Essay

Kondo Effect from the TGD perspective

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Abstract

Kondo effect is due to a scattering of conduction electrons from valence electrons of magnetic impurity atoms and implies logarithmic increase of the resistance of the conductor at the zero temperature limit. Anderson's impurity model combined with the renormalization group explains the logarithmic increase of the quadratic coupling between conduction electron and valence electron. The Kondo effect occurs in the non-perturbative regime of the Anderson model and this implies several analogies with QCD and hadron physics. With a motivation coming from the QCD analogy and TGD view of hadrons, the Kondo effect is discussed from the TGD view point by introducing notion of the magnetic body carrying dark matter as $h_{eff} > h$ phase, assignable to the impurity spin. The conduction electrons forming the electron cloud around the impurity spin and neutralizing would be actually dark valence electrons. It is assumed that Nature is theoretician friendly. As the perturbation series ceases to converge, either the quantum coherence is lost or the value of Planck constant h increases to $h_{eff} > h$ to guarantee its convergence. Also the generalization of Nottale's hypothesis from gravitational to electromagnetic situation is assumed. In the recent situation the relevant coupling parameter would be Q^2e^2, where Q is the total charge of the valence electron cloud around the impurity: after the transition the coupling parameter would be universally $\beta_0/4\pi$, $\beta_0 = v_0/c < 1$. This transition would happen in the Kondo effect and lead to the formation of spin singlets as analogs of hadrons in color confinement. The dark valence electrons would be analogs of sea partons and the impurity electrons would be counterparts of valence quarks in this picture. This picture also allows us to understand heavy fermions as analogs of constituent quarks and Kondo insulators. This picture also provides new insights to hadron physics.

1 Introduction

I have tried to learn some condensed matter physics from the TGD point of view and have even written a book [4] containing a chapter [9], which summarizes my most recent efforts.

Although I have only a superficial understanding of condensed matter physics, I can agree with Anderson when he says that there is no theory of condensed matter physics. There are models based typically on a Hamiltonian in a spin lattice but somehow it seems that there is no attempt to understand the basic physics. With TGD as a background, the reductionist view does not force me to believe that condensed matter physics is mere complexity, so that I cannot avoid the intuition that a lot of new physics is waiting to be discovered.

1.1 Kondo effect

I realized that the Kondo effect (`https://rb.gy/gm5bom`) could involve new physics predicted by TGD. Kondo effect relates to the scattering of s-orbital conduction electrons scattering on d-orbital electrons of magnetic impurities. A low temperature phenomenon is in question. Electrical resistivity is given by the general formula

$$\rho(T) = \rho_0 + aT^2 + c_m log(\frac{\mu}{T}) + bT^5 \quad . \tag{1.1}$$

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Here T^2 term comes from Fermi liquid properties, T^5 term corresponds to the scattering from lattice vibrations and the logarithmic Kondo term corresponds to the scattering from the magnetic impurities. Kondo term increases at low temperatures and diverges logarithmically at the $T = 0$ limit and begins negligible at a temperature of a few Kelvins. The reason is resonant scattering at low temperatures. This would suggest formation of resonances.

Transformation of a conduction electron to an impurity electron and vice versa is essential as also the bilocal Coulomb interaction term for the conduction electrons, which makes the system non-linear in the description using electron's oscillator operators. Hybridization as a mixing valence electrons and conduction electrons occurs: valence electron transforms to an impurity electron or vice versa in the 2-vertex. Screening of the impurity spin by the spins of the conduction electron cloud takes place so that a system with a vanishing spin is formed is essential. At high temperatures impurity electrons appear as free particles.

There is an analogy with low energy QCD. Logarithmic increase of the resistivity towards $T = 0$ is analogous to the logarithmic increase of the QCD coupling strength α_s as the mass scale corresponding to QCD Λ is approached and hadronization takes place. Resonances in Kondo scattering correspond to the formation of hadrons. At high temperatures the impurity spins seem to behave like free spins and there is an analogy with asymptotic freedom. The screening of impurity spin by conduction electrons is analogous to color confinement.

