Exploration

The TGD View of the Recently Discovered Gravitational Hum as Gravitational Diffraction

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Abstract

Scientists from the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) have now officially made the first detections of the gravitational wave background. This gravitational hum was not detected by Earth bound instruments. Rather, they make themselves manifest as periodic changes of the spinning rates of pulsars with the frequencies of the gravitational waves involved. The periods are of order year. In the LIGO experiment the periods are measured as fractions of a second. The strength of gravitational hum is unexpectedly large. The basic insight of the TGD view is that diffraction produces very high intensities but only in preferred directions and that in diffraction the amplitude of the scattered field is proportional to the square N^2 of the number N of scatterers rather than N. The identification of dark matter as phases of the ordinary matter with an arbitrarily large value of Planck constant suggests the existence of tessellations of the hyperbolic 3-space identifiable as light-cone proper time hyperboloid of Minkowski space. Gravitational diffraction could occur in astrophysical scales at these tessellations.

1 Introduction

Year 2022 initiated a revolution in cosmology when James Webb telescope started to function [4][**?**]. This is not the only big step of progress. The latest breakthrough related to the detection of gravitational hum was announced June 29 2023 [1] (`rb.gy/e226v`: see also `rb.gy/vcm28` and `rb.gy/i4msf`).

1.1 The discovery

Scientists from the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) have now officially made the first detections of the gravitational wave background. This gravitational hum was not detected by Earth bound instruments. Rather, they make themselves manifest as periodic changes of the spinning rates of pulsars with the frequencies of the gravitational waves involved. The periods are of order year. In the LIGO experiment the periods are measured as fractions of a second.

The wavelength of the oscillations makes itself visible as correlations between the variations of the spinning rates for pulsars having relative distances measured using a light year as a unit. The wavelength of the oscillations is measured in light years. Where could this length scale come from? What might make bells ringing is that the star nearest to the Sun is at a distance of 4 light years and the typical distance between stars is 5 light years.

The unexpectedly large amplitude of the oscillations motivates the hypothesis that pairs of galactic supermassive blackholes or interacting groups of them could generate the gravitational hum. There are candidates for these pairs but no established pair. The group hypothesis seems to work better.

1.2 TGD based model for the gravitational hum

The strength of gravitational hum is unexpectedly large. The basic insight of the TGD view is that diffraction produces very high intensities but only in preferred directions and that in diffraction the

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amplitude of the scattered field is proportional to the square N^2 of the number N of scatterers rather than N.

TGD indeed predicts the possibility of lattice-like structures as tessellations of the hyperbolic space identifiable as light-cone proper time= constant surface of M^4. The 4 regular honeycombs corresponding to cubic, icosahedral, and dodecahedral tessellations and one honeycomb, icosa-tetrahedral honeycomb, involving several different Platonic solids tetrahedra, octahedra and icosahedral with triangular faces serve as candidates for the tessellation in question. The icosa-tetrahedral tessellation is unique and I have proposed that it might define a universal realization of the genetic code so that genetic code would be much more than a biochemical accident. The details of this realization are discussed in [6, 5]. Which of these tessellations is involved, remains an open question.

2 TGD explanation in terms of astrophysical gravitational quantum coherence and diffraction in hyperbolic tessellation

TGD suggests a radically different hypothesis based on TGD view of gravitational quantum coherence an diffraction in a hyperbolic tessellation.

1. TGD predicts quantum gravitational coherence in astrophysical scales characterized by gravitational Planck constants $h_{gr} = GMm/\beta_0$ characterizing big mass M and small mass m. $\beta_0 = v_0/c < 1$ ia velocity parameter. The Equivalence Principle is realized as the independence of the gravitational Compton length $\Lambda_{gr} = GM/\beta_0 = r_s/2\beta_0$ on mass m.

   (a) For the Sun with $\beta_0 \simeq 2^{-11}$ gravitational Λ_{gr} is 1/2 of Earth radius. According to the TGD proposal, which explains the Cambrian explosion in terms of rapid increase of the Earth radius by factor 2, this scale is the radius of Earth before the explosion [2].

   (b) For Earth with $\beta_0 = 1$, the scale is .45 cm and the size scale of a snowflake, which is a zoomed version of the unit cell of the ice crystal: a fact which still remains a mystery.

