Article

Higher Dimensional Plane Symmetric Cosmological Model with Linear Equation of State

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Abstract

In this paper, we investigated higher dimensional plane symmetric cosmological model with linear equation of state. To decipher solutions, we considered the variation law of mean Hubble parameter and constant deceleration parameter which gives two different physically viable modes. The first solution yields a singular model for $n \neq 0$ while the second gives a non-singular model for $n = 0$. We discussed the physical behaviour of both models by using physical parameters.

Keywords: Higher dimension, plane symmetric, cosmological model, linear equation.

1. Introduction

The scientific study of cosmology has long captivated the imagination of researchers and scientists alike, seeking to unravel the mysteries of the universe's origin, evolution, and ultimate fate. Over the years, numerous theoretical frameworks and mathematical models have been proposed to explain the large-scale structure and dynamics of our cosmos. Within this rich tapestry of cosmological theories, the Bianchi type I cosmological model represent a significant and intriguing avenue for exploring the large-scale structure and dynamics of universe. The field of cosmology has witnessed remarkable advancements. The modern astronomical observational data proclaimed that universe is not only merely expanding but also accelerating [1-8]. This implies the existence of total negative pressure for the universe.

In recent years, considerable attention has been directed toward understanding the nature of mysterious component of energy known as dark energy which is believed to constitute a significant fraction of the energy content of the universe. The equation of state parameter which characterizes the relationship between the pressure and energy density of a cosmic fluid plays a crucial role in modelling dark energy and its physical aspects. Among the various proposed forms of the linear equation of state has gained prominence due to its simplicity and ability to reproduce certain observed phenomena.

Studies of higher dimensional cosmological models are crucial in addressing various aspects of early cosmological questions. Exploring higher dimensional space-time suggest that our universe may have been significantly smaller in its early stages, contrasting with its current observed size. They provide valuable insights into the behaviour of the universe at different scales and in early cosmic epochs, potentially helping us explain phenomena that are challenging to understand within the confines of four dimensions. Samanta et al. [9] have investigated five-dimensional Bianchi type -I cosmological model generated by a cloud of string with particles attached to them in Lyra manifold. Mete and Elkar [10] have

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examined higher dimensional plane symmetric cosmological model with time dependent cosmological constant within the framework of Saez and Ballester theory of gravitation. Ladke et al. [11] have analysed Bianchi type -V space-time in $f(R,T)$ theory of gravity for five-dimension. Mohurley et al. [12] have obtained five-dimensional Bianchi type – I cosmological models filled with perfect fluid in the frame work of Saez and Ballester theory. Mete et al. [13] have presented anisotropic, homogeneous higher dimensional plane symmetric cosmological model with two fluid sources in general relativity.

Higher dimensional investigation was done by several authors [14-19]. Hatkar and Katore [20] have examined Kalunza-Klein cosmological models with polytropic equation of state in Lyra Geometry. Adhav et al. [21-23] have explored various cosmological models with equation of state in General theory of Relativity. Shaikh [24] have obtained plane symmetric cosmological model with linear equation of state. Pawar et al. [25] have explored plane symmetric cosmological model with magnetized strange quark matter in Lyra manifold. Agrawal and Pawar [26] have investigated plane symmetric cosmological model incorporating quark and strange quark matter in $f(R,T)$ theory. Chirde et al. [27] have studied LRS Bianchi type I cosmological models with barotropic perfect fluid and cosmic string in the framework of f(T) theory of gravitation.

Hatkar et al. [28] have explored Bianchi type I cosmological model with matter and radiation fluids in scale covariant theory of gravitation. Both non-interacting and interacting cases of two fluids are examined. In case of interacting fluid energy transfer from matter to radiation is observed. Pawar and Mapari [29] have investigated plane symmetric cosmological model for interacting fluid in $f(R,T)$ gravity. They found both negative and positive deceleration parameters in different time intervals, indicating universe transition phase. Thakre et al. [30] have examined a deterministic solution of five-dimensional plane symmetric cosmological model incorporating quadratic equation of state in $f(R,T)$ theory of gravity

In this research paper, we aim to provide a comprehensive analysis of the plane symmetric cosmological model with a linear equation of state. We will explore its mathematical formulation, physical implications and observational consequences. By studying the dynamics and evolution of the universe within this framework we hope to shed light on the interplay between anisotropy matter content and dark energy. Anisotropy plays a pivotal role in the initial phases of the universe's development, making the examination of both anisotropic and homogeneous cosmological models a matter of great significance. In this particular research paper, we have delved into the study of higher dimensional cosmological model within the framework of General Relativity, incorporating a linear equation of state. Furthermore, our investigation extends to scrutinizing the geometrical and physical characteristics inherent to these models. The remainder of this paper is structured as follows: Section 1 presents a detailed review of the relevant literature and theoretical background discussing the cosmological principles the linear equation of state. Section 2 focuses on the mathematical formulation of the model elucidating the field equations. In Section 3, we analyse exact solutions of field equations Section 4 discussed the physical quantities for both cosmological models. Finally, Section 5 summarizes and concludes the results.