Kondo's original model was based on third order perturbative calculation. Anderson's impurity model, combined with Wilsonian renormalization theory, provides another approach and in this model the Kondo effect occurs in the regime, where the perturbation series fails to converge. Since hybridization terms are quadratic, this must be caused by the Coulomb interaction term. The analogy with QCD becomes more obvious.

How could one understand the increase of the hybridization coupling towards low temperatures? Is it a secondary phenomenon caused by the non-linearity of the Hamiltonian? Somehow the increase of the Coulomb term induced by the increase of the size of the volume containing the conduction electron cloud should cause the increase of the coupling and lead to the analog of hadronization as a formation of spin singlets?

The total Coulomb interaction energy is proportional to Q^2, Q the total charge of the conduction electron cloud, increases and if the system is quantum coherent it could lead to the failure of perturbation theory containing $Q^2\alpha$ as a coupling parameter.

1.2 TGD view based on the notion of magnetic body and h_{eff} hierarchy

Consider now the system from the TGD point of view assuming the view that impurity spin as valence electron is accompanied by a magnetic body (MB) consisting of magnetic flux tubes representing the approximate dipole field and that the conduction electrons interacting with impurity spin can have a value of effective Planck constant h_{eff}, which is larger than h so that quantum coherence in length scales longer than the atomic length scales becomes possible at the MB and induces ordinary coherence of ordinary matter.

It is good to list the general ideas first.

1. Impurity electrons are accompanied by magnetic flux tubes and some fraction of conduction electrons ends up at the flux tubes as dark *valence* electrons, which are localized in a longer scale. The formation of resonances corresponds to the formation of conduction electron clouds around impuritie as association of conduction electrons to the magnetic flux tubes and is analogous to the formation of hadrons.

   Magnetic moments sum up to zero and spin screening takes place as an analog of color confinement. Magnetic moment interaction becomes strong at the limit $T = 0$.

2. The original theoretical motivation for the $h_{eff} = nh_0$ hierarchy is the following. When the interaction strength $\alpha alpha = Q_1Q_2g^2/4pi\hbar$ for a quantum coherent system consisting of charges Q_1

and Q_2 becomes too large, the perturbation series fails to converge. Nature is however theoretician friendly and induces the phase transition $h \rightarrow h_{eff}$ making a perturbation theory possible. The size of the MB and associated quantum coherence length inducing coherence at the level of ordinary matter, increases and bound states with larger size become possible.

3. Nottale's proposal [1] [7, 6, 5] for the notion of gravitational Planck constant generalizes to other interactions. When the perturbation series fails, the electromagnetic counterpart of gravitational Planck constant would increase to $\hbar_{eff} = \hbar_{em} = Q_1Q_2\alpha/\beta_0$, $\beta_0 = v_0/c < 1$. The perturbative coupling parameter for quantum coherent states would be $Q_1Q_2e^2/\hbar_{em} = \beta_0/4\pi$ and would be universal. This would be the situation for all interactions. The value of β_0 is by number theoretical arguments proposed to be an inverse integer [13].

4. h_{eff} should increase as some relevant coupling constant increases. Number theoretic interpretation implies that more complex states are generated. $n = h_{eff}/h_0$ corresponds to the degree of extension of rationals defined by a given polynomial [L1, L2]. For a given polynomial, the largest and smallest ramified primes associated with an extension with dimension $n = h_{eff}/h_0$ are physically preferred and could be seen as fixed points of the coupling constant evolution assignable to this polynomial [14]. Also for the set of polynomials with fixed degree k and coefficients smaller than k, one can identify smallest and largest p-adic length scales. They define fixed points of coupling constant evolution in this set.

2 TGD view of Kondo effect

TGD view of the Kondo effect relies on the basic notions discussed in the introduction. It is best to start by making questions.

2.1 Why the spin confinement occurs only at low temperatures?

One should understand why the spin confinement occurs at low temperatures only.

1. The value of h_{eff} associated with the MB of the impurity spins surrounded by valence spin cloud should increase at the limit $T = 0$. h_{eff} and p-adic length scale are correlated so that also p-adic length scale measuring the size of the system would increase: larger spin singlets are formed.

2. The energies of states increase with h_{eff}. The increase of h_{eff} requires energy feed and h_{eff} can decrease spontaneously. Thermal energy can serve as an energy feed by inducing h_{eff} increasing transitions. Why does this increase not occur at higher temperatures and lead to larger spin singlet states than at low temperatures?

