

Article

LRS Bianchi Type-I Cosmological Model in Modified $f(R, T)$ V. G. Mete^{*1}, V. M. Ingle¹ & A.T. Valkunde²¹Department of Mathematics, R.D.I.K. & K.D. College, Badnera, India²Department of Science, Government Polytechnic, Khamgaon, India**Abstract**

In this paper, we have studied LRS Bianchi type-I cosmological model for $f(R, T)$ gravity. In order to present simplest mode of evolution, here we discussed in the background of a generic viable non-minimally coupled $f(R, T) = \alpha_1 R^m T^n + \alpha_2 T(1 + \alpha_3 T^p R^q)$ gravity model. Here we used the case $f(R, T) = R + \alpha_2 T$. The exact solution of the field equations in respect of LRS Bianchi type-I space time filled with perfect fluid in frame work of $f(R, T)$ gravity are derived. The physical and kinematical behaviors of the model are also studied.

Keywords: LRS Bianchi type-I, perfect fluid, $f(R, T)$, modified gravity.

1. Introduction

Currently, observational experiments show that the Universe is undergoing an accelerated expansion [1-10]. The “dark energy” (DE) which makes a negative pressure and thus gives rise to the accelerated expansion of the Universe. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment suggests that 73% content of the Universe is in the form of dark energy, 23% is in the form of non-baryonic dark matter and rest 4% is in the form of usual baryonic (normal) matter as well as radiation. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, Tachyon, quintessence, phantom and so on. There are two major approaches to tackle this problem of cosmic acceleration either by introducing a dark energy component in the Universe and study its dynamics or by interpreting it as a failure of general relativity and consider modifying Einstein’s theory of gravitation termed as modified gravity approach.

Modified theories of gravitation are used to explain the mysterious nature of dark energy. The modification of Einstein-Hilbert action may be the correct approach to explain the evolution of the universe. Noteworthy amongst them are $f(R)$ modified theory of gravity formulated by Nojiri and Odintsov [11]. Recently, Harko et al. [12] developed $f(R, T)$ modified theory of gravity,

*Correspondence: V. M. Ingle, Department of Mathematics, R.D.I.K. & K.D. College, Badnera- 444701, India.

E-mail: vishalinglevmi@gmail.com

where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor.

Now by considering the metric-dependent Lagrangian density L_m , the corresponding field equation for $f(R, T)$ gravity is obtained from the Hilbert–Einstein variation principle in the following manner. The action for $f(R, T)$ theory of gravity is

$$S = \int \sqrt{-g} \left(\frac{1}{16\pi G} f(R, T) + L_m \right) d^4x, \tag{1}$$

where L_m is the usual matter Lagrangian density of matter source, $f(R, T)$ is an arbitrary function of Ricci scalar R and the trace T of the energy–momentum tensor T_{ij} of the matter source, and g is the determinant of the metric tensor g_{ij} . The energy–momentum tensor T_{ij} from Lagrangian matter is defined in the form

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}, \tag{2}$$

and its trace is $T = g^{ij}T_{ij}$. Here, we have assumed that the matter Lagrangian L_m depends only on the metric tensor component g_{ij} rather than its derivatives. Hence, we obtain

$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}}. \tag{3}$$

The $f(R, T)$ gravity field equations are obtained by varying the action S with respect to metric tensor ($g_{\mu\nu}$).

$$f_R(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} + [g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j]f(R, T) = 8\pi T_{ij} - f_T(R, T)T_{ij} - f_T(R, T)\theta_{ij}, \tag{4}$$

where

$$f_R = \frac{\delta f(R, T)}{\delta R}, f_T = \frac{\delta f(R, T)}{\delta T}, \Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}}.$$

Here ∇ is the covariant derivative and T_{ij} is usual matter energy-momentum tensor derived from the Lagrangian L_m . It is mentioned here that these field equations depend on the physical nature

of the matter field. Many theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are possible; However, Harko et al.[12] gave three classes of these models

$$f(R, T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T) \end{cases} \quad (5)$$

The individual field equation for $f(R, T)$ gravity is given as

1. $f(R, T) = R + 2f(T)$.

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij} \quad (6)$$

2. $f(R, T) = f_1(R) + f_2(T)$.

