Article

Role of Inhomogeneous Equation of State with Bulk Viscous Fluid in Bianchi Type II Universe

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Abstract

In the present investigation, we find the solution of the Einstein field equations with dark energy in the form of inhomogeneous equation of state with bulk viscous fluid in Bianchi type II space-time. To solve the survival field equations, here we consider expansion scalar is proportional to the shear scalar. For the detail study of the model, we have discussed the three different cases depending upon the nature of cosmological term (Λ), the bulk viscous fluid (ξ) and the equation of state parameter (ω). Also we analyzed the physical and geometrical characteristics of the cosmological model for each case. It is observed that our resultant models are in accelerating phase which favours the current observations of SNIa and CMBR.

Keywords: Bianchi type II metric, inhomogeneous equation of state, bulk viscous fluid, cosmological term.

1. Introduction

The present scenario is that the universe is subject to acceleration which can be explained in the form of dark energy i.e. an ideal fluid with usual matter having an uncommon equation of state. To describe the cosmic acceleration expansion, there are various kinds of dark energy models are introduced. To explain the fact, cosmologist tried to impose a negative pressure which converts as dark energy. Nowadays many researchers has been attracted towards the dark energy concepts i.e. Extended gravity [1-6], Modifying Equation of State [MEoS].

Various authors extensively discussed the perfect fluid models with different equation of state [7,8] in terms of Chaplygin gas, Generalized Chaplygin gas, Inhomogeneous equation of state and barotropic fluid dark energy etc. In relativity and cosmology, the equation of state is the relationship among combined matter, temperature, pressure, energy and energy density for any region of space plays an important role. Babicher et.al. [9] have proposed a linear equation of state of more general form $p=\alpha\rho-\rho_0$ to overcome the drawback of hydrodynamic instability. Such results described the hydrodynamically stable dark energy behaviours.

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Now many cosmologists has attracted towards viscosity mechanism which shows the more realistic models. Bulk viscosity driven inflation is primarily due to the negative bulk viscous pressure giving rise to a total negative effective pressure which may overcome the pressure because of usual matter distribution in the universe and provide an impetus to drive it apart. Bulk viscous models have prime roles in getting inflationary phases of the universe [10-14]. Misner [15, 16] observed that the strong dissipation due to neutrino viscosity may reduce the anisotropy of the black body radiation. In cosmology, this mechanism can describe the unusual high entropy per baryon in present events suggested by Weinberg [17, 18]. Some authors have presented the bulk viscosity associated with the grant unified transition may lead to an inflationary scenario [19-22]. The effect of bulk viscosity on cosmological evolution has been investigated by a number of authors in the framework of general relativity [23, 24].

To construct the cosmological models, Bianchi type II space time plays a fundamental role to explain the early stages of evolution of the universe. Asseo and Sol [25] studied the importance of the bianchi type II universe. Lorenz [26] investigated the exact solutions for LRS Bianchi type II space time. In this way this universe has been discusses by a number of authors. Recently Sonia Sharma [27] found that Bianchi II cosmological model with bulk viscous fluid in general relativity. Boutros [28] presented Bianchi type II space time with perfect fluid by a generating technique. Tyagi & Sharma [29] obtained the solution of Bianchi Type-II Bulk Viscous String Cosmological Models in General Relativity. LRS Bianchi type II cosmological model with a Decaying lambda term studied by Tiwari et.al. [30]. Recently some authors [31-37] studied the results of modified equation of state in various theories of gravitation.

With this inspirable effective research work, in the present paper we investigate the effect of inhomogeneous equation of state with bulk viscous fluid in the frame work of Bianchi type II universe. The paper is organized as follows: The brief introduction is given in section 1. The Bianchi type II space time and corresponding Einstein field equations are introduced in section 2. In section 3 the solution of the models is obtained by considering the physical condition $X_1^2 = X_2$ for three different cases. The physical and kinematical parameters of the models are obtained and discussed in section 4. The conclusion is summarized in the last section.

2. The Metric and Field Equations

Here we consider Bianchi type II model of the form

$$ds^2 = -dt^2 + X_1^2(dx^2 + dz^2) + X_2^2(dy - xdz)^2 \tag{1}$$

where X_1 and X_2 are the scale factors depends on cosmic time *t*.

The Einstein field equations are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{2}$$

where R is the Ricci scalar, R_{ij} is Ricci tensor, T_{ij} is the energy momentum tensor respectively.

The energy momentum tensor is given by

$$T_{ij} = (\rho + \bar{p})u_i u^j + \bar{p} g_{ij} \tag{3}$$

where $\bar{p} = p - \xi\theta$ (4)

Here ρ is the density, p is the pressure, ξ is the coefficient of bulk viscosity and θ is expansion scalar.

By following [9] we assume the inhomogeneous equation of state in the form

$$p = \omega\rho + \Lambda(t) \tag{5}$$

where Λ is a function of time t and $\omega > 1$.

