Article

Memory Space-Time

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Abstract

Spacetime nonlocality which in time domain results in memory effect can be considered as a fundamental characteristic of the universe we live in. We introduce memory spacetime and explore its consequences. We investigate this idea through the framework of fractional dynamics and using the powerful tool of fractional calculus. In particular we show that in this framework we can reproduce the recently reported experimental mass for the hidden-charm pentaquarks.

Keywords: Memory spacetime; Fractional dynamics; Exotic baryons.

1. Introduction

Spacetime nonlocality which in time domain results in memory effect can be considered as a fundamental characteristic of the universe we live in. Generally, we can consider four possible types of spacetime: a. local space-local time, b. local space- nonlocal time, c. nonlocal space-local time, d. nonlocal space- nonlocal time where "a" is the traditional view of spacetime i.e., local and without memory but "b", "c" and "d" are nonlocal ones.

Among them the case of "c" is again memoryless but "b" and "d" are nonlocal spacetimes with memory and we call the case of "d" as memory spacetime. This memory spacetime can exhibit all different kind of memory effect including gravitational memory effect (refers to the permanent shift in space-time caused by passing gravitational waves), electromagnetic memory effect (lasting changes in electromagnetic fields or wave propagation) etc. Straightforward consequence of spacetime nonlocality is emerging of fractional operators in the formulation of our problems i.e., instead of the traditional operators we have Left (forward) and right (backward) Riemann–Liouville (RL) partial fractional derivatives for space and time derivative defined as:

$$
{}_{a_\mu}\partial_\mu^{\alpha_\mu} f(x^0,...,x^d) = \frac{1}{\Gamma(n_\mu-\alpha_\mu)} \partial_{x^\mu}^{n_\mu} \int_{a_\mu}^{x} \frac{f(x^0,...,x^{\mu-1},u,x^{\mu+1},...,x^d)}{(x^\mu-u)^{1+\alpha_\mu-n_\mu}} du \quad \text{(left RL)} \tag{1}
$$

$$
{}_{\mu}\partial_{b_\mu}^{\beta_\mu} f(x^0,...,x^d) = \frac{(-1)^{n_\mu}}{\Gamma(n_\mu-\beta_\mu)} \partial_{x^\mu}^{n_\mu} \int_{x}^{b_\mu} \frac{f(x^0,...,x^{\mu-1},u,x^{\mu+1},...,x^d)}{(u-x^\mu)^{1+\beta_\mu-n_\mu}} du \quad \text{(right RL)} \tag{2}
$$

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of order α_μ , β_μ (which are positive real or even complex numbers) of a real valued function f of $d+1$ real variables $x^0, x^1, ..., x^d$ with respect to x_μ [1-3]. Also $\partial^{n_\mu}_{x^\mu}$ is the ordinary partial derivative of integer order n. The second one i.e., the backward fractional operators in particular in time domain also can be used for describing possible consequences of the recently proposed theories for the CPT reflection of our universe [4,5]. Also, in the memory space-time one can construct fractional generalized field theories [6-8]. In addition, fractional quark theory (FQT) can be introduced in this framework [9]. For this purpose, at first a fractional group theory has been introduced in [10]. In this framework the author introduced $SO^\alpha(3N)$ as a fractional generalization of the standard rotation group $SO(3N)$ and then define the set of Casimir operators C_k^2, where the index k indicates the Casimir operator associated with $SO^\alpha(k)$, as:

$$C_k^2 = \frac{1}{2}\sum_{i,j}^{k}\left(L_{ij}^\alpha\right)^2 \qquad k = 2, ..., 3N \tag{3}$$

where L_{ij}^α are the generators of infinitesimal fractional rotations in the i, j plane in R^N with ($i, j = 1, ..., 3N$) defined as [10,11]:

$$L_{ij}^\alpha = \hat{X}_i^\alpha \hat{P}_j - \hat{X}_j^\alpha \hat{P}_i^\alpha = -i\hbar\left(x_i^\alpha \partial_j^\alpha - x_j^\alpha \partial_i^\alpha\right) \tag{4}$$

And finally with the definitions $\hat{L}_z(\alpha) = C_2$ and $\hat{L}^2(\alpha) = C_3^2$, we will have eigenvalues of Casimir operator in terms of the quantum numbers L and M as [10,11]:

$$\hat{L}_z(\alpha)\left|LM\right\rangle = \pm\hbar\frac{\Gamma(1+\left|M\right|\alpha)}{\Gamma(1+(\left|M\right|-1)\alpha)}\left|LM\right\rangle \qquad M = 0, \pm1, \pm2, ..., \pm L \tag{5}$$

$$\hat{L}^2(\alpha)\left|LM\right\rangle = \pm\hbar^2\frac{\Gamma(1+(L+1)\alpha)}{\Gamma(1+(L-1)\alpha)}\left|LM\right\rangle \qquad \mathrm{L} = 0, +1, +2, ... \tag{6}$$

Now introducing the model Hamilton operator H^α as:

$$H^\alpha = m_0 + a_0 L^2(\alpha) + b_0 L_z(\alpha) \tag{7}$$

where coefficients m_0, a_0 and b_0 are free parameters, one can derive in lowest order an analytic expression for the splitting of the energy levels and mass spectrum of a non-relativistic charged fractional spinless particle in a constant fractional magnetic field as [10,11]:

$$E_R^\alpha = m_0 + a_0\frac{\Gamma(1+(L+1)\alpha)}{\Gamma(1+(L-1)\alpha)} \pm b_0\frac{\Gamma(1+\left|M\right|\alpha)}{\Gamma(1+(\left|M\right|-1)\alpha)} \tag{8}$$

that such particle can be associated with a quark and the fractional magnetic field with a color magnetic field and finally this model should allow a description of the hadron spectrum. It is showed that the above formula reproduces the full baryon spectrum with an error less than 1%.