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + [g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j]f_1(R) = 8\pi T_{ij} - f_2'(T)T_{ij} - f_2'(T)\theta_{ij} + \frac{1}{2}f_2(T)g_{ij}. \quad (7)$$

If $L_m = p$ then $\theta_{ij} = -2T_{ij} - pg_{ij}$. (8)

We can see that the result depends on the choice of $f(R, T)$ model. So we need to choose a viable $f(R, T)$ model in order to represent our results in a meaningful way. The $f(R, T)$ model which we have selected for discussion by Sharif and Zubair [13]

$$f(R, T) = \alpha_1 R^m T^n + \alpha_2 T(1 + \alpha_3 T^p R^q) \quad (9)$$

where α_i 's are positive real numbers, whereas m, n, p, q assumes some value greater than or equal to 1. We will analyze our results considering different cases of above mentioned model and we will precede our further discussion under following three cases

1. $f(R, T) = R + \alpha_2 T$ for $\alpha_1 = 1, m = 1, n = 0, \alpha_3 = 0$ (10)

2. $f(R, T) = \alpha_1 R + \alpha_2 T + \alpha_4 T^2$ for $m = 1, n = 0, \alpha_4 = \alpha_1 \alpha_3, p = 1, q = 0$ (11)

3. $f(R, T) = \alpha_1 R + \alpha_2 T(1 + \alpha_3 T R^2)$ (12)

Using equations (6), (7) and (8) along with equations (10), (11) and (12), we have

Model-I: $f(R, T) = R + \alpha_2 T$ for $\alpha_1 = 1, m = 1, n = 0, \alpha_3 = 0$

$$R_{ij} - \frac{1}{2}Rg_{ij} = [8\pi + \alpha_2]T_{ij} + \left[p\alpha_2 + \frac{1}{2}\alpha_2 T \right] g_{ij} \quad (13)$$

This theory of gravity has attracted a lot of research interest in recent times [14-19]. Sahoo and his collaborators have extensively investigated different aspects of this modified gravity theory and have reconstructed some $f(R, T)$ cosmological models for anisotropic universes [20-25]. Houndjo [26] have investigated the cosmological reconstruction in the $f(R, T)$ modified theory of gravitation. Reconstruction of cosmological models in the $f(R, T)$ theory of gravitation is also studied by Jamil et al. [27]. $f(R, T)$ gravity has been extensively studied in the literature by several eminent researchers [28-47].

Motivated by the above work, the present paper aims to study dynamics of LRS Bianchi type I cosmological model in $f(R, T)$ theory of gravitation.

2. Metric, Field Equations and Solutions

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations and relative ease of solutions made Bianchi space times useful in constructing models of spatially homogeneous and anisotropic cosmologies.

The LRS Bianchi type-I line element is

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2), \quad (14)$$

where A and B are the scale factors and function of cosmic time t only [48-49]. We have assumed the stress energy tensor of matter as

$$T_{ij} = (p + \rho)u_i u_j + pg_{ij}, \quad (15)$$

where $u_i = (0,0,0,1)$ is the four-velocity vector in co-moving coordinate system satisfying $u_i u_j = -1$.

2.1 Model: $f(R,T) = R + \alpha_2 T$ for $\alpha_1 = 1, m = 1, n = 0, \alpha_3 = 0$.

Using equations (10), (13) and (14), the field equations are obtained as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = [8\pi + \frac{7}{2}\alpha_2]p - \frac{1}{2}\alpha_2\rho, \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = [8\pi + \frac{7}{2}\alpha_2]p - \frac{1}{2}\alpha_2\rho, \tag{17}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \frac{5}{2}\alpha_2 p - [8\pi + \frac{3}{2}\alpha_2]\rho, \tag{18}$$

where dot represents derivatives with respect to time. The deceleration parameter is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}, \tag{19}$$

where a is the average scale factor. We have three equation's (16)-(18) involving four parameters as A, B, p, ρ . In order to solve these equations, we assume the time varying

deceleration parameter as $q = -1 + \frac{\beta}{1+a^\beta}$, where $\beta > 0$ is a constant. Bearing in mind the

relation between scale factor and redshift, we have $a(t) = \frac{1}{(1+z)}$, which yields $q = \frac{\beta}{(\frac{1}{z+1})^\beta + 1} - 1$

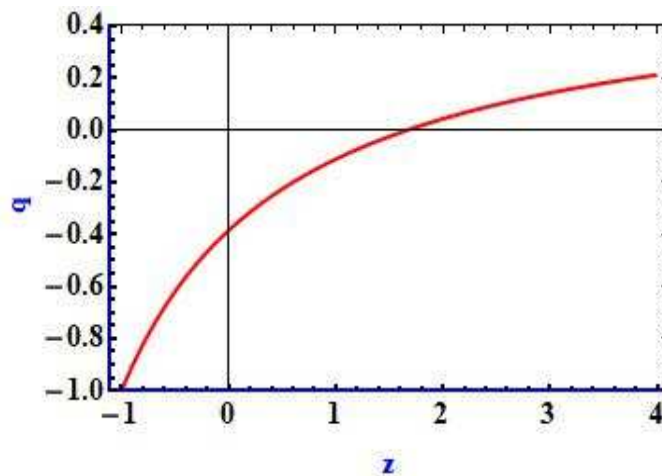


Fig. 1. Variation of q against z for $\beta = 1$.

