Article

New Uniform Motion & Fermi–Walker Derivative of Symplectic Regular Curves

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Abstract

In this this paper, we have dealed optical model for symplectic curve to make new optical conditions with optical motions by using Fermi-Walker transportation such as Uniformly optical accelerated motion, unchanged optical direction motion and uniformly circular optical motion We also investigate optical modelling relations between Fermi-Walker transportation and optıcal motions in symplectic space.

Keywords: Frenet vectors, equiform frame, Fermi-Walker derivative.

1. Introduction

In the literature there are many studies on Fermi-Walker transport and Fermi-Walker derivative. Simple description for the construction of Fermi-Walker transported frames out of an arbitrary set of tetrad fields was presented and a research on Fermi-Walker transports has been expanded to Minkowski spacetime in Ref. [1, 5]. Many researches expressed their applications and being parallel in the Fermi-Walker sense. In addition, the geodesic curves of magnetic curves within a given magnetic field.Since it is a generalized form of the Fermi-Walker derivative for magnetic curves calculation plays an important role. In addition, Fermi-Walker magnetic curves by calculating the energy of its derivatives, basic definitions such as mass-energy and motion-energy concepts can be understood better.

Classical mechanics forms the basis of symplectic geometry. Especially ,The center of Hamilton and Kahler manifolds, which have an important application area in mathematics and physics have an important application area in Mathematics and physics, is connected to symplectic geometry. Symplectic geometry also has many applications fields such as optics, dynamic systems, integrable systems, algebraaic geometry, low-dimensional topology, microlocal analysis, partial differential equations. Symplectic geometry is studied by geometers [6-11].

In this study, we give optical model for symplectic curve to make new optical conditions with optical motions by using Fermi-Walker transportation such as Uniformly optical accelerated motion, unchanged optical direction motion and uniformly circular optical motion We also investigate optical modelling relations between Fermi-Walker transportation and optıcal motions in symplectic space.

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2. Preliminaries

In this part, some basic definition and properties of symplectic space are given.

For, any vectors , inner product can be written as

$$u = (x^1, x^2, \dots, x^n, y^1, \dots, y^n),\ \ v = (\xi^1, \dots, \xi^n, \eta^1, \dots, \eta^n) \in R^{2n}$$

$$< u, v >= \varphi(u, v) = \sum_{i=1}^{n} (x_i \eta_i \wedge y_i \xi_i)$$

Standard symplectic form of symplectic space can be written as [7]

$$\varphi = \sum dx_i \wedge dy_i$$

Symplectic space with symplectic inner product can be written as

$$< u, v > = \varphi(u, v) = \sum_{i=1}^{2} (x_i \eta_i - y_i \xi_i)$$

$$= x_1 \eta_1 + x_2 \eta_2 - y_1 \xi_1 - y_2 \xi_2$$

where $u = \{x_1, x_2, y_1, y_2\}$ and $v = \{\xi_1, \xi_2, \eta_1, \eta_2\}$

Let $z(s)$ be a symplectic curve parametrized by arc length respect to $\{a_1, a_2, a_3, a_4\}$ a symplectic frame. Then, Frenet equations can be given as

$$a_1{}'(s) = a_3(s)$$
$$a_2'(s) = H_2(s) a_4(s)$$
$$a_3'(s) = k_1(s) a_1(s) + a_2(s)$$
$$a_4'(s) = a_1(s) + k_2(s) a_2(s)$$

where $H_2(s) = cost(\neq 0)$ [6].

3. Uniformly optical motion of Symplectic Space

Definition 3.1. X complies UOAM (uniformly optical accelerated motion) if and only if

$$\widetilde{\nabla}\left(\nabla_{a_1} X\right) = 0$$

where $\widetilde{\nabla}$ means a Fermi-Walker connection [5]

$$\widetilde{\nabla}_{a_1} X = \nabla_{a_1} X - \langle a_1, X \rangle \nabla_{a_1} a_1 + \langle \nabla_{a_1} a_1, X \rangle a_1$$

Definition 3.2. X complies UODM (unchanged optical direction motion) if and only if [4]

$$\left|\nabla_{a_1} X\right|^2 = const.$$

and

$$\left|\widetilde{\nabla}(\nabla_{a_1} X)\right|^2 = const.$$

Definition 3.3. X complies UOCM (uniformly optical circular motion) if and only if [5]

