduction

The general features of Friedmann cosmological models with multiple fluids should be investigated in light of recent observational findings. The prevailing view in modern cosmology is that the universe's total energy density is currently dominated by the densities of two substances: (i) dark matter, which exhibits the same gravitational pull as regular matter, and (ii) dark energy, which is a form of vacuum energy with a negative pressure [1].

The universe is typically modelled using mixes of non-interacting, perfect fluids [2]. This indicates that there is no conversion between the components and that they are believed to have evolved separately in compliance with recognised conservation laws.The fact that this is the only situation that might possibly exist is not supported by observational data, though. This opens the door for us to think about cosmic models that make sense and include fluids that interact. This situation involves large energy transfers between these fluids.

Thanks to these energy exchanges, in some cosmological models it is possible, for example, to give a reasonable description for the observed current time acceleration of the universe [3] and for the coincidence problem [4-5], since some mechanisms could exist for converting one fluid into another. There are many other cosmological situations where this exchange of energy was considered. For instance, the interaction between dust-like matter and radiation has been

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explored for the first time by Tolman [6] and Davidson [7]. It's interesting to note that Davidson only considered positive pressures because there wasn't any scientific support for negative stresses in intergalactic space at the time. The creation of new matter or the radiation-producing breakdown of massive particles are two more cosmic scenarios that Davidson has studied. For other examples, see Barrow and Clifton [8] and the references cited therein.

The impact of viscosity on the evolution of cosmological models has been researched by Misner [9–11] and Weinberg [12–13].According to this premise, dissipative processes are predicted to be crucial to the development of the early universe. The history of the early universe and various astrophysical phenomena have both been thoroughly studied using full causal bulk viscous thermodynamics [14, 15]. However, due to the complicated nature of the evolution equations, very few exact cosmological solutions of the gravitational field equations are known in the framework of the full causal theory [16].

In this study, we begin by discussing the key elements of a flat FRW cosmology model with decreasing cosmological terms and introducing the assumptions. These presumptions motivate our inquiry into the universe's evolution. Recent cosmology observational data are found to be consistent with the model.

2. Metric & Field Equation

We consider homogeneous and isotropic spatially flat Rabertson-Walker line element of the form

$$ds^2 = -dt^2 + S^2(t)(dx^2 + dy^2 + dz^2), \tag{1}$$

where S(t) is the scale factor. We assume the matter content of source field to be bulk viscous fluid expressed by energy-momentum tensor

$$T_{ij} = (\rho + p)v_i v_j + \bar{p} g_{ij} \ , \tag{2}$$

where ρ is proper energy density and $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta v^i_{;i} \tag{3}$$

satisfying equation of state

$$p = (\omega - 1)\rho; \ \ 1 \le \omega \le 2. \tag{4}$$

Here p is the isotropic pressure, ζ bulk viscosity and v^i is the four-velocity vector satisfying $v^i v_i = -1$.

In commoving coordinates $v^i = \delta^i_4$. From Einstein field equations

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j + \Lambda g_i^j \ , \tag{5}$$

in gravitational units 8πG = c = 1 with time-dependent cosmological term Λ(t), we obtain the equations

$$\overline{p} - \Lambda = -2\frac{\ddot{S}}{S} - \frac{\dot{S}^2}{S^2}, \tag{6}$$

$$\rho + \Lambda = 3\frac{\dot{S}^2}{S^2}, \tag{7}$$

Equations (6) and (7) can be written in terms of H and q as

$$\overline{p} - \Lambda = (2q-1)H^2, \tag{8}$$

$$\rho + \Lambda = 3H^2, \tag{9}$$

where H is the Hubble parameter and q is the deceleration parameter defined as

$$H = \frac{\dot{S}}{S}, \tag{10}$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{S\ddot{S}}{\dot{S}^2} \tag{11}$$

where dot (.) represents ordinary derivative .

From equations (3), (8) and (9), we have

$$q = 2 - \frac{1}{2H^2}\left[(2-\omega)\rho + 2\Lambda + 3\zeta H\right], \tag{12}$$

implying that the bulk viscosity lowers the value of deceleration parameter.

The vanishing divergence of Einstein tensor gives rise to

$$\dot{\rho} + 3(\rho + \overline{p})H + \dot{\Lambda} = 0. \tag{13}$$

3. Solution & Discussion

The system of equations (4), (6) and (7) are three independent equations in five unknowns S, p, ρ, Λ and ζ . In order to obtain the deterministic solution, we require two more conditions.

We assume that the functional relation between Hubble parameter H and cosmic time *'t'* is given by Singh et al.[17]

$$H = m + n\coth t, m > 0, n > 0, \tag{14}$$

which represents decelerating expansion in the initial time followed by accelerating expansion recent time. Also , we assume

$$\Lambda = \Lambda_0 \rho \tag{15}$$

where Λ_0 is constant[18].

For this assumption, we obtain scale factor S, spatial volume V, expansion scalar θ and deceleration parameter q as

$$S = (\sinh t)^n e^{mt}, \tag{16}$$

$$V = (\sinh t)^{3n} e^{3mt}, \tag{17}$$

$$\theta = 3(m + n\coth t), \tag{18}$$

$$q = -1 + \frac{n}{(m\sinh t + n\cosh t)^2}. \tag{19}$$

Matter density ρ, vacuum density Λ and bulk viscosity ζ for the model are given by

$$\rho = \frac{3}{1+\Lambda_0}(m + n\coth t)^2, \tag{20}$$

$$\Lambda = \frac{3\Lambda_0}{1+\Lambda_0}(m + n\coth t)^2, \tag{21}$$

$$\zeta = (m + n\coth t)\left[\frac{\omega}{1+\Lambda_0} - \frac{2n}{3(m\sinh t + n\coth t)^2}\right]. \tag{22}$$

Our observation that the spatial volume V is zero at time zero while the expansion scalar is infinite demonstrates that the universe begins to evolve at time zero with a volume of zero and an infinite rate of expansion. At $t = 0$, θ, ρ and Λ all are infinitely large. For the large value of *t,* $\theta \to 3(m+n)$, $\rho \to \frac{3}{1+\Lambda_0}(m+n)^2$, $\Lambda \to \frac{3\Lambda_0}{1+\Lambda_0}(m+n)^2$ and $\zeta \to (m+n)\left[\frac{\omega}{1+\Lambda_0} - \frac{2}{3n}\right]$. We discovered that vacuum energy Λ, which was enormously huge at the beginning of time, relaxed to a genuine cosmological constant in more recent times. We found that vacuum energy Λ being very large at initial epoch relaxes to genuine cosmological constant at recent time. Matter density ρ tends to genuine constant for recent values of *t*. At $t = 0$, $q = -1 + \frac{1}{n} > 0$ provided $0 < n < 1$ and for $t = \infty, q = -1$. Therefore, the model of universe describes earlier decelerating and current time accelerating universe passing through transition phase.

4. Conclusion

In this article, we have studied the spatially flat Friedmann cosmological models for bulk viscous fluid under the assumption that there is a functional relationship between the Hubble parameter H and cosmic time t, which represents the model of the universe decelerating expansion in the initial time followed by accelerating expansion at the present time.Vacuum energy density Λ is a decaying function of time and it approaches a small value at recent time.Recent data are found to be consistent with the model of the universe.

Received September 3,2025; Accepted December 12, 2025

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