2. Metric and Field Equations

Higher dimensional plane symmetric cosmological model given by [4]

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2(dz^2 + d\omega^2) \tag{1}$$

where A, B, C are cosmic scale factors which are functions of cosmic time t and fifth coordinate is taken as space-like.

The energy momentum tensor is given by

$$T_{\nu}^{\chi} = diag\left[T_1^1, T_2^2, T_3^3, T_4^4, T_5^5\right] = dia[-p, -p, -p, -p, \rho] \tag{2}$$

where p is the pressure and ρ is the energy density of fluid

In this context, we utilize linear equation of state [30] as $p = \alpha\rho - \beta$ (3)

The Einstein field equations, in natural limits $(8\pi G = 1)$ and $c = 1$ are

$$G_{\kappa\nu} = R_{\kappa\nu} - \frac{1}{2} R g_{\kappa\nu} = -T_{\kappa\nu} \tag{4}$$

where $g_{\kappa\nu} u^{\kappa} u^{\nu} = 1$; $u^{\kappa} = (1,0,0,0,0)$ is five – velocity vector; R is the Ricci scalar; $T_{\kappa\nu}$ is energy-momentum tensor and$R_{\kappa\nu}$ is the Ricci tensor.

In co-moving co-ordinate system, the field equation (4) for metric (1) with the help of equation (2) and (3) can be written as

$$\frac{\ddot{B}}{B} + 2\frac{\ddot{C}}{C} + 2\frac{\dot{B}\dot{C}}{BC} + \left(\frac{\dot{C}}{C}\right)^2 = -(\alpha\rho - \beta) \tag{5}$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{C}}{C} + 2\frac{\dot{A}\dot{C}}{AC} + \left(\frac{\dot{C}}{C}\right)^2 = -(\alpha\rho - \beta) \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = -(\alpha\rho - \beta) \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{A}\dot{C}}{AC} + 2\frac{\dot{B}\dot{C}}{BC} + \left(\frac{\dot{C}}{C}\right)^2 = \rho \tag{8}$$

where dot denotes the differentiation with respect to time

3. Solution of the field equations

Since the above system of equations (5) – (8) are four non-linear equations consist of five unknowns. Hence to solve system completely an extra condition is necessary, so using special law of variation for the Hubble parameter given by Berman (1983,1992) given by

$$H = ma^{-n} = m(ABC^2)^{\frac{-n}{4}} \tag{9}$$

where $m > 0$ and $n \geq 0$

The corresponding Ricci Scale is given by

$$R = \frac{2\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{4\ddot{C}}{C} + 2\left(\frac{\dot{C}}{C}\right)^2 + 4\left(\frac{\dot{A}\dot{C}}{AC}\right) + 2\left(\frac{\dot{A}\dot{B}}{AB}\right) + 4\left(\frac{\dot{B}\dot{C}}{BC}\right) \tag{10}$$

where $m > 0$ and $n \geq 0$ are constant.

Subtracting eqn. (20) from eqn. (21), we get

$$\frac{B}{A} = k_1 exp\left[d_1 \int \frac{dt}{a^4}\right] \tag{11}$$

Subtracting eqn. (21) from eqn. (22), we get

$$\frac{C}{A} = k_2 exp\left[d_2 \int \frac{dt}{a^4}\right] \tag{12}$$

Subtracting eqn. (22) from (21), we get

$$\frac{A}{C} = k_3 exp\left[d_3 \int \frac{dt}{a^4}\right] \tag{13}$$

where k_1, k_2, k_3 & $d_{1,}d_2, d_3$ are constant of integration which satisfies the relation

$$k_1 k_2 k_3 = 1, d_1 + d_2 + d_3 = 1$$

$$A = a\, X_1\;\; exp\left[D_1 \int \frac{dt}{a^4}\right] \tag{14}$$

$$B = a\, X_2 a exp\left[D_2 \int \frac{dt}{a^4}\right] \tag{15}$$

$$C\; = a\, X_3\; exp\left[D_3 \int \frac{dt}{a^4}\right] \tag{16}$$

where $X_1 X_2 {X_3}^2 \;=\; 1$ & $D_1 + D_2 + 2D_3 \;=\; 0$

4. Some Important Physical Quantities

Spatial volume V of the universe is defined as $V \;=\; A^2BC$

$$a^4 \;=\; V\;\; then\, a \;=\; (A^2BC)^{\frac{1}{4}} \tag{17}$$

The generalized mean Hubble's parameter H is given as

$$H \;=\; \frac{1}{4}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{18}$$

$H_x \;=\; \frac{\dot{A}}{A}\, , H_y = \frac{\dot{B}}{B}, H_z \;= H_\omega = \frac{\dot{C}}{C}$ are directional Hubble parameters in the x, y, z axes direction respectively.