   (c) For the galactic black hole with $\beta_0 = 1$, Λ_{gr} is about $1.2 \times 10^7 km = 1.2 \times 10^{-2}$ light seconds and corresponds to a frequency of about 100 Hz, the upper bound of EEG frequencies by the way (which might put bells ringing!). For $\beta_0 = 1$, Λ_{gr} happens to correspond to the radius of the lowest Bohr orbit for Sun Λ_{gr} in the Bohr orbit model for planetary orbit (another bell ringing!) and defines a lower bound for the quantum coherence scale.

   (d) For the Milky Way with a mass of $2 \times 10^{12} M_{Sun}$, Λ_{gr} is about 2×10^4 second and still much shorter than a few years' scale.

2. Where does the wavelength of order of distance between stars emerge? TGD strongly suggests that the tessellations (lattices) associated with hyperbolic 3-spaces define light-cone proper time a= constant surface play a key role in all scales, in particular in biology.

   There could exist a fractal hierarchy of tessellations (`rb.gy/yqd1l`) formed by astrophysical objects of various mass scales. Could the stars with average distance of 5 light years form tessellations of this kind analogous to lattices in a condensed matter system. The wavelength for the diffracted gravitational waves in cubic tessellation would have the upper bound 2d, d the lattice constant, which would be now about 5 light years.

3. There is empirical evidence for these tessellations. So called cosmic fingers, discovered by Halton Arp [7] [3], correspond to astrophysical objects appearing at single light of sight (first mystery) and having redshift coming as multiples of a basic redshift (second mystery). This could serve as a direct signature of the hyperbolic counterpart for a line of atoms located along a lattice. Redshift is proportional to distance and also to the recession velocity, which would therefore be quantized in the observed manner.

4. What kind of tessellations could be involved? There is an infinite number of tessellations for H^3 but only 4 regular uniform honeycombs. For two of them the unit cell is dodecahedron, for the remaining two it is icosahedron (I) or cube (C). Note that in Euclidian 3-space one has just one regular honeycomb consisting of cubes.

   There are also more general uniform honeycombs involving several cell types. There is a unique "multicellular" honeycomb for which all cells are Platonic solids: for other tessellations all cells are not Platonic solids (`rb.gy/t3c88`) . This is icosatetrahedral (or more officially, tetrahedral-icosatetrahedral) honeycomb for which the cells are tetrahedrons, octahedrons, and icosahedrons. All faces are triangles and I have proposed a universal realization of genetic code in which genetic codons correspond to the triangular faces of icosahedra and tetrahedra.

The key prediction is gravitational diffraction in this cosmic lattice.

1. In lattice diffraction, the diffracted amplitude concentrates in specific directions corresponding to the reciprocal lattice. Something analogous should happen for tessellations in hyperbolic 3-space. Already the concentration to beams would mean an amplification effect (note that the lowest order prediction for the intensity of the radiation does not depend on the value of effective Planck constant).

   Furthermore, by quantum coherence the scattered amplitude is proportional to N^2 rather than N, where N is the number of atoms in the lattice, now stars in the tessellation. Could these two amplification effects explain why the observed effect is so much larger than expected? Professionals could easily find whether this idea fails at the quantitative level.

   The TGD view suggests that the dark gravitational radiation propagates along the monopole flux tubes connecting stars.

2. In the ordinary diffraction from a cubic lattice in Euclidean space E^3 , the condition of constructive interference for the two rays scattered from to neighboring points of the cubic lattice states, requires that the difference of lengths for the paths travelled is a multiple of the wavelength of the incoming radiation. This gives the Bragg condition: $sin(\theta) = n\lambda/2d$, where θ is the glance angle defined as the angle of incoming ray with respect to the normal direction of the lattice plane. The condition gives $\lambda < 2d/n$ and implies $\lambda < 2d$ for $n = 1$. Therefore the diffraction occurs only for frequency $\omega \geq n\omega_n$, $\omega_n > c/2d$.

   In the case of gravitational radiation, this would give for a cubic lattice $\lambda < 2d/n$, $d \sim 5$ light years, which conforms with the scale of a few years for the periods. The lower bound for the period T would be about $T_{min} = 10$ years. The condition that the scattered beams connect lattice points, gives an additional quantization condition to the glance angle θ. Most naturally it would correspond to a line connecting lattice points.

2.1 About honeycombs in hyperbolic 3-space

This section, written in 2023, represents some new understanding related to the tessellations of H^3 known as honeycombs.

2.1.1 Some preliminaries

Some preliminaries are needed in order to understand Wikipedia articles related to tessellations in general.

1. Schläfli symbol $\{p, r\}$ (`rb.gy/j36tg`) tells that the possibly existing Platonic solid $\{p, r\}$ has r p-polygons as faces meeting at each vertex. For instance, icosahedron $\{3, 5\}$ has 5 triangles as faces meeting at each vertex.

