Article

N-Dimensional Bianchi Type-I Cosmological Model With Time Varying Deceleration Parameter in $f(R,T)$ Theory

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Abstract: In this paper, we have obtained exact solution of n-dimensional Bianchi type-I spacetime with perfect fluid distribution in the $f(R,T)$ theory, for a special choice of $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$. The solution of the corresponding field equations is obtained by assuming a periodically varying deceleration parameter. The physical and geometrical properties of the model have been discussed using some physical parameters.

Keywords: Periodic varying, deceleration parameter, Variable Lambda, f(R,T) theory

1 Introduction

Einstein's theory of general relativity is the foundation of the modern physics, and it describes most of the gravitational theories of the universe, but it does not suitable for some of the important problems in cosmology like as the accelerating expansion phase of the universe. It is now proved from theoretical and observational facts that universe is not only expanding but also in an accelerating phase. The astronomical observation of type-Ia supernova experiments [1, 2, 3] suggests that the observable universe is expanding. Observation such as cosmic background radiation [4], large-scale structure [5] and PLANK collaborations [6] provides indirect evidence for late time accelerated expansion of the universe.
In order to explain the accelerated expansion of the universe, there are several cosmological models have been proposed by various authors. Explanation of the current expansion of the universe comes from modified theories of gravity. These modifications are based on the Einstein-Hilbert action to obtain alternative theories of Einstein such as $f(R)$ gravity [7], $f(T)$ gravity [8], $f(G)$ gravity [9], $f(R,G)$ gravity [10], where R, T, G are the scalar curvature, the torsion scalar and the Gauss-Bonnet scalar respectively. Bertolami *et al.* [11] have proposed a generalization of $f(R)$ modified theory of gravity, by explicit coupling of arbitrary function of Ricci scalar R with the matter Lagrangian density L_m. The generalisation of $f(R)$ gravity by introducing the trace of energy momentum tensor has become the most popular theory to describe the nature of expansion of the universe, known as $f(R,T)$ theory of gravity. This theory is proposed by Harko[12], where gravitational Lagrangian is the arbitrary function of the scalar curvature (R) and trace of energy momentum tensor (T).
Many authors have studied many problems of the cosmology in this theory. Ahmed and Pradhan[13] have investigated the cosmological models with a cosmological constant in $f(R,T)$ theory for different Bianchi type spacetime. Sahoo and Sivakumar[14] have studied LRS Bianchi type-I cosmological model in $f(R,T)$ theory of gravity with variable $\Lambda(T)$. Aditya *et al.*[15] have studied different Bianchi type of spacetime in $f(R,T)$ gravity with variable cosmological constant. Tiwari *et al.*[16] have examined the $f(R,T)$ gravity using a time dependent cosmological term.
Higher Dimensional cosmological model plays an important role in many phases of early stage of cosmological problems. Kaluza and Kelin [17, 18] have done noticeable work by introducing an idea of higher dimension spacetime of the universe. Rathore and Mandawat [19] have investigated five dimensional Bianchi type-I string cosmological model in Brans-Dicke theory. Samanta and Dhal[20] have studied

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higher dimensional cosmological model with perfect fluid in $f(R,T)$ theory of gravity. Khurdiya and Singh[21] have discussed five-dimensional exact solution of Bianchi type-V spacetime in the $f(R,T)$ theory.
The time varying deceleration parameter is important in evolution of the universe and phase transition in expansion of the universe. Taking a variable deceleration parameter that depends on the Hubble parameter, Tiwari *et al.*[22] considered the LRS Bianchi type-I model in the $f(R,T)$ theory. Sahoo *et al.* [23] studied a periodic varying deceleration parameter in $f(R,T)$ gravity. Tiwari and Sofuoglu [24] have considered a quadratically varying deceleration parameter in $f(R,T)$ theory of gravity. Tiwari *et al.*[25] have studied time varying deceleration parameter in Bianchi type-I spacetime by taking $f(R,T) = R + 2f(T)$. Motivating from the above analysis, we have obtained exact solution of N-dimensional Bianchi type-I in $f(R,T)$ gravity by adopting a particular form of $f(R,T)$ function as $f(R,T) = f_1(R) + f_2(T)$. This choice leads us to an evolving cosmological constant (Λ) which depend on trace of the energy momentum tensor. For finding an exact solution of the field equations of the model, we use the periodical time varying deceleration parameter. The physical and geometrical properties are also discussed for the model.

