Article

Bianchi Type I Bulk Viscous Fluid Cosmological Models with Varying Cosmological Term Λ

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Abstract

Bulk viscous Bianchi type I Cosmological models with varying cosmological term Λ in general relativity are investigated. To get a determinate Einstein exact solution, we have assumed the condition that cosmological term $\Lambda = \Lambda_0 H$, deceleration parameter q =n (constant) and $\zeta = \zeta_0 + \zeta_1 H$, where Λ_0, ζ_0 and ζ_1 are constant, ρ is the energy density and ζ is the coefficient of bulk viscosity. The deceleration parameter q is constant throughout the model of the universe. The cosmological term Λ is a decreasing function of time. The ratio σ/θ is initially highly anisotropic and late time isotropic.Further the physical and geometrical significance of the model of universe are discussed.

Keywords: Bulk viscosity, Bianchi Type-I, deceleration parameter, energy density, gravitation, cosmological term.

1. Introduction

One of the most notable recent observational discoveries is the problem of the cosmological constant, which is of great interest to all researchers. The cosmological constant Λ was originally given by Einstein in the field equation. In an evolving universe, it seems natural to consider this constant as a function of time. The term Λ occurs naturally in general relativistic quantum field theory and is interpreted as the energy density of the vacuum [1]. Some workers [2,3] claimed a dependence on Λ. Variable G and Λ universe models have been studied by many researchers for uniform and isotropic FRW line elements [4,5]. Also, Schutzhold [6,7] recently proposed that the vacuum energy density is proportional to the Hubble parameter and decreases as Λ ≈ mH. where m ≈ 150 MeV is the energy scale of the chiral phase transmission of QCD.

The shear and nonlinear bulk viscosities of the Bianchi Type I inflationary universe model have been studied. Models with a dynamic cosmological term Λ(t) are gaining popularity because they solve the cosmological constant problem in a natural way. There is significant observational evidence in the discovery of Einstein's cosmological constant Λ, or the components of the matter content of the universe that change slowly with time and space to behave like Λ. Recent

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cosmological projects [8-11] suggest the existence of a positive cosmological term Λ of magnitude $\Lambda \left(Gh/c^3\right) \approx 10^{-123}$. These observations about the size and redshift of type Ia supernovae suggest that our Universe may be an accelerating function of cosmic density in the form of the cosmological term Λ. Many studies on this subject are included in Zeldovich [12], Weinberg [13], Dolgov [14], Betrolomi [15], Ratra and Peebles [16]. Several dynamic models have been proposed in which the cosmological constant Λ varies with cosmological time t to explain the density decline. These models lead to an effective cosmological constant that, as the universe expands, drops from its initially huge value to the small value observed today. In the last two decades, cosmological models with different damping laws for variations in the cosmological term have been considered [17,18].

Borges and Carnerio [19] also considered an isotropic uniform planar space filled with matter and a cosmic term proportional to H that obeys the vacuum equation of state. Recently, Tiwari and Divya Singh [20] studied anisotropic Bianchi type I models with various Λ terms. Tiwari and Sonia [21] studied the absence of shear in a Bianchi III-type string cosmological model with volume viscosity and time-dependent Λ. Banerjee et al. proposed a Bianchi type I solution for the rigid body case where the shear viscosity is a power function of the energy density. Examined. [22], while the volume viscosity as a power function of energy density and the rigid body model were studied by Huang [23]. The effect of bulk viscosity with time-dependent bulk viscosity coefficients on the development of isotropic FRW models was studied in the context of an open thermodynamic system by Desikan [24]. This work was further extended to the anisotropic Bianchi model by Krori and Mukherjee[25]. A cosmological solution with nonlinear bulk viscosity has been reported by Chimento et al. [26]. A model with both shear and bulk viscosities has been published by Gavrilove et al. [27]. Gron [28] reviewed the work on the viscous universe model.

In this paper, we have study the role of constant deceleration parameter in Bianchi type- I space time with bulk viscous fluid.The law of variation explicitly determines the scale factors. Cosmological consequences of the resulting models have been discussed.

2. Metric & Field Equation

The spatially homogeneous and anisotropic Bianchi type-I space time is described by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1}$$

where *A*, *B* and *C* are cosmic scale factors.