$$\widetilde{\nabla}(\left|\nabla_{a_1}X\right|^{-1}\nabla_{a_1}X)=0$$

Theorem 3.1

♦ The vector field a_1 complies the UOAM iff

$$k_1=0$$

♦ The vector field $a_2$complies the UOAM iff

$$H_2=H_2k_1=H_2'=0$$

♦ The vector field $a_3$complies the UOAM iff

$$(k_1'-k_1)=k_1=H_2=0$$

♦ The vector field $a_4$complies the UOAM iff

$$k_2k_3=-k_3'-{k_3}^2=0$$

The general vector fields X complies iff

$$[x_1'+x_4+k_1x_3]=[x_2'+x_3+x_4\,k_1]=(H_2x_2+x_4')=0$$

Proof. By definition of Fermi –Walker derivatives and using frame field

$\nabla_{a_1}a_1,\nabla_{a_1}a_2,\nabla_{a_1}a_3,\nabla_{a_1}a_4$ are obtained by

$$\widetilde{\nabla}_{a_1}(\nabla_{a_1}a_1)=\widetilde{\nabla}_{a_1}(a_3)=a_3-\langle a_1,a_3\rangle a_3+\langle a_3,a_3\rangle a_1$$
$$=k_1a_1+a_2-a_3$$

$$\widetilde{\nabla}_{a_1}(\nabla_{a_1}a_2)=\widetilde{\nabla}_{a_1}(H_2a_4)=H_2'\,a_4+H_2a_1+H_2k_1a_2-\langle a_1,H_2a_4\rangle a_3+\langle a_3,H_2a_4\rangle a_1$$
$$=H_2a_1+H_2k_1a_2+H_2'\,a_4$$

$$\widetilde{\nabla}_{a_1}(\nabla_{a_1}a_3)=\widetilde{\nabla}_{a_1}(k_1a_1+a_2)=k_1'\,a_1+k_1a_3+H_2a_4-\langle a_1,k_1a_1+a_2\rangle a_3+\langle a_3,k_1a_1+a_2\rangle a_1$$
$$=(k_1'-k_1)a_1+k_1a_3+H_2a_4$$

$$\widetilde{\nabla}_{a_1}(\nabla_{a_1}a_4)=\widetilde{\nabla}_{a_1}(a_1+k_1a_2)=a_3+k_1'\,a_2+k_1a_2H_2-\langle a_1,a_1+k_1a_2\rangle a_3+\langle a_3,a_1+k_1a_2\rangle a_1$$
$$=-a_1+(k_1'+k_1H_2)a_2+a_3$$

For the general vector field X, we conclude

$$\nabla_{a_1}X=\nabla_{a_1}(x_1a_1+x_2a_2+x_3\,a_3+x_4\,a_4)$$
$$=(x_1'+x_3k_1+x_4)a_1+(x_4k_1+x_2'+x_3)a_2+(x_1+x_3')a_3+(H_2x_2+x_4')\,a_4$$

Therefore , we can express the Fermi-Walker derivative of ∇_TX ,

$$\widetilde{\nabla}_{a_1}(\nabla_{a_1}X)=(x_1'+x_3k_1+x_4)a_1+(x_4k_1+x_2'+x_3)a_2+(x_1+x_3')a_3+(H_2x_2+x_4')\,a_4$$
$$[x_1'+x_4+k_1x_3]a_1+[x_2'+x_3+x_4\,k_1]a_2+(H_2x_2+x_4')\,a_4$$

which means that the theorem is proved.

Theorem 3.2. The vector field a_1 complies the UODM iff

♦ ${k_1}^2 = const.$

♦The vector field a_2 complies the UODM iff

${H_2}^2 = const.$

and

${H_2}^2 + {H_2}'^2 + (H_2 k_1)^2 = const.$

♦ The vector field a_3 complies the UODM iff

${k_1}^2 = const.$

and

$(k_1' - k_1)^2 + {k_1}^2 + {H_2}^2 = const.$

♦The vector field a_4 complies the UODM iff

${k_1}^2 = const.$

and

$(k_1' + k_1 H_2)^2 = const.$

♦ The general vector field X complies the UODM iff

$(x_1' + x_3 k_1 + x_4)^2 + (x_4 k_1 + x_2' + x_3)^2 + (x_1 + x_3')^2 + (H_2 x_2 + x_4')^2 = const.$

and

$\left[x_1' + {k_1}^2 x_1 + k_1 x_2' - k_1 k_2 x_3\right]^2 + [x_1 k_1 + x_2' - x_3 k_2 + x_1' \ k_1]^2 + [(x_2 k_2 + x_3' + x_3 k_3 - x_4 k_3)]^2 + {x_4}'^2 = const.$

Proof. By some short calculations yields we can easily see that proof is completed.

Theorem 3.6.