We can see good agreement with recent observations where $t \rightarrow \infty$. Also, in our model, $q = -0.5$ as $z = 0$ while the current value of q is expected to be around -0.55 [50] as depicted in figure 1.

The scale factor and metric potentials are expressed as

$$a = [e^{\beta t} - 1]^{\frac{1}{\beta}} \tag{20}$$

$$A = [e^{\beta t} - 1]^{\frac{2}{\beta}}, B = [e^{\beta t} - 1]^{\frac{1}{2\beta}} \tag{21}$$

The energy density is obtained as

$$\rho = \frac{-[18\pi + 6\alpha_2] - [18\pi + \frac{5}{2}\beta\alpha_2 + 6\alpha_2][2(1+z)^{3\beta} + (1+z)^{2\beta}] + \frac{5}{2}\beta\alpha_2[(1+z)^\beta]}{4[16\pi^2 + 10\pi\alpha_2 + \alpha_2^2]} \tag{22}$$

The pressure is found to be

$$p = \frac{6\pi + [6\pi - 8\pi\beta - \frac{3}{2}\alpha_2\beta][2(1+z)^{3\beta} + (1+z)^{2\beta}] + [8\pi + \frac{3}{2}\alpha_2]\beta[(1+z)^\beta]}{4[16\pi^2 + 10\pi\alpha_2 + \alpha_2^2]} \tag{23}$$

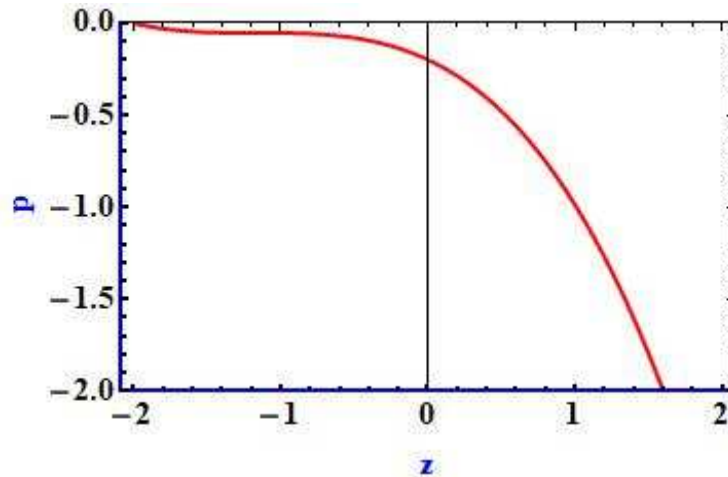


Fig. 2. Variation of pressure against z for $\alpha_2 = -15$ and $\beta = 1$

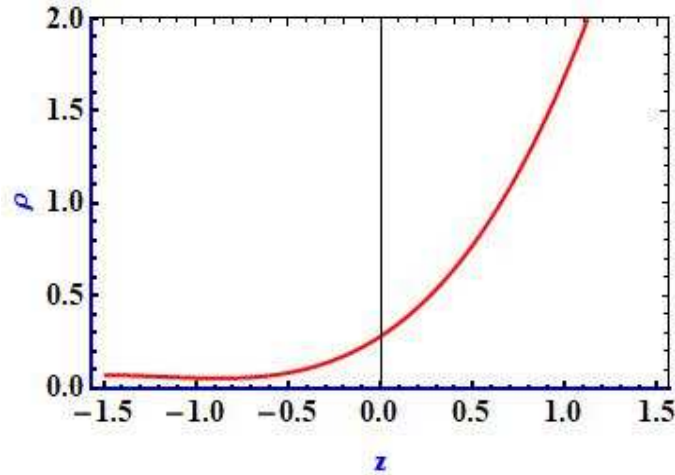


Fig. 3. Variation of energy density against z for $\alpha_2 = -15$ and $\beta = 1$.