   Since this does not occur, thermal energy must exceed the binding energy of the state above a critical temperature and make it unstable. This requires that the binding energy of the state must decrease with increasing h_{eff}. Atomic binding energies satisfy this condition. They are proportional to $1/\hbar_{eff}^2$ and approach zero like $1/\hbar_{eff}^2$ and are stable only below a critical temperature determined by the ground state energy. Something similar should happen also now, which suggests that atomic binding energy of dark valence electrons is important.

2.2 What interactions should be taken into account?

In the Anderson's impurity model, Coulomb interaction between valence electrons and the interaction describing the hybridization are taken into account. In the TGD framework the situation can be more complex.

1. Certainly the coupling between dark valence electrons is important. In the presence of quantum coherence, one could perhaps approximate this interaction by using interaction strength Q^2e^2, where Q is the total charge of dark valence electrons of the cloud. Could one assume that Q^2e^2 defines in a good approximation the effective Planck constant as $\hbar_{em} = Q^2\alpha/\beta_0$? Could $\hbar_{em}$ define the binding energy scale of the dark valence electrons and reduce it by a factor $(\hbar/\hbar_{em})^2$ so that a thermal instability would be the outcome and the model would be in many respects be similar to the Anderson's impurity model.

   Since the spin spin-interaction is essentially the interaction energy between two magnetic moments, it should be proportional to $\hbar_{eff}$ and would increase as dark valence electrons with increasing values of $\hbar_{eff}$ are stabilized. The stable value of $\hbar_{eff}$, determined by the Coulomb interaction terms for conduction electrons, should depend logarithmically on the temperature. The generalization of the Nottale hypothesis [1] to the electromagnetic case implies $\hbar_{eff} \propto Q^2\alpha/\beta_0$. Therefore the parameter Q^2/β_0 measuring also the charge of the dark valence electron cloud should increase logarithmically with the inverse temperature.

2. In Anderson's impurity model, the attractive interaction between the conduction electrons and atoms is not taken into account. Can one forget the presence of atoms in the TGD framework? Assume that the dark valence electrons of the cloud behave like a single quantum coherent unit with total charge Q. The interaction strength for the mutual interactions of valence electrons would be Q^2e^2.

   If each dark valence electron is associated with a single atom (rather than with the atoms associated with the cloud), the effective charge Q_{eff} of the atom screened by the inner electrons is equal to $Q_{eff} = -1$ in the first approximation. The interaction strength for dark valence electron charge and atomic charge would be $QQ_{eff}e^2 = -Qe^2$. The interaction strength for the mutual interaction of valence electrons would dominate. Situation would be similar to that in the Anderson model.

2.3 Hadron physics analogy in the TGD framework

Ordinary hadron physics need not be enough as an analogy of Kondo effect in the TGD framework.

1. TGD predicts a hierarchy of scaled up versions of hadron physics associated with Mersenne primes and their Gaussian analogs [2, 3]. Color confinement would occur always but at high energies the scale of confinement decreases as the size of the quarks decreases. The MB of the hadron would not disappear at high energies but its size decreases in a stepwise manner at p-adic length scales corresponding to Mersennes.

2. The possibility of having $h_{eff} > h$ allows to have a situation in which the Compton length and geometric size of say M_{89} hadron with 512 times higher mass than that of ordinary M_{107} hadron is the same as that of ordinary hadron [2, 3].

   The model assumes that the p-adic length scale is not completely fixed by h_{eff}. The transition $h \rightarrow h_{eff} = 512h$ for M_{89} hadrons could serve as a TGD counterpart of color deconfinement for quarks, whose masses have increased by this factor.

   The number theoretic vision allows us to formulate this idea in a rather detailed way [14].

   (a) A given extension of rationals with dimension $n = h_{eff}/h_0$ allows several ramified primes, which define possible p-adic length scales [14] tentatively identified as p-adic lengths scales associated with the many-particle state assigned with the polynomial. For elementary particles there would be only a single ramified prime.

   (b) For a given value of p-adic length scale, one can have several values of $h_{eff}/h_0 = n$. Particles with different values of $h_{eff}/h_0 = n$ can have the same unique ramified prime. For two particles with the same ramified prime but with different values n_i, the ratio of p-adic length scales would be n_1/n_2.