♦ The vector field a_1 complies the UOCM iff

$$\tilde{\nabla}(\left|\nabla_{a_1} X\right|^{-1} \nabla_{a_1} X) = 0$$

♦ The vector field a_2 complies the UOCM if

$$\tilde{\nabla}(\left|\nabla_{a_1} X\right|^{-1} \nabla_{a_1} X) = 0$$

♦ The vector field a_3 complies the UOCM iff

$$\tilde{\nabla}(\left|\nabla_{a_1} X\right|^{-1} \nabla_{a_1} X) = 0$$

♦ The vector field a_4 complies the UOCM iff

$$\tilde{\nabla}(\left|\nabla_{a_1} X\right|^{-1} \nabla_{a_1} X) = 0$$

Proof. Using Frenet frame and definition 3.3, it can easily proven.

Example 4.2.8. Let's consider the following curve

$$z(s) = \frac{1}{\sqrt{5}}\left(shs, \frac{1}{2}s^2 + 2s, chs, \frac{1}{2}s^2 - 2s\right).$$

Figure 1. Then,

$$a_1(s) = z'(s) = \frac{1}{\sqrt{5}}(chs, s + 2, shs, s - 2)$$

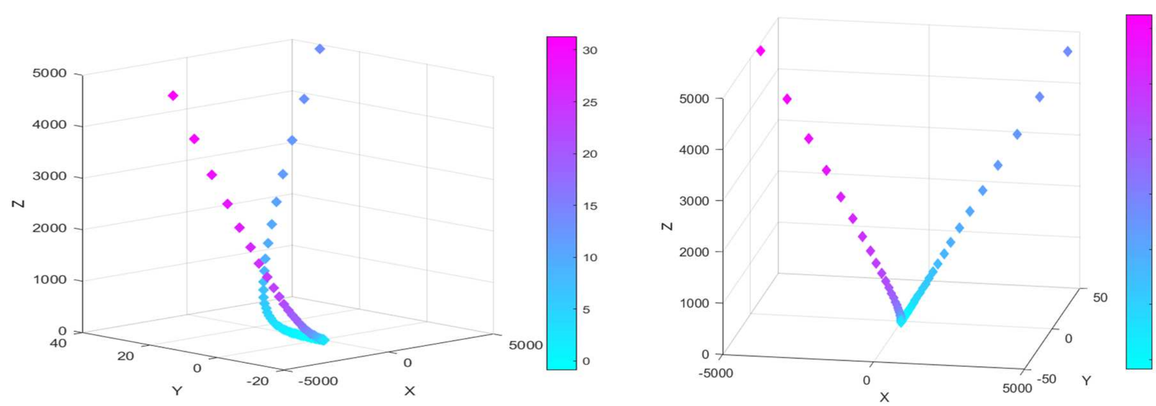
and

$$a_3(s) = z''\,(s) = \frac{1}{\sqrt{5}}(shs, 1, chs, 1).$$

Thus $z(s)$ is a symplectic regular curve. Additionally,

$$a_2(s) = \frac{1}{\sqrt{5}}(\frac{4}{5}chs, -\frac{1}{5}(s + 2), \frac{4}{5}shs, -\frac{1}{5}(s - 2),$$

$$a_4(s) = \frac{5}{4\sqrt{5}}(4shs, -1, 4chs, -1).$$



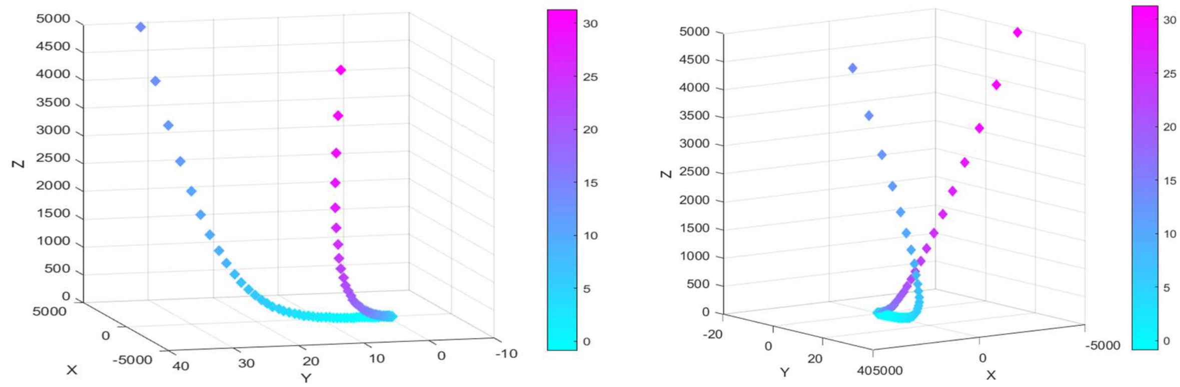


Figure 1. Symplectic regular curve $z(s)$

5. Conclusion

We obtain UOAM, UOCM and UODM for symplectic space using Fermi-Walker transportation. In our opinion optical radial model and optical motions are applied different-type space-time.

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