The Einstein's equations with varying Λ in suitable units are

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j + \Lambda g_i^j, \tag{2}$$

where energy momentum tensor T_i^j in the presence of bulk viscosity is taken in the form

$$T_i^j = (\rho + \overline{p})v_i v_j + \overline{p}g_i^j, \tag{3}$$

where $\bar{p} = p - \xi\theta$. (4)

Here ρ, p, ξ and θ are the energy density, isotropic pressure, bulk viscosity and expansion scalar respectively. The flow vector v^i satisfies the condition

$$v_i v^i = -1. \tag{5}$$

In co-moving system of coordinates $T_1^1 = \bar{p},\ T_2^2 = \bar{p},\ T_3^3 = \bar{p},\ T_4^4 = \rho$ .For the metric (1) and energy moment tensor (3) in the commoving system of coordinates, the field equation (2) yields

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\bar{p} + \Lambda, \tag{6}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\bar{p} + \Lambda, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\bar{p} + \Lambda, \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho + \Lambda. \tag{9}$$

From equations (6)-(9), we obtained

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 0. \tag{10}$$

Eliminating $\bar{p}$ and Λ from (6)-(8) and integrating, we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC}, \tag{11}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{ABC}, \tag{12}$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_1 + k_2}{ABC}. \tag{13}$$

where k_1 and k_2 are constant of integrations.

Spatial volume V as an average scale factor of the model (1) may be defined as

$$V = S^3 = ABC. \tag{14}$$

From equations (12)-(14), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{2k_1 + k_2}{3S^3}, \tag{15}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{k_2 - k_1}{3S^3}, \tag{16}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{k_1 + 2k_2}{3S^3}. \tag{17}$$

Integrating it, we get

$$A = m_1 S \exp\left[\left(\frac{2k_1 + k_2}{3}\right)\int \frac{dt}{S^3}\right], \tag{18}$$

$$B = m_2 S \exp\left[\left(\frac{k_2 - k_1}{3}\right)\int \frac{dt}{S^3}\right], \tag{19}$$

$$C = m_3 S \exp\left[-\left(\frac{k_1 + 2k_2}{3}\right)\int \frac{dt}{S^3}\right]. \tag{20}$$

Where m_1, m_2, m_3 are constant of integration.

We introduce volume expansion θ and σ as usual

$$\theta = v^i_i \quad \text{and} \quad \sigma^2 = \frac{1}{2}\left(\sigma_{ij} \times \sigma^{ij}\right), \tag{21}$$

σ_{ij} being shear tensor.

In the above the semicolon stands for covariant differentiation. For the Bianchi type-I metric expression for the dynamical scalar come out to be

$$\theta = 3\frac{\dot{S}}{S}, \tag{22}$$

$$\sigma = \frac{k}{\sqrt{3}S^3}. \tag{23}$$

Here $k^2 = k_1^2 + k_2^2 + k_3^2$ In analogy with FRW universe, we define a generalized, Hubble parameter H and the generalized deceleration parameter q as

$$H = \frac{\dot{S}}{S}, \tag{24}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 - \frac{\dot{H}}{H^2}. \tag{25}$$

Equations (6) – (9) can be written in terms of R, σ and q as

$$\bar{p} = H^2(2q-1) - \sigma^2 + \Lambda, \tag{26}$$

$$\rho = 3H^2 - \sigma^2 - \Lambda. \tag{27}$$

From (26), we observe that $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Therefore a positive Λ restricts the upper limit of anisotropy whereas a negative more room the anisotropy.

3. Solution & Discussion

The equations (6) - (9) are fourequations involving seven unknown term A, B, C, p, ρ, Λ, ζ so in order to close the system, we need three extra condition.

We assume the form of deceleration parameter q given by

$$q = \frac{-S\ddot{S}}{\dot{S}^2} = Const. = n, \tag{28}$$

yields

$$S = [S_0(t+t_0)]^{\frac{1}{1+n}} = (S_0 T)^{\frac{1}{1+n}}. \tag{29}$$

Where S_0 and t_0 are constant of integration and $T = t_0 + t$.

Coefficient of bulk viscosity ζ in the form [29].

$$\zeta = \zeta_0 + \zeta_1 H. \tag{30}$$

Next, we consider $\Lambda = \Lambda_0 H$,

where $\Lambda_0 > 0$ is constant [30].