2 N-Dimensional Field Equation in $f(R,T)$ Theory of gravity

The $f(R,T)$ theory of gravity is the generalization or modification of general relativity. $f(R,T)$ gravity was formulated by Harko[12] whose field equations are derived from the action principal.

$$S = \int \left(\frac{1}{16\pi} f(R,T) + L_m \right) \sqrt{-g}\, d^n x \tag{2.1}$$

Where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor T_{ij} of the matter. L_m is the matter Lagrangian density and g is the metric determinant of fundamental tensor g_{ij} and we consider $G = c = 1$.
By varying the above equation (2.1) with raspect to g_{ij}, we obtain the field equation of $f(R,T)$ gravity in covariant tensor form as

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} + (g_{ij}\Box - \nabla_i \nabla_j) f_R(R,T) = 8\pi T_{ij} - (T_{ij} + \Theta_{ij}) f_T(R,T) \tag{2.2}$$

where $f_R(R,T) \equiv \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f(R,T)}{\partial T}$, $T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\partial(\sqrt{-g} L_m)}{\partial g^{ij}}$, $\Box \equiv \nabla^i \nabla_i$ is the D'Alemberts operator, ∇_i is the covariant derivative and R_{ij} is the Ricci tensor and Θ_{ij} is defined as

$$\Theta_{ij} = g_{ij} L_m - 2T_{ij} - 2g^{lk} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{lk}} \tag{2.3}$$

Here the energy-momentum tensor is considered to be perfect fluid which is defined as

$$T_{ij} = (p + \rho) u_i u_j - p g_{ij} \tag{2.4}$$

where $u^i = (0,0,0,.....1)$ is the velocity in the co-moving coordinates which satisfies the condition $u^i u_i = 1$. ρ and p are the energy density and pressure of the fluid, respectively. Here replacing the matter Lagrangian as $L_m = -p$ [26, 27] in equation (2.3)

$$\Theta_{ij} = -p g_{ij} - 2T_{ij} \tag{2.5}$$

and the field equation (2.2), take the following form

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} + (g_{ij}\Box - \nabla_i \nabla_j) f_R(R,T) = 8\pi T_{ij} + (T_{ij} + p g_{ij}) f_T(R,T) \tag{2.6}$$

Three explicit functional form of $f(R,T)$ which have been considered by Harko [12] are as follows

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{2.7}$$

In this paper, we have considered that $f(R,T) = f_1(R)+f_2(T)$. Therefore, the gravitational field equation (2.6) leads to

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_1'(R) = 8\pi T_{ij} + f_2'(T)T_{ij} + \left(f_2'(T)p + \frac{1}{2}f_2(T)\right)g_{ij} \tag{2.8}$$

Where prime denotes differentiation with respect to argument. For the sake of simlicity, we consider a particular form of the function $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$, where λ_1 and λ_2 are arbitrary parameters. So that $f(R,T) = \lambda_1 R + \lambda_2 T$.
Now, the equation (2.8) becomes

$$\lambda_1 R_{ij} - \frac{1}{2}\lambda_1 R\, g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)\lambda_1 = 8\pi T_{ij} + \lambda_2 T_{ij} + \lambda_2\left(p + \frac{T}{2}\right)g_{ij} \tag{2.9}$$