The expansion Scalar θ is given by,

$$\theta \;=\; 4H \;\; = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{19}$$

The mean anisotropic parameter and Shear Scalar is defined as

$$A_m \;= \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_{i-H}}{H}\right)^2 \text{ and} \tag{20}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} {H_i}^2 -\; 4H^2\right) = \frac{4}{2}AH^2 \tag{21}$$

The measure of cosmic accelerated expansion of universe given by decelerating parameter q is

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{22}$$

From Equation (9) and (18), we have

$$\dot{a} = ma^{-n+1} \tag{23}$$

On integration equation (23), we get

$$a = (mnt + c_1)^{\frac{1}{n}}, n \neq 0 \tag{24}$$

$$a = c_2 \exp(mt), n = 0 \tag{25}$$

where c_1, c_2 are constant of integration

4.1 Model for $n \neq 0$

In this section we study the five-dimensional Bianchi type – I universe for $n \neq 0$

Using the eq^n (24) in (14) - (16), the scale factor obtained as follows:

$$A = X_1 (mnt + c_1)^{\frac{1}{n}} \exp\left[\frac{D_1 (mnt+c_1)^{\frac{n-4}{n}}}{m(n-4)}\right], n \neq 4 \tag{26}$$

$$B = X_2 (mnt + c_1)^{\frac{1}{n}} \exp\left[\frac{D_2 (mnt+c_1)^{\frac{n-4}{n}}}{m(n-4)}\right], n \neq 4 \tag{27}$$

$$C = X_3 (mnt + c_1)^{\frac{1}{n}} \exp\left[\frac{D_3 (mnt+c_1)^{\frac{n-4}{n}}}{m(n-4)}\right], n \neq 4 \tag{28}$$

where D_1, D_2, D_3 are the constant of integration.

It must be stated that, the scale factors accept constant value at initial time, after which they evolve with time without a singularity and eventually diverge to infinity.

The directional Hubble parameter $H_x, H_y, H_{z,} H_\omega$ are given as

$$H_x = m(mnt + c_1)^{-1} + D_1 (mnt + c_1)^{\frac{-4}{n}} \tag{29}$$

$$H_y = m(mnt + c_1)^{-1} + D_2 (mnt + c_1)^{\frac{-4}{n}} \tag{30}$$

$$H_\omega = H_Z = m(mnt + c_1)^{-1} + D_3 (mnt + c_1)^{\frac{-4}{n}} \tag{31}$$

At initial epoch the directional Hubble parameters H_x, H_y, H_z are finite whereas when $t \to \infty$ they are constant.

Mean Hubble parameter H is given by

$$H = \frac{m}{mnt + c_1} \tag{32}$$

Anisotropy parameter of the expansion is

$$\Delta = \left[\sum_{i=1}^{4} \left(\frac{H_i - H}{H}\right)^2\right] = \frac{1}{4m^2}\left(D_1{}^2 + D_2{}^2 + 2D_3{}^2\right)(mnt + c_1)^{\frac{2n-8}{n}} \tag{33}$$

Dynamical scalar is given by

$$\theta = 4H = \frac{4m}{mnt+c_1} \tag{34}$$

The Dynamical scalar is constant throughout the evolution

Deceleration parameter q is given by,

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = n - 1 \tag{35}$$

The values of n decide the nature of universe, when $= 0$, the deceleration parameter gives $q = -1$ represent the quickest rate growth of universe. The decelerating parameter is always positive for $n > 1$ indicating standard decelerating model whereas negative sign of q i.e., $0 < n < 1$ indicates inflation. When $n = 1$ the universe expands at a constant rate corresponds to $q = 0$.