Using (29) in (18) – (20), we get following expression for the scale factor

$$A = m_1 (S_0 T)^{\frac{1}{1+n}} \exp\left[\frac{2k_1 + k_2}{2}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right], \tag{31}$$

$$B = m_2 (S_0 T)^{\frac{1}{1+n}} \exp\left[\frac{k_2 - k_1}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right], \tag{32}$$

$$C = m_3 (S_0 T)^{\frac{1}{1+n}} \exp\left[\frac{-k_1 - 2k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right]. \tag{33}$$

By the transformation, $T = t_0 + t,\ m_1 x = X,\ m_2 y = Y,\ m_3 z = Z$ the metric (1) becomes

$$ds^2 = dT^2 + (S_0 T)^{\frac{2}{1+n}} \exp 2\left[\frac{2k_1 + k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] dX^2 + (S_0 T)^{\frac{2}{n+1}} \exp 2\left[\frac{k_1 - k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] dY^2$$
$$+ (S_0 T)^{\frac{2}{1+n}} \exp 2\left[\frac{-k_1 - k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] dZ^2 . \tag{34}$$

For model (33), expansion scalar θ, special volume V, shear scalar σ

$$\theta = \frac{3}{(1+n)\ T} , \tag{35}$$

$$V = (S_0 T)^{\frac{3}{1+n}} , \tag{36}$$

$$\sigma = \frac{k}{\sqrt{3}(S_0 T)^{\frac{3}{1+n}}}, \tag{37}$$

Matter densityρ, isotropic pressure p, cosmological term Λ, for the model take the form

$$\rho = \left[\frac{3}{(1+n)^2 T^2}\right] - \left[\frac{k^2}{3(S_0 T)^{6/(1+n)}}\right] - \left[\frac{\Lambda_0}{(1+n)T}\right], \tag{38}$$

$$p = \left[\frac{2n-1}{(1+n)^2 T^2}\right] - \left[\frac{k^2}{3(S_0 T)^{6/(1+n)}}\right] - \left[\frac{\Lambda_0}{(1+n)T}\right] + \frac{3}{(1+n)T}\left[\zeta_0 + \frac{\zeta_1}{(1+n)T}\right], \tag{39}$$

$$\Lambda = \frac{\Lambda_0}{(1+n)T}, \tag{40}$$

$$\zeta = \zeta_0 + \frac{\zeta_1}{(1+n)T}. \tag{41}$$

We see that the spatial volume is zero and the expansion scalar is infinite at T = 0, implying that the universe begins with zero volume and an infinite rate of expansion. The model begins to expand with a big-bang at T = 0. At T = 0, ρ , p, Λ and ζ are all infinitely large. ρ , Λ and p are zero whereas $\zeta \to \zeta_0$ for large values of T. The cosmological term Λ appears to be a decreasing function of time T. It starts out very large and shrinks to a small value for large t. This is consistent with recent findings from supernovae Ia observations. For large values of T, matter density tends to be a true constant.
We have the model (33).

$$\frac{\sigma}{\theta} = \frac{k(1+n)}{3\sqrt{3}S_0^{\frac{3}{1+n}}T^{\frac{2-n}{1+n}}}. \tag{42}$$

We observe that $\sigma / \theta \to 0$ as $T \to \infty$. Thus, the models approach isotropy at late times.

4. Conclusion

We investigated homogeneous and anisotropic Bianchi type I cosmological models with constant bulk viscosity in general relativity in this paper. In our proposal, the deceleration parameter q is proposed to be a constant function. The method is ad hoc, however, as it is not based on any established field theories. Einstein's field equations can be precisely solved. The expansion σ / θdrops to zero in the late stages as the scale factor rises. We explore a straightforward variation rules for the cosmological term Λ that, among the a functional forms that are feasible, lead to a cosmological scenario that is consistent with well-established aspects of modern cosmology. Throughout the evaluation, the model represents a decelerating universe for n > 0. Ifn = 0, we get H = 1 / T and q = 0, implying that every galaxy moves at the same rate. The precise solution described in this paper may be useful for a better understanding of the universe's origin and evaluation.

Received August 20, 2024; Accepted December 4, 2024

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