Schläfli symbol generalizes to higher dimensions. The analog of Platonic solid $\{p,r,q\}$ possibly in 4-dimensions and assignable to 3-sphere has q 3-faces which are Platonic solids $\{p,r\}$. This description is purely combinatorial and is recursive. For instance, one can start from 3-D dimensional Platonic solid $\{p,q\}$ with 3-D objects in dimension 4 by replacing p with p,r. One can also project this object to dimension 3. In this manner one obtains a projection of 4-cube (tesseract) $\{4,3,3\}$ for which 3 cubes $\{4,3\}$ meet at each vertex ($2^4=16$ of them) and which has 8 3-cubes as faces as a 3-D object.

In the case of hyperbolic tessellations also strange looking Schläfli symbols $\{(p,q,r,s)\}$ are encountered: icosa-tetrahedral tessellation involving only Platonic solids has symbol $\{(3,3,5,3)\}$. My understanding is that this object corresponds to $\{3,3,5,3\}$ as an analogue of Platonic solid associate with 4-sphere in 5-D Euclidian space and that the fundamental region of this tessellation in H^3 is analogous to a 3-D projection of this object. At a given vertex 3 objects $\{3,3,5\}$ meet. For these objects 5 tetrahedrons meet at a given vertex.

2. Vertex figure is a further central notion. It represents a view of the fundamental region of tessellation from a given vertex. The vertices of the figure are connected to this vertex. It does not represent the entire fundamental region. For instance, for a cube (octahedron) it contains only the 3 (4) nearest vertices. For icosa-tetrahedral tessellation the vertex figure is icosidodecahedron (`rb.gy/3u4pq`). The interpretation of the vertex symbol of the hyperbolic icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`) is a considerable challenge.

3. One cannot avoid Coxeter groups and Coxeter symbols (`rb.gy/48qhg`) in the context of tessellations. They code the structure of the symmetry group of say Platonic solid (tessellation of S^2). This symmetry group is generated by reflections with respect to some set of lines, usually going through origin. For regular polygons and Platonic solids is its discrete subgroup of rotation group.

   The Coxeter group is characterized by the number of reflection hyperplanes H_i and the reflections satisfying $r_i^2=1$. The products $r_{ij}=r_ir_j$ define cyclic subgroups of order c_{ij} satisfying $r_{ij}^{c_{ij}}=1$. Coxeter group is characterized by a diagram in which vertices are labelled by i. The orders of the cyclic subgroups satisfy $c_{ij}\geq 3$. For c_{ij} the generators r_i and r_{ij} commute. For $c_{ij}=2$ the vertices are not connected, for $c_{ij}=3$ there is a line and for $c_{ij}>3$ the number c_{ij} is assigned with the line. For instance, hyperbolic tessellations are characterized by 4 reflection hyperplanes.

   For instance, for p-polygon the Coxeter group has 2 generators and the cyclic group has order p. For Platonic solids the Coxter group has 3 generators and the orders of cyclic subgroups are 3, 4, or 5. For icosa-tetrahedral tessellation the order is 4.

2.1.2 The most interesting honeycombs in hyperbolic 3-space

H^3 allows an infinite number of tessellations. There are 9 types of honeycombs. This makes 76 uniform hyperbolic honeycombs involving only a single polyhedron (`hrb.gy/rs9h5`).

4 of these honeycomes are *regular*, which means that they have identical regular faces (Platonic solids) and the same numbers of faces around vertices. The following list gives the regular uniform honeycombs and their Schläfli symbols $\{p,q,r\}$ telling that each edge has around it regular polygon $\{p,q\}$ for which each vertex is surrounded by q faces with p vertices.

1. H1: 2 regular forms with Schläfli symbol {5,3,4} (dodecahedron) and {4,3,5} (cube).
2. H2: 1 regular form with Schläfli symbol {3,5,3}(icosahedron)
3. H5: 1 regular form with Schläfli symbol {5,3,5} (dodecahedron).

There is a large number of uniform honeycombs involving several cell types. There exists however a "multicellular" honeycomb, which is completely unique in the sense that for it all cells are Platonic solids. This icosa-tetrahedral (or more officially, tetrahedral-icosahedral) honeycomb has tetrahedrons, octahedrons, and icosahedrons as its cells. All faces are triangles. The icosa-tetrahedral honeycomb is of special interest since it might make possible the proposed icosa-tetrahedral realization of the genetic code (`rb.gy/h8xx0`).