Since $(g_{ij}\Box - \nabla_i\nabla_j)\lambda_1 = 0$, we get

$$\lambda_1 G_{ij} = 8\pi T_{ij} + \lambda_2 T_{ij} + \lambda_2\left(p + \frac{T}{2}\right)g_{ij} \tag{2.10}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor.
Therefore, we have

$$G_{ij} - \frac{\lambda_2}{\lambda_1}\left(p + \frac{T}{2}\right)g_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)T_{ij} \tag{2.11}$$

Since Einstein equation with cosmological constant

$$G_{ij} - \Lambda g_{ij} = -8\pi T_{ij} \tag{2.12}$$

To make the same sign in the right hand side of the equation (2.11) and (2.12), we put a restriction on small value of $\lambda_1 < 0$ and we keep this choice of λ_1 throughout. The term $\frac{\lambda_2}{\lambda_1}\left(p + \frac{T}{2}\right)$ can now be regarded as a cosmological constant. Hence, we have

$$\Lambda \equiv \Lambda(T) = \frac{\lambda_2}{\lambda_1}\left(p + \frac{T}{2}\right) \tag{2.13}$$

The dependence of cosmological constant (Λ) on the trace T of the energy momentum tensor T_{ij} have been proposed by [28] where the cosmological constant in the gravitational Lagrangian is a function of trace of energy momentum tensor. In this paper we have considered the perfect fluid distribution case, so the trace of energy momentum tensor $T = \rho - (n-1)p$ for our model. Therefore, equation (2.13) becomes to

$$\Lambda = \frac{\lambda_2}{\lambda_1}\left[\frac{\rho - (n-3)p}{2}\right] \tag{2.14}$$

Now, from the equations (2.11) and (2.13) we have

$$G_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right) T_{ij} + \Lambda g_{ij} \tag{2.15}$$

3 Metric and Field Equations in V_n

In this section we find the field equations for n-dimensional Bianchi type-I spacetime in $f(R,T)$ theory of gravity. The line element of Bianchi type-I in V_n is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)\sum_{i=1}^{n-3} du_i^2 \tag{3.1}$$

Where A, B and C are function of cosmic time t. The corresponding Ricci Scalar is given by

$$R = 2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + (n-3)\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + (n-3)\frac{\dot{B}\dot{C}}{BC} + (n-3)\frac{\dot{A}\dot{C}}{AC} + \frac{(n-3)(n-4)}{2}\frac{\dot{C}^2}{C^2}\right] \tag{3.2}$$

Here dot (.) represents a derivative with respect to cosmic time t. From equation (2.15), we have cosmological field equations for the metric (3.1) as follows
For $nn-$component

$$\frac{\dot{A}\dot{B}}{AB} + (n-3)\frac{\dot{B}\dot{C}}{BC} + (n-3)\frac{\dot{A}\dot{C}}{AC} + \frac{(n-3)(n-4)}{2}\frac{\dot{C}^2}{C^2} = -\left(\frac{8\pi+\lambda_2}{\lambda_1}\right)\rho - \Lambda \tag{3.3}$$

For $11-$component

$$\frac{\ddot{B}}{B} + (n-3)\frac{\ddot{C}}{C} + (n-3)\frac{\dot{B}\dot{C}}{BC} + \frac{(n-3)(n-4)}{2}\frac{\dot{C}^2}{C^2} = \left(\frac{8\pi+\lambda_2}{\lambda_1}\right)p - \Lambda \tag{3.4}$$

For $22-$component

$$\frac{\ddot{A}}{A} + (n-3)\frac{\ddot{C}}{C} + (n-3)\frac{\dot{A}\dot{C}}{AC} + \frac{(n-3)(n-4)}{2}\frac{\dot{C}^2}{C^2} = \left(\frac{8\pi+\lambda_2}{\lambda_1}\right)p - \Lambda \tag{3.5}$$