(c) If the particle belongs to a many-particle system possessing several ramified primes, also the ramified prime and the p-adic length scale characterizing the particle can change.

(d) Assuming this picture, one can formulate the assumption that M_{89} hadrons appear as their dark variants in the situations, where indications for their existence can be found [2, 3]. Dark variant of M_{89} hadron would have h_{eff} related by factor 512 to its non-dark variant.

3. Color confinement involves the increase of the strong coupling strength α_s. The proposed vision predicts a parton-hadron phase transition in which α_s is replaced by the coupling parameter $\beta_0/4\pi$ of the perturbation theory based on hadrons as fundamental objects.

   One can apply the analogy between color confinement and the Kondo effect also in the opposite direction. Valence quarks are analogous to the impurity electrons whereas sea quarks and gluons are analogous to the conduction electrons, which have transformed to dark valence electrons. Valence quarks would correspond to ordinary matter and sea partons to dark matter at MB. This suggests a new approach to hadron physics.

   As a matter of fact, the notion of a color magnetic body as a structure much larger than hadron itself has already made its appearance in TGD. The reason is that the Compton wavelengths of u and d current quarks are much longer than nucleus size and even nuclear size so that their proper place is naturally at the MB if the Compton length has geometric size as a classical space-time correlate.

4. Galois confinement [L3, 10, 11, 9] generalizes the notion of color confinement in the TGD framework, and is an essential element of the number theoretical view of TGD. It can be formulated at the level of 4-momenta of fermions and corresponds to momentum space description. Obviously, this description is especially well-suited in condensed matter physics.

   The extreme form of Galois confinement [L3, 15, 8, 12, 10, 11] states that all bound states are Galois confined. The Galois bound states are characterized by their binding energy. For a given extension they can exist only below a certain temperature and have a temperature dependent sice. The size of MB should increase and binding energy should decrease with the size so that the stable size of MB decreases with increasing temperature.

Consider now the application of this picture to the Kondo effect.

1. The hadronic analogy suggests that the logarithmic increase of the coupling between valence electron and impurity spins is a secondary phenomenon induced by the increase of $\hbar_{em}$ to which the magnetic interaction energy is proportional. One cannot speak of conduction electron scattering anymore. Rather, one should speak of the scattering of spin singlets as analogs of hadrons, whose size decreases at high temperatures as they approach impurity spins.

2. Assume that the sizes of MB of the spinless bound state increase at the $T = 0$ limit. The reduction of the size of MB would be due to both due the discrete p-adic length scale evolution and $h_{eff} = h_{em}Q^2\alpha/\beta_0$ evolution of the coupling strength for mutual interactions of valence-electrons as a quantum coherent unit. At high temperatures one would obtain free impurity spins as analogs of free quarks. Here one must take into account the possibility that some fraction of spins are genuinely free simply because the dark valence electron clouds contain only a fraction of valence electrons.

2.4 Some applications of the Kondo effect

Kondo effect according to TGD involves only very general assumptions such as hierarchy of Planck constants, the idea about $h \to h_{eff}$ when perturbation theory fails, and the instability of the bound states with $h_{eff} > h$ against too large thermal perturbations. Therefore the Kondo effect should generalize and have a strong resemblance with color confinement.

The Kondo effect indeed has several applications.

1. In heavy fermion systems (https://rb.gy/hpkcyh) the effective mass of electrons can increase by several orders. Also in hadron current quarks transform to much more massive consituent quarks, which involve not only the current quark but also the flux tube with color magnetic energy and the mass of the sea partons.

   Could the situation be the same now? The impurity spin valence electron is bound to conduction electrons, which have transformed to a sea of dark valence electrons. The effective mass of the impurity electron involves also the contribution from the magnetic energy and dark valence electrons.

2. In Kondo insulators (https://rb.gy/ikl33x) the valence electrons are bound to the flux tubes associated with the impurities and localization occurs. Current would not flow. There is an analogy with the state with no free quarks, which carry no color currents. Note however that now only spin currents vanish. This could allow charge currents as currents formed by the bound states of impurities and dark valence electron clouds.

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