From the Wikipedia article about icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`) one learns the following.

1. The Schläfli symbol of icosa-tetrahedral honeycomb is $\{(3,3,5,3)\}$. This combinatorial symbol allows several geometric representations. The inner brackets would refer to the interpretation as an analogue of the Platonic solid assignable to a 4-sphere of Euclidian 5-space. At each vertex 3 objects of type $\{3,3,5\}$ would meet. At the vertex of $\{(3,3,5)\}$ in turn 5 tetrahedrons meet.

2. Icosa-tetrahedral honeycomb involves tetrahedron $\{(3,3\}$, octahedron $\{(3,4)\}$, an icosahedron $\{(3,5)\}$ as cells. That there are no other honeycombs involving several Platonic solids and only them as cells makes this particular honeycomb especially interesting. Octahedron with Schläfli symbol $\{3,4\}$ can be also regarded as a rectified tetrahedron havig Schläfli symbol $r\{3,3\}$.

3. The vertex figure of icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`), representing the vertices a lines connecting them is icosidodecahedron (`rb.gy/q5w62`), which is a "fusion" of icosahedron and dodecahedron having 30 vertices with 2 pentagons and 2 triangles meeting at each, and 60 identical edges, each separating a triangle from pentagon. From a given vertex VF=60 vertices connected to this vertex by an edge can be seen. In the case of cube, octahedron, and dodecahedron the total number of vertices in the polyhedron is 2(VF+1). It is true also now, one would have 122 vertices in the basic structural unit. The total number of vertices for the disjoint polyhedra is 6+4+12= 22 and since vertices are shared, the number of polyhedra in the basic unit must be rather large.

4. The numbers called "cells by location" could correspond to numbers 30, 20, and 12 for octahedrons, tetrahedrons and icosahedrons respectively inside the fundamental region of the tessellation defining the honeycomb. That the number of icosahedrons is smallest, looks natural. These numbers are quite large. The counts around each vertex are given by (3.3.3.3), (3.3.3), *resp.* (3.3.3.3.3) for octahedra, tetrahedra, *resp.* icosahedra and tell the numbers of vertices of the faces meeting at a given vertex.

5. What looks intriguing is that the numbers 30, 20, and 12 for octahedrons (O), tetrahedrons (T) and icosahedrons (I) correspond to the numbers of vertices, faces, and edges for I. As if the fundamental region would be obtained by taking an icosahedron and replacing its 30 vertices with O, its 20 faces with T and its 12 edges with I, that is by using the rules *vertex* $\rightarrow$ *octahedron*; *edge* $\rightarrow$ *I*, *face* $\rightarrow$ *T*. These 3-D objects would be fitted together along their triangular faces.

   Do the statements about the geometry and homology of I translate to the statements about the geometry and homology of the fundamental region? This would mean the following replacements:

   (a) "2 faces meet at edge" $\rightarrow$ "2 T:s share face with an I".
   (b) "5 faces meet at vertex" $\rightarrow$ "5 T:s share face with an O".
   (c) "Edge has 2 vertices as ends" $\rightarrow$ "I shares a face with 2 different O:s".
   (d) "Face has 3 vertices $\rightarrow$ "T shares a face with 3 different O:s".
   (e) "Face has three edges" $\rightarrow$ "T has a common face with 3 I:s".

2.1.3 An attempt to understand the hyperbolic honeycombs

The following general observations might help to gain some understanding of the honeycombs.

1. The tessellations of E^3 and H^3 are in many respects analogous to Platonic solids as 2-D objects. The non-compactness implies that there is an infinite number of cells for tessellations. It is important to notice that the radial coordinate r for H^3 corresponds very closely to the hyperbolic angle and its values are quantized for the vertices of tessellation just like the values of spherical coordinates are quantized for Platonic solids.

   The tessellations for E^3 are scale covariant. For a fixed radius of H^3 characterized by Lorentz invariance cosmic time this is not the case. One can however scale the value of a.

2. What distinguishes between regular tessellations in E^3 and H^3 is that the metric of H^3 is non-flat and has negative curvature. H^3 is homogeneous space meaning that all points are metrically equivalent (this is the counterpart of cosmological principle in cosmology). Since both spaces have rotations as symmetries, this does not affect basic Platonic solids as 2-D structures assignable with 2-sphere if the edges are identified as geodesic lines of S^2. Quite generally, isometries characterize the tessellations, whose fundamental region corresponds to coset space of H^3/Γ by a discrete group of the Lorentz group acting as isometries of H^3.