For $33,\ 44, \ldots.., (n-1)(n-1)-$components

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + (n-4)\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + (n-4)\frac{\dot{B}\dot{C}}{BC} + (n-4)\frac{\dot{C}\dot{A}}{CA} + \frac{(n-4)(n-5)}{2}\frac{\dot{C}^2}{C^2} = \left(\frac{8\pi+\lambda_2}{\lambda_1}\right)p - \Lambda \tag{3.6}$$

The field equation for components $33,\ 44, \ldots.., (n-1)(n-1)$ are identical because of metric function C is common along $u_1,\ u_2, \ldots.., u_{n-3}$ directions in the metric (3.1).
Here we have four independent field equations in five unknowns A, B, C, p and ρ. Therefore, in order to get deterministic solution, we consider time-dependent periodic varying deceleration parameter (PVDP) [29] as follows

$$q = m\cos(kt) - 1 \tag{3.7}$$

where m and k are positive real numbers.
Now, using the definition of deceleration parameter as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{3.8}$$

We have

$$\frac{d}{dt}\left(\frac{1}{H}\right) = m\cos(kt) \tag{3.9}$$

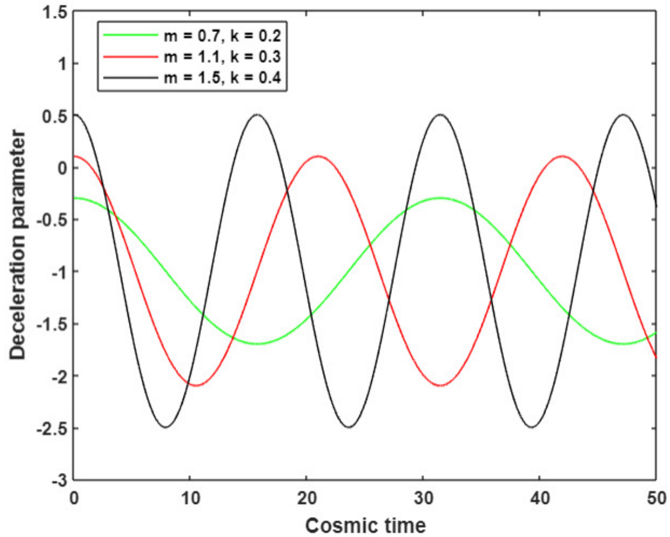

Figure 1: Deceleration parameter (q) versus cosmic time (t)

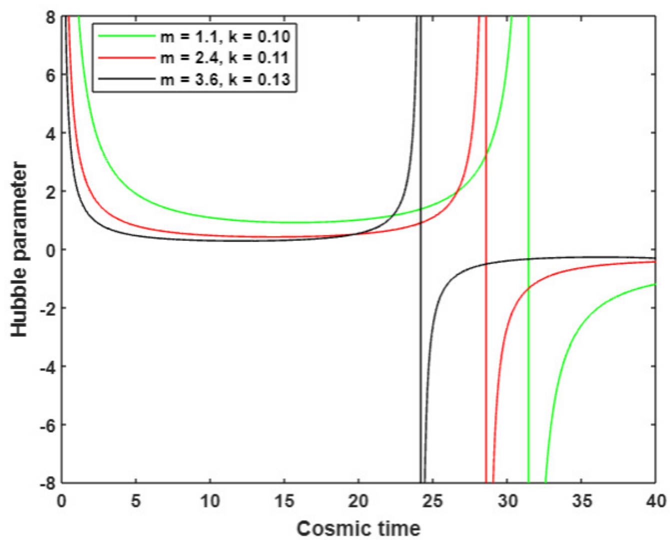

Figure 2: Hubble parameter (H) versus cosmic time (t)