Shear scalar

$$\sigma^2 = \frac{({D_1}^2+{D_2}^2+2{D_3}^2)}{2(mnt+c_1)^{\frac{8}{n}}} \tag{36}$$

Thus, the energy density of the universe

$$\rho = 4m^2(mnt+c_1)^{-2} + m\,(mnt+c_1)^{\frac{-4}{n}-1}(D_1+3D_2+4D_3) + \left[(mnt+c_1)^{\frac{-4}{n}}\right]^2 \left(D_1D_2 + 2D_2D_3 + {D_3}^2\right) \tag{37}$$

$$p = 4\,\alpha\, m^2(mnt+c_1)^{-2} + m\,(mnt+c_1)^{\frac{-4}{n}-1}(D_1+3D_2+4D_3) + \left[(mnt+c_1)^{\frac{-4}{n}}\right]^2 \left(D_1D_2 + 2D_2D_3 + {D_3}^2\right) - \beta \tag{38}$$

4.2 Model for $n = 0$

In this section we study the five-dimensional Bianchi type – I universe for $n = 0$

Using the eq^n (23) in (14) - (16), the scale factor obtained as follows:

Then scale factors become

$$R_1 = X_1c_2\, exp(mt) exp\left(\frac{-D_1\, exp(-4mt)}{4m{c_2}^4}\right) \tag{39}$$

$$R_2 = X_2c_2\, exp(mt) exp\left(\frac{-D_2\, exp(-4mt)}{4m{c_2}^4}\right) \tag{40}$$

$$R_3 = X_3c_2\, exp(mt) exp\left(\frac{-D_3\, exp(-4mt)}{4m{c_2}^4}\right) \tag{41}$$

The directional Hubble parameter $H_x, H_y, H_{z,}H_\omega$ are given as

$$H_x = m + D_1{c_2}^{-4}\, e^{-4mt} \tag{42}$$

$$H_y = m + D_2 {c_2}^{-4}\, e^{-4mt} \tag{43}$$

$$H_\omega = H_z = m + D_3 {c_2}^{-4}\, e^{-4mt} \tag{44}$$

Mean Hubble parameter H is given by $H = m$ (45)

Anisotropy parameter of the expansion is

$$\Delta = \frac{1}{4m^2}\left(\frac{{D_1}^2+{D_2}^2+2{D_3}^2}{{c_2}^8}\right)\ exp(-8mt) \tag{46}$$

Dynamical scalar is given by

$$\theta = 4H = 4m \tag{47}$$

Deceleration parameter q is given by,

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{4}{m} - 1 \tag{48}$$

Shear Scalar is given by

$$\sigma^2 = \frac{1}{2}\left(\frac{{D_1}^2+{D_2}^2+2{D_3}^2}{{c_2}^8}\right)\ exp(-8mt) \tag{49}$$

Thus, the energy density of the universe

$$\rho = \frac{3}{8}\frac{b^2}{t^2} + \frac{1}{a^2 t^{2b}}\left(2\, D_1 D_2 + {D_1}^2 + 2\, D_1 D_3 + D_2 D_3\right) \tag{50}$$

Thus, the pressure of the universe

$$p = \frac{\alpha\left(3a^2b^2t^{2b} + 8\,t^2\left(2\,D_1D_2 + {D_1}^2 + 2\,D_1D_3 + D_2D_3\right)\right) - 8\beta a^2 t^{2(b+1)}}{8a^2t^{2(b+1)}} \tag{51}$$

5. Conclusion

In the present paper, we have obtained deterministic solution of higher dimensional Biachi type I space - time. These field equations have been solved using special law of variation of Hubble's Parameter that yields a constant value of deceleration parameter. These solutions result in two distinct outcomes. In first outcome, for $n \neq 0$ solution represent singular model with power law expansion and second outcome represent non-singular model with exponential expansion of universe for $n = 0$ .The physical behaviour and some important kinematical parameters of the cosmological model are discussed below.

(i) For $n \neq 0$,

Cosmological model has initial singularity at $t = -\frac{c_1}{mn}$ .The scale factors A, B, C and volume V vanishes at this point of singularity. It has been found that the average Hubble parameter H, shear scalar σ^2 , expansion scalar θ, and mean anisotropy parameter Δ are all infinite at this point of singularity. An infinite expansion scalar implies that the universe starts with zero volume experiencing infinite rate of expansion. As the cosmic time increases i.e., as $t \to \infty$ isotropy condition is satisfied. Thus, model tends towards isotropy for large values of t. Energy density vanishes at the point of singularity whereas pressure approaches to constant for large time. The negative sign for pressure indicates expansion of universe in late time. Therefore, model describe a late time universe that exhibits shear, lack of rotation and ongoing expansion.

(ii) For $n = 0$,
This model represents non-singular universe. The scale factors A, B, C and volume increases exponentially with cosmic time t. The average Hubble parameter H, expansion Scalar θ are constant. The shear scalar σ^2 and mean anisotropy parameter Δ are finite for finite values of t. In particular, energy density $\rho \to 0$ and pressure $p \to -\beta$ as $t \to \infty$ which shows that the universe started its evolution in an infinite past with strong pressure and energy density.

Therefore, both models describe a late time universe that exhibits shear, lack of rotation and ongoing expansion. The results obtained in these models are reassembles with the result obtained by [11,12]

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