3. The modifications induced by the replacement $E^3 \to H^3$ relate to the 3-D aspects of the tessellation. This is because the metric is non-flat in the radial direction. The negative curvature implies that the geodesic lines diverge. One can use a counterpart of the standard spherical coordinates and in these coordinates the solid angles assignable to the vertices of Platonic solid are smaller than in E^3. Also the hyperbolic planes H^2 emerging from edges of the tessellation of H^3 diverge in normal direction the angles involved are smaller.

It is useful to start from the description of the Platonic solids. They are characterized combinatorially by integers and geometrically by various kinds of angles. Denote by p the number of vertices/edges of the face and by q the number of faces meeting at vertex.

1. Important constraints come from the topology and combinatorics. Basic equations for the numbers V ,E, and F for the number of vertices, edges and faces are purely topological equations $VE+F=2$, and the equation $pF=2E=qV$. Manipulation of these equations gives $1/r+1/p=1/2+1/E$ implying $1/r+1/p>1/2$. Since p and q must be at least 3, the only possibilities for $\{p,q\}$ are $\{3,3\}$, $\{4,3\}$, $\{3,4\}$, $\{5,3\}$, and $\{3,5\}$.

2. The angular positions of the vertices at S^2 are basic angle variables. In H^3 hyperbolic angle assignable to the radial coordinate is an additional variable of this kind analogous to the position of the unit cell in the E^3 tessellation. The cosmological interpretation is in terms of redshift.

3. There is the Euclidian angle ϕ associated with the vertex of the face given by π/p. Here there is no difference between E^3 and H^3.

4. The angle deficit δ associated with the faces meeting at a given vertex due to the fact that the faces are not in plane in which case the total angle would be 2π. δ is largest for tetrahedron with 3 faces meeting at vertex and therefore with the sharpest vertex and smallest for icosahedron with 5 triangles meeting at vertex. This notion is essentially 3-dimensional, being defined using radial geodesics, so that the δ is not the same in H^3. In H^3 δ is expected to be larger than in E^3.

5. There is also the dihedral angle θ associated with the faces as planes of E^3 meeting at the edges of the Platonic solid. θ is smallest for a tetrahedron with 4 edges and largest for a dodecahedron with 20 edges so that the dodecahedron is not far from the flat plane and this angle is not far from π. The H^3 counterpart of θ is associated faces identified as hyperbolic planes H^2 and is therefore different.

6. There is also the vertex solid angle Ω associated with each vertex of the Platonic solid $\{p, q\}$ given by $\Omega = q\theta - (q-2)\pi$. For tessellations in E^3 the sum of these angles is 4π. In H^3 its Euclidian counterpart is larger than 4π.

7. The face solid angle is the solid angle associated with the face when seen from the center of the Platonic solid. The sum of the face solid angles is 4π. For Platonic solid with n vertices, one has $\Omega = 4\pi/n$. The divergence of the geodesics of H^3 implies that this angle is smaller in H^3: there is more volume in H^3 than in E^3.

E^3 allows only single regular tessellation having cube as a unit cell. H^3 allows cubic and icosahedral tessellations plus two tessellations having a dodecahedron as a unit cell. Why does E^3 not allow icosahedral and dodecahedral tessellations and how the curvature of H^3 makes them possible? Why is the purely Platonic tetra-icosahedral tessellation possible in H^3?

The first guess is that these tessellations are almost but not quite possible in E^3 by looking at the Euclidian constraints on various angles. In particular, the sum of dihedral angles θ between faces should be 2π in E^3, the sum of the vertex solid angles Ω at the vertex should be 4π. Note that the scaling of the radial coordinate r decreases the dihedral angles θ and solid angles Ω. This flexibility is expected to make possible so many tessellations and honeycombs in H^3. The larger the deviation of the almost allowed tessellation, the larger the size of the fundamental region for fixed a.

Consider now the constraints on the basic parameters of the Platonic solids (`rb.gy/1cuav`) in E^3 while keeping their H^3 counterparts in mind.

1. The values of didedral angle for tetrahedron, cube, octahedron, dodecahedron, and icosahedron are

   $$[\theta(T), \theta(C), \theta(O), \theta(D), \theta(I)] \approx [70.3°, 90°, 109.47°, 116.57°, 138.19°] \ .$$

   Note that $r = 5$ tetrahedra meeting at a single edge in E^3 woul almost fill the space around the edge. In E^3 $r = 4$ cubes can meet at the edge. In H^3 r should be larger. This is indeed the case for the cubic honeycomb $\{4, 3, 5\}$ having $r = 5$.