The integration of equation (3.9) gives the Hubble parameter H as

$$H = \frac{k}{k_1 + m\,\sin(kt)} \tag{3.10}$$

where k_1 is a constant of integration. Without loss of generality, we can choose $k_1 = 0$. Hence, Hubble parameter becomes

$$H = \frac{k}{m\,\sin(kt)} \tag{3.11}$$

The spatial volume V and the average scale factor $\alpha(t)$ are defined respectively as under

$$V = ABC^{(n-3)} \tag{3.12}$$

$$\alpha(t) = \left(ABC^{(n-3)}\right)^{\frac{1}{n-1}} \tag{3.13}$$

We also define the generalized Hubble parameter H in the form

$$H = \frac{1}{n-1}\sum_{i=1}^{n-1} H_i \tag{3.14}$$

Where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_{i+2} = \frac{\dot{C}}{C}$ are the directional Hubble parameter in the direction of x, y and u_i axis respectively for $i = 1, 2....., (n-3)$. Now, By using equations (3.13) and (3.14), we have

$$H = \frac{\dot{\alpha}(t)}{\alpha(t)} \tag{3.15}$$

On integrating the equation (3.11), we find the average scale factor $\alpha(t)$ as follows

$$\alpha(t) = a_0\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{1}{m}} \tag{3.16}$$

where, a_0 is constant of integration.

4 Exact solution of Bianchi type-I space time in V_n

Subtracting equation (3.4) from (3.5), equation (3.5) from (3.6) and equation (3.4) from equation (3.6) we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + (n-3)\frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{4.1}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{A}}{A} + (n-4)\frac{\dot{C}}{C}\right)\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \tag{4.2}$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} + (n-4)\frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) = 0 \tag{4.3}$$

On solving above equations, we get

$$\frac{A}{B} = p_1 \exp\left(q_1 \int \frac{dt}{\alpha^{n-1}}\right) \tag{4.4}$$

$$\frac{B}{C} = p_2 \exp\left(q_2 \int \frac{dt}{\alpha^{n-1}}\right) \tag{4.5}$$

$$\frac{A}{C} = p_3 \exp\left(q_3 \int \frac{dt}{\alpha^{n-1}}\right) \tag{4.6}$$

where p_1, p_2, p_3 and q_1, q_2, q_3 are integration constants.
Now From equation (3.13), the metric coefficients can be written as

$$A(t) = \alpha_1\, \alpha \exp\left(\beta_1 \int \frac{dt}{\alpha^{n-1}}\right) \tag{4.7}$$

$$B(t) = \alpha_2\, \alpha \exp\left(\beta_2 \int \frac{dt}{\alpha^{n-1}}\right) \tag{4.8}$$

$$C(t) = \alpha_3\, \alpha \exp\left(\beta_3 \int \frac{dt}{\alpha^{n-1}}\right) \tag{4.9}$$

where

$$\alpha_1 = \left(p_1^{(n-2)} p_2^{(n-3)}\right)^{\frac{1}{n-1}},\ \alpha_2 = \left(p_1^{-1} p_2^{(n-3)}\right)^{\frac{1}{n-1}},$$
$$\alpha_3 = \left(p_1^{-1} p_2^{-2}\right)^{\frac{1}{n-1}} \tag{4.10}$$

and

$$\beta_1 = \frac{(n-2)q_1 + (n-3)q_2}{(n-1)},\ \beta_2 = \frac{-q_1 + (n-3)q_2}{(n-1)},$$
$$\beta_3 = -\frac{q_1 + 2q_2}{(n-1)} \tag{4.11}$$

The constants $\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$ and $\beta_1,\ \beta_2,\ \beta_3,\ \beta_4$ satisfy the following two relations:

$$\alpha_1 \alpha_2 \alpha_3^{(n-3)} = 1,\ \beta_1 + \beta_2 + (n-3)\beta_3 = 0 \tag{4.12}$$