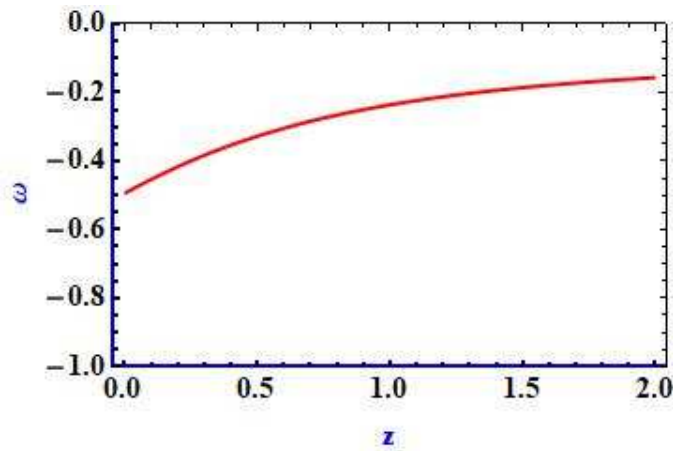


Fig. 4. Variation of EoS parameter against z for $\alpha_2 = -15$ and $\beta = 1$.

It is clear from Fig. 2 that the energy density of the Universe is an increasing function of redshift (z) and tends to a constant value in the future (i.e. $z \rightarrow -1$). For pressure (see Fig. 3), it is a decreasing function of redshift (z) and takes negative values throughout cosmic evolution. At the beginning of time it starts with very large negative values and later it approaches zero. The negative pressure is evidence of an accelerating phase of the Universe as shown by recent observations, and thus the validity of our model. The value of the EoS parameter for dark energy, or what is called in modified theories of gravity by cosmic acceleration, is negative. From this figure 4, we see that the behavior of the EoS parameter is similar to the quintessence model i.e. $-1 < \omega < -1/3$.

3. Physical properties

The spatial volume is given by

$$V = a^3 = AB^2 \tag{24}$$

The above equation indicates that the spatial volume is zero at initial time $t = 0$. It shows that the evolution of our universe starts with big bang scenario. It is further noted that from (24) the average scale factor becomes zero at the initial epoch. Hence, both models have a point-type singularity. The spatial volume increases with time. The Hubble's parameter H , expansion scalar and shear scalar are

$$H = \frac{1}{3}(H_x + H_y + H_z) = \frac{e^{\beta t}}{[e^{\beta t} - 1]} \tag{25}$$

$$\theta = 3H = 3e^{\beta t} \frac{1}{[e^{\beta t} - 1]} \tag{26}$$

$$\sigma^2 = \frac{1}{2} \left(H_x^2 + H_y^2 + H_z^2 - \frac{\theta^2}{3} \right) = \frac{3}{4} e^{2\beta t} \frac{1}{[e^{\beta t} - 1]^2} \tag{27}$$

The anisotropy parameter

$$\Delta = \frac{1}{3} \sum_{x=1}^3 \left(\frac{H_x - H}{H} \right)^2 = 6 \left(\frac{\sigma}{\theta} \right)^2 = \frac{1}{2} \tag{28}$$

We can observe that the Hubble factor, scalar expansion and shear scalar diverge at $t = 0$ and they become finite as $t \rightarrow \infty$ [51-54]. The anisotropic parameter becomes constant for our models. From the above mentioned equation, it can be observed that our models are expanding and accelerating the universe, which starts at a big bang singularity.

4. Jerk parameter

The jerk parameter is considered as one of the important quantities for describing the dynamics of the universe. Jerk parameter is dimensionless third derivative of scale factor a with respect to cosmic time t and is defined as

$$j = \frac{a^2}{\dot{a}^3} \frac{d^3 a}{dt^3} \tag{29}$$

$$j = q + 2q^2 - \frac{\dot{q}}{H} \tag{30}$$

Hence we have,

$$\Rightarrow j = 1 - \frac{3\beta}{(1+z)^{-\beta} + 1} + \frac{\beta^2}{(1+z)^{-\beta} + 1} + \frac{\beta^2}{[(1+z)^{-\beta} + 1]^2} \tag{31}$$