   For $r = 3$ icosahedrons the sum dihedral angles exceeds 2π which conforms with the that $\{3, 5, 3\}$ defines an icosahedral tessellation in H^3.

   For the $r = 4$ dodecahedra meeting at the edge the total dihedral angle is larger than 360°: $r = 4$ is therefore a natural candidate in H^3. There are indeed regular dodecahedral honeycombs with Schläfli symbol $\{5, 3, r\}$, $r = 4$ and $r = 5$. Therefore it seems that the intuitive picture is correct.

2. The values of the vertex solid angle Ω for cube, dodecahedron, and icosahedron are given by the formula $\Omega = q\theta - (q-2)\pi$ giving

   $$[\Omega(C), \Omega(D), \Omega(I)] \approx [1.57080, 2.96174, 2.63455].$$

   The sum of these angles should be 4π for a tessellation in E^3. In E^3 This is true only for 8 cubes per vertex ($\Omega = \pi/2$) so that the cubic honeycomb is the only Platonic honeycomb in E^3. The minimal number of cubes per vertex is 9 in H^3. It is convenient to write the values of the vertex solid angles for D and I as

   $$[\Omega(D), \Omega(I)] = [0.108174, 0.209651] \times 4\pi \ .$$

   The number of D:s *resp.* I:s must be at least 10 *resp.* 5 for dodecahedral *resp.* icosahedral honeycombs in H^3.

3. The basic geometric scales of the Platonic solids are circumradius R, surface area A and volume V. The circumradius is given by $R = (a/2)\tan(\pi/q)\tan(\theta/2)$, where a denotes the edge length. The surface area A of the Platonic solid $\{p, q\}$ equals the area of face multiplied by the number F of faces: $A = (a/2)^2 Fpcot(\pi/p)$. The volume V of the Platonic time is F times the volume of the pyramid whose height is the length a of the face: that is $V = FaA/3$.

   Choosing $a/2$ as the length unit, the circumradii R , total face areas A an the volumes V of the Platonic solids are given by

$$[R(T), R(C), R(O), R(D), R(I)] = [\sqrt{3}/2, \sqrt{3}, \sqrt{2}, \sqrt{3}\phi, \sqrt{3-\phi}\phi] \ ,$$

$$[A(T), A(C), A(O), A(D), A(I)] = [4\sqrt{3}, 24, 2\sqrt{3}, 12\sqrt{25+10\sqrt{5}}, 20\sqrt{3}] \ ,$$

   and

$$\begin{aligned}&[V(T), V(C), V(O), V(D), V(I)] \approx [\sqrt{8}/3, 8, \sqrt{128}/3, 20\phi^3/(3-\phi), 20\phi^2/3] \\ &\approx [.942809, 8, 3.771236, 61.304952, 17.453560] \ .\end{aligned}$$

What can one say about icosa-tetrahedral tessellation?

1. Consider first the dihedral angles θ. The values of dihedral angles associated T, O, and I in H^3 are reduced from that in E^3 so that their sum in E^2 scene must be larger than 2π. Therefore at least one of these cells must appear twice in H^3. It could be T but also O can be considered. For $2T + O + I$ and $T + 2O + I$ the sum would be $388.26°$ *resp.* $427.43°$ in E^3. $2T + O + I$ *resp.* $T + 2O + I$could correspond to 4 cells ordered cyclically as ITOT resp. IOTO.

2. The values of the vertex solid angle Ω for tetrahedron, octahedron, and icosahedron are given by $[\Omega(T), \Omega(O), \Omega(I)] = [0.043870, 0.108174, 0.209651]4\pi$ If the numbers of T, O and I are $[n(T), n(O), n(I)]$, one must have $[n(T)\Omega(T), +n(O)\Omega(O) + n(I)\Omega(I) > 4\pi$ in H^3.

   If the number of the cells for the fundamental domain are really $[N(T), N(O), N(I)] = [30, 20, 12]$, the first guess is that $[n(T), n(O), n(I)] \propto [N(T), N(O), N(I)$ is approximately true. For $[n(T), n(O), n(I)] = [2, 3, 1]n(I)$, one obtains $\Omega = n(T)\Omega(T) + n(O)\Omega(O) + n(I)\Omega(I) = n(I) \times .629 \times 4\pi$. This would suggest $n(I) = 2$ giving $[n(T), n(O), n(I)] = [4, 6, 2]$

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