Substituting equation (3.16) in equations (4.7)-(4.9), we obtain

$$A(t) = \alpha_1\, a_0 \left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{1}{m}} \exp\left(\beta_1 \int \frac{dt}{a_0^{n-1}\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{n-1}{m}}}\right) \tag{4.13}$$

$$B(t) = \alpha_2\, a_0 \left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{1}{m}} \exp\left(\beta_2 \int \frac{dt}{a_0^{n-1}\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{n-1}{m}}}\right) \tag{4.14}$$

$$C(t) = \alpha_3\, a_0 \left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{1}{m}} \exp\left(\beta_3 \int \frac{dt}{a_0^{n-1}\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{n-1}{m}}}\right) \tag{4.15}$$

5 Some Importantant Physical Quantities

The spatial volume (V) and expansion scalar (θ) are defined respectively as under

$$V = \alpha^{n-1} = a_0^{n-1}\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{n-1}{m}} \tag{5.1}$$

$$\theta = (n-1)H = \frac{(n-1)k}{m\,\sin(kt)} \tag{5.2}$$

The anisotropy parameter (A_m) is defined as

$$A_m = \frac{1}{n-1}\sum_{i=1}^{n-1}\left(\frac{H_i - H}{H}\right)^2 \tag{5.3}$$

Since, the directional Hubble parameters are

$$H_1 = \frac{k}{m\,\sin(kt)} + \frac{\beta_1}{a_0^{n-1}\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{n-1}{m}}} \tag{5.4}$$

$$H_2 = \frac{k}{m\,\sin(kt)} + \frac{\beta_2}{a_0^{n-1}\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{n-1}{m}}} \tag{5.5}$$

$$H_{i+2} = \frac{k}{m\,\sin(kt)} + \frac{\beta_3}{a_0^{n-1}\left[\tan\left(\frac{kt}{2}\right)\right]^{\frac{n-1}{m}}} \tag{5.6}$$

where, $i = 1$ to $(n-3)$.
So, we get anisotropy parameter as

$$A_m = \frac{1}{n-1}\left[\frac{\beta_1^2+\beta_2^2+(n-3)\beta_3^2}{a_0^{2(n-1)}\left[\tan\left(\frac{kt}{2}\right)\right]^{2(n-1)/m}}\right]\frac{m^2\sin^2(kt)}{k^2} \tag{5.7}$$

Shear tensor σ_{ij} is defined as

$$\sigma_{ij} = \frac{1}{2}\left(\nabla_i u_j + \nabla_j u_i\right) - \frac{1}{n-1}\,\theta g_{ij} \tag{5.8}$$

Shear scalar σ^2 in term of Hubble parameter is given by

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{n-1} H_i^2 - (n-1)H^2\right] \tag{5.9}$$

Shear scalar σ^2 in term of anisotropy parameter is defined as

$$\sigma^2 = \left(\frac{n-1}{2}\right)H^2 A_m \tag{5.10}$$

Using equations (3.11) and (5.7) in (5.10), we obtain

$$\sigma^2 = \frac{\beta_1^2+\beta_2^2+(n-3)\beta_3^2}{2a_0^{2(n-1)}\left[\tan\left(\frac{kt}{2}\right)\right]^{2(n-1)/m}} \tag{5.11}$$

Using Equations (3.3)-(3.6) and (4.7)-(4.9), we obtain the pressure (p) and energy density (ρ) of the universe

$$\begin{aligned} p(t) = \frac{\lambda_1}{(8\pi+\lambda_2)(16\pi+n\lambda_2)}\Bigg[&\frac{(8\pi+\lambda_2)(n-1)(n-2)k^2}{m^2\,\sin^2(kt)} \\ &- \frac{(16\pi+3\lambda_2)(n-2)k^2\,\cos(kt)}{m\,\sin^2(kt)} + \frac{1}{a_0^{2(n-1)}\left[\tan(\frac{kt}{2})\right]^{\frac{2(n-1)}{m}}} \\ &\Big\{(16\pi+3\lambda_2)(\beta_1^2+\beta_2^2) + (16\pi+2\lambda_2)\beta_1\beta_2 \\ &- (n-3)\Big((n-4)8\pi+(n-5)\lambda_2\Big)\beta_3^2\Big\}\Bigg] \end{aligned} \tag{5.12}$$