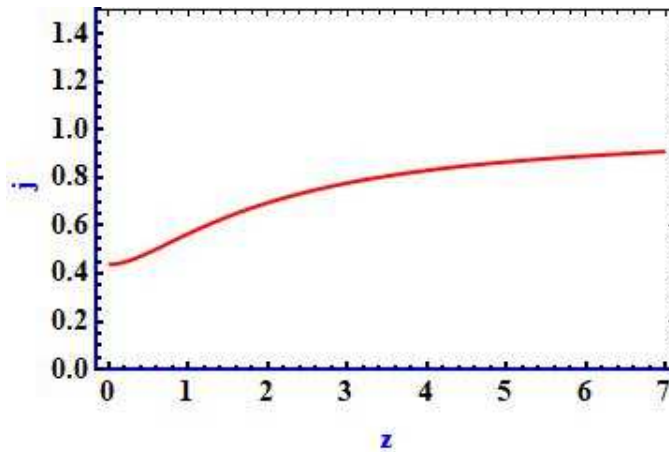


Fig. 5. Behavior of jerk parameter j versus z with $\beta = 1.5$

Jerk is a positive parameter. This denotes a quickening of growth. It's interesting to note that the jerk parameter never reaches unity at $z = 0$, which obviously contradicts the Λ CDM model.

5. Statefinder diagnostic

The statefinder parameters are important to discuss the cosmological aspects of models which are introduced in Refs. [55-56], originally. The state-finder pair $\{r, s\}$ is defined as

$$r = \frac{\ddot{a}}{aH^2}, \quad r = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}, \quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{32}$$

The values of the state-finder parameter for our model are

$$r = 1 - \frac{3\beta}{(1+z)^{-\beta} + 1} + \frac{\beta^2[(1+z)^{-\beta} + 2]}{[(1+z)^{-\beta} + 1]^2} \tag{33}$$

$$s = \frac{1}{\{6\beta - 9[(1+z)^{-\beta} + 1]\}[(1+z)^{-\beta} + 1]} \{2\beta^2((1+z)^{-\beta} + 2) - 6\beta[(1+z)^{-\beta} + 1]\}. \quad (34)$$

We get different dark energy models for different combinations of r and s : For Λ CDM $\rightarrow (r = 1, s = 0)$ For SCDM $\rightarrow (r = 1, s = 1)$ For HDE $\rightarrow (r = 1, s = 2/3)$ For CG $\rightarrow (r > 1, s < 0)$ For Quintessence $\rightarrow (r < 1, s > 0)$. Our model satisfies the Λ CDM scenario of the universe.

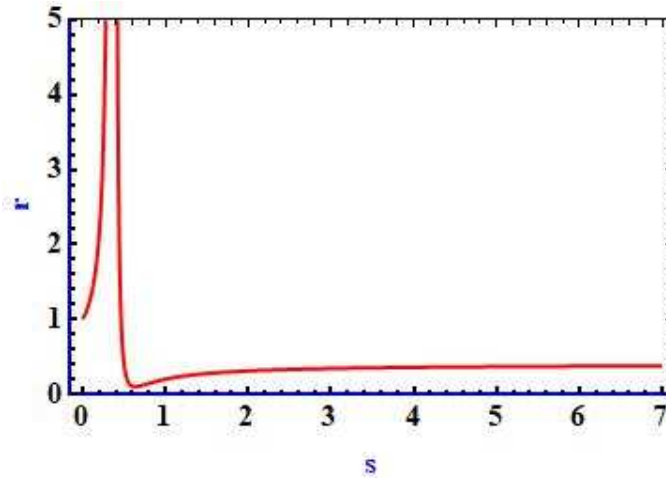


Fig. 6 : r vs s .

6. Conclusion

In this paper, we have considered LRS Bianchi type-I cosmological model in the presence of perfect fluid and generic viable non-minimally coupled

$$f(R, T) = \alpha_1 R^m T^n + \alpha_2 T(1 + \alpha_3 T^p R^q)$$

gravity model, where the gravitational Lagrangian is given by an arbitrary function of Ricci scalar (R) and of the trace of the stress-energy tensor (T). In this paper, the gravitational field equation has been established by taking $f(R, T) = f_1(R) + f_2(T)$. The cosmic acceleration in $f(R, T)$ gravity results not only from a geometrical effect, but also from a matter contribution. The deceleration parameter describes the rate of expansion and acceleration or deceleration of the universe. If $q > 0$, the universe is at a decelerated phase, else $q < 0$ corresponds to an accelerated phase.

The deceleration parameter depicts a transition from positive in the past to negative in the present showing the current accelerated expansion of the universe. One can observe that the energy density is positive throughout the universe whereas the pressure is always negative. The

negative pressure indicates the expanding accelerated phase of the universe. It is observed that the EoS parameter remains in the quintessence phase supporting the acceleration in the universe.

Received August 20, 2023; Accepted September 30, 2023

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