2. Exotic Baryons

In this section we consider the special case of exotic baryons in the above-mentioned framework i.e., fractional quark theory and we examine whether it can play a role in future exotic baryon spectroscopy. Recently the LHCb collaboration reported the updated list of exotic baryons as follows [12 and Refs. therein]:

	Exotic baryons	**Mass** (MeV)	**Width** (MeV)
1	$P_{c\bar{c}}(4312)^{+}$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$
2	$P_{c\bar{c}s}(4338)^{0}$	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 0.4$
3	$P_{c\bar{c}}(4380)^{+}$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$
4	$P_{c\bar{c}}(4440)^{+}$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$
5	$P_{c\bar{c}}(4457)^{+}$	$4457 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$
6	$P_{c\bar{c}s}(4459)^{0}$	$4458 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$

Table 1. The masses and decay widths of the hidden-charm pentaquark-like states reported by the LHCb Collaboration.

The optimum fit parameter set α $, m_0$ $, a_0$ $, b_0$ of Eq. (8) for baryon spectroscopy were obtained respectively as: $0.112, -17171.6\ [MeV]$, $10971.8\ [MeV], 8064.6\ [MeV]$ [10,11]. Surprisingly the fractional baryon mass spectra for the values of : $L = 66$ and $M = -34$ will result in $4310.6\ [MeV]$ for $P_{c\bar{c}}(4312)^{+}$, $L = 16$ and $M = 7$ will result in $4337.2\ [MeV]$ for $P_{c\bar{c}s}(4338)^{0}$, $L = 14$ and $M = 12$ will result in $4384\ [MeV]$ for $P_{c\bar{c}}(4380)^{+}$, $L = 14$ and $M = 13$ will result in $4441.6\ [MeV]$ for $P_{c\bar{c}}(4440)^{+}$, $L = 154$ and $M = -115$ will result in $4456.3\ [MeV]$ for $P_{c\bar{c}}(4457)^{+}$, $L = 118$ and $M = -97$ will result in $4460.5\ [MeV]$ for $P_{c\bar{c}s}(4459)^{0}$ which all of them are completely close to the observed and experimentally calculated masses of pentaquarks listed in table (1) and again with an error less than 1%.

3. Discussion & Conclusion

A final theory of nature has to consider space-time nonlocality which in time domain leads to memory effect described by memory kernel. We call such this spacetime a “memory space-time”. Based on this theory it seems that memory is a fundamental quantity of our nature. One can ask why we need a nonlocal theory of our nature at all? Answer to this question in term of the possible applications of a nonlocal quantum field theory (NFQT) can be contained the following important reasons and advantages [13 and the Refs. therein]: 1- unlike perturbatively renormalizable theories, NFQT can be formulated in an arbitrary dimensional spacetime.

This allows for greater flexibility in modeling physical phenomena across different dimensional spaces. 2- NFQT provides frameworks to describe high-energy physics phenomena such as scalar particles and quantum electrodynamics, where conventional local theories may face challenges. 3- research in NFQT extends to curved spacetime and quantum gravity, exploring interactions in contexts beyond traditional local field theories. 4- NFQT connects with statistical physics through the grand canonical partition function/nonlocal, nonpolynomial Euclidean QFT scattering matrix duality. This duality allows for applying statistical physics methods in QFT and vice versa, enhancing computational and theoretical techniques. 5- By considering the traditional scalar field theory, we will always find that the entanglement entropy is UV divergent however NFQT makes it possible to obtain finite results [14].

Knowing the above-mentioned reasons now one can ask that how we can construct a natural NFQT. Of course, there will be many different approaches for this aim, however the best tool to apply this idea to the formulation of physics of our nature is the rich tool of fractional calculus because in this framework we can intrinsically incorporate the space nonlocality and the memory effect in the formulation of our theories. So, a straightforward consequence of this argument will be a fractional generalized field theory. There will be many applications of such this theory and one of them is the factional theory of quark physics. By use of this application one can derive a formula for the fractional baryon mass spectra. In this work we showed that this powerful framework can reproduce the experimental mass for the exotic baryons recently reported by the LHCb collaboration.

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References

[1] V. E. Tarasov, Fractional Dynamics (Springer, 2011).

[2] H. Nasrolahpour, Prespacetime J. **10 (1)** (2019) 27-33.

[3] M. J. Lazo, Physics Letters A **375** (2011) 3541–3546.

[4] L. Boyle et al., Phys. Rev. Lett. **121**(2018) 251301.

[5] L. Boyle et al., Ann. of Phys. **438** (2022)168767

[6] H. Kleinert, EPL **100** (2012) 10001.

[7] V. E. Tarasov, Advances in High Energy Physics (2014) 957863.

[8] V. E. Tarasov, Modern Physics Letters A **36(14)** (2021) 2140001.

[9] H. Nasrolahpour, Prespacetime J. **10 (7)** (2019) 860-865.

[10] R. Herrmann, Fractional Calculus (World Scientific Press, 2013).

[11] R. Herrmann, Phys. Lett. A **372** (2008) 5515.

[12] S. Navas et al. (Particle Data Group), Phys. Rev. D **110** (2024) 030001.

[13] N. A. Ignatyuk et al., Symmetry **15** (2023) 1823.

[14] R. Landry and J. W. Moffat, arXiv:2309.06576v3 [hep-th] 2024.