The energy density of the universe becomes

$$\begin{aligned}\rho(t) = \frac{-\lambda_1}{(8\pi+\lambda_2)(16\pi+n\lambda_2)}\Bigg[&\frac{(8\pi+\lambda_2)(n-1)(n-2)k^2}{m^2\,\sin^2(kt)} \\ &- \frac{\lambda_2(n-2)(n-3)k^2\,\cos(kt)}{m\,\sin^2(kt)} + \frac{1}{a_0^{2(n-1)}\left[\tan(\frac{kt}{2})\right]^{\frac{2(n-1)}{m}}} \\ &\Big\{\lambda_2(n-3)(\beta_1^2+\beta_2^2) - (16\pi+2\lambda_2)\beta_1\beta_2 \\ &+(n-3)\Big((n-2)8\pi+(2n-5)\lambda_2\Big)\beta_3^2\Big\}\Bigg]\end{aligned} \tag{5.13}$$

Now, from equations (2.14), (5.12) and (5.13), we obtain the expression for cosmological constant as

$$\begin{aligned}\Lambda(t) = \frac{-\lambda_2}{(16\pi+n\lambda_2)}\Bigg[&\frac{(n-1)(n-2)^2\,k^2}{2m^2\,\sin^2(kt)} \\ &- \frac{(n-2)(n-3)k^2\,\cos(kt)}{m\,\sin^2(kt)} + \frac{1}{a_0^{2(n-1)}\left[\tan(\frac{kt}{2})\right]^{\frac{2(n-1)}{m}}} \\ &\Big\{(n-3)(\beta_1^2+\beta_2^2)+(n-2)\beta_1\beta_2 \\ &- \frac{(n-3)(n^2-6n+10)}{2}\beta_3^2\Big\}\Bigg]\end{aligned} \tag{5.14}$$

6 Conclusion

In this paper, we have obtained n-dimensional exact solution of Bianchi type-I spacetime in $f(R,T)$ modified theory of gravity for an appropriate choice of the function $f(R,T) = f_1(R) + f_2(T) = \lambda_1 R + \lambda_2 T$ with variable $\Lambda(T)$. We have considered a periodically variable deceleration parameter. Quantities which are of cosmological importance for the model are also evaluated.

n-dimensional model of the universe have singularities periodically at $t = \frac{2r\pi}{k}$ $(r = 0, 1, 2, ...)$. These singularities are point type [30] because metric coefficients are vanished at these points.

From the equation (3.16), we observe that the average scale factor (α) is diverges initially. It increases with cosmic time periodically.

Figure (1) shows the behavior of PVDP with time for different values of the constants m and k. The peak of the PVDP is depending on m which can be increased by the factor m. The periodicity of the PVDP is depending on k which can be viewed as a parameter of cosmic frequency. In this model, we can observe from figure (1) for a particular case $m = 0.7,\ k = 0.2$ the universe starts with a period of decelerating phase of expansion $(q > 0)$ and later with accelerating phase of expansion $(q < 0)$ and then goes to super exponential phase of expansion $(q < -1)$ in cyclic history.

Figure (2) shows the variation of the Hubble parameter with respect to cosmic time for different m and k. Hubble parameter is important observational parameters in cosmology which play an important role in the expansion history of the Universe. From the graph of Hubble parameter it is observed that it is decreasing functions of cosmic time and tend to zero for a specific time after that it is increasing and changes periodically. From the equation (5.7) and for $n-1 > m$, we observed that at $t = 0$, anisotropy parameter (A_m) diverge which indicating that the universe was anisotropic. After large time $A_m \to 0$, this shows that the universe in the model turns isotropic at late time, and this process happening periodically. All the cosmological parameters θ, σ, p, ρ and Λ are tends to ∞ at initial stage, and they preserve their periodic behavior with cosmic time.

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