The field equation (2) for the line element (1) with the help of (3) and (4) can be written as

$$\frac{\ddot{X}_1}{X_1} + \frac{\ddot{X}_2}{X_2} + \frac{\dot{X}_1\dot{X}_2}{X_1X_2} + \frac{X_2^2}{4X_1^4} = -(p - \xi\theta) \tag{6}$$

$$\frac{2\ddot{X}_1}{X_1} + \frac{\dot{X}_1^2}{X_1^2} - \frac{3X_2^2}{4X_1^4} = -(p - \xi\theta) \tag{7}$$

$$\frac{\dot{X}_1^2}{X_1^2} + \frac{2\dot{X}_1\dot{X}_2}{X_1X_2} - \frac{X_2^2}{4X_1^4} = \rho \tag{8}$$

An over dot denotes differentiation with respect to time t.

The number of unknowns in the field equations is more than equations; hence we need to obtain an exact solution of the field equations. Therefore, we assumed that the expansion scalar (θ) in the model is proportional to shear scalar (σ^2) which leads to the following condition

$$X_1^2 = X_2 \tag{9}$$

Here we assume the linear form of bulk viscosity and cosmological term as follows

$$\xi = \xi_0 + \xi_1 H \tag{10}$$

and

$$\Lambda = \Lambda_0 + \Lambda_1 H \tag{11}$$

In this paper, by using the inhomogeneous equation of state with linear form of cosmological term and the bulk viscous fluid here we discuss the following three different cases depending on the nature of ξ, Λ and ω as follows:

Case I: $\Lambda = \Lambda_0 + \Lambda_1 H$, $\xi = \xi_0 + \xi_1 H$, $\omega \neq 1$

Case II: $\Lambda = \Lambda_0$, $\xi = \xi_1 H$, $\omega \neq 1$

Case III: $\Lambda = \Lambda_1 H$, $\xi = \xi_0$, $\omega = 1$

where Λ_0 and ξ_0 zero are constants.

Case I: In this case, we consider the linear form of bulk viscosity and cosmological term and $\omega \neq 1$

Solving eq (6-8) with the help of (5), (9) (10) and (11), we get

$$\frac{\ddot{X}_2}{X_2} - \left\{\frac{16\xi_1}{27} - \frac{(5\omega+1)}{6}\right\}\frac{\dot{X}_2^2}{X_2^2} + \left\{\frac{4(\Lambda_1 - 3\xi_0)}{9}\right\}\frac{\dot{X}_2}{X_2} + \left\{\frac{4\Lambda_0 + (1-\omega)}{6}\right\} = 0 \tag{12}$$

By solving eq (12), one can obtain

$$X_2 = K_2 \exp\left\{\frac{Z_2\, t}{2(Z_1 - 1)}\right\}\left[\sec\left(Z_4 (K_1 + \frac{t}{2})\right)\right]^{\frac{1}{Z_1 - 1}} \tag{13}$$

where K_1 and K_2 are integrating constants.

$$Z_1 = \frac{16\xi_1}{27} - \frac{(5\omega+1)}{6}, \quad Z_2 = \frac{4(\Lambda_1 - 3\xi_0)}{9}, \quad Z_3 = \frac{4\Lambda_0 + (1-\omega)}{6} \; Z_4 = \sqrt{4Z_3 - 4Z_1 Z_3 - Z_2^2}$$

Case II: In this case, we consider the bulk viscosity ξ is a function of Hubble parameter *H*, the cosmological term Λ is constant and $\omega \neq 1$ now simplifying eq (12), we get

$$X_2 = \left[l_1 \sin(l_2 t + K_3\right]^{\frac{1}{(M_1+1)}} \tag{14}$$

where K_3 is integrating constant

$$l_1 = \sqrt{\frac{K_3(M_1+1)}{M_2}}, \quad l_2 = N_0(M_1+1)\sqrt{M_2}, \; M_1 = \left(\frac{27(5\omega+1) - 96\xi_1}{162}\right) M_2 = \left(\frac{4\Lambda_0 + (1-\omega)}{6}\right)$$

$$N_0 = \left(\sqrt{\frac{K_3(1+M_1)}{M_2}}\right)$$

Case III: In this case, we consider the cosmological term Λ is a function of Hubble parameter *H*, the bulk viscosity ξ is constant and $\omega = 1$ by solving eq (12) gives the solution

$$X_2 = \left\{D_1 - \frac{3}{\Lambda_1} e^{\left(D_0 - \frac{2\Lambda_1}{3} t\right)}\right\}^{\frac{1}{2}} \tag{15}$$

where D_0 and D_1 are integrating constants.

3. The Physical and Geometrical Aspects

Case I) For line element (1), the expressions for the Hubble parameter (H) , the expansion scalar (θ), the density (ρ) , the cosmological term (Λ), the pressure (p) and the bulk viscosity (ξ) , the mean anisotropy parameter (Δ) and deceleration parameter (q) are given by

$$H = \frac{2\dot{X}_2}{3X_2} = \frac{1}{3(Z_1-1)}\left[Z_2 + 2Z_4 \tan\left(Z_4(K_1+\frac{t}{2})\right)\right] \tag{16}$$

$$\theta = \frac{2}{3(Z_1-1)}\left[Z_2 + 2Z_4 \tan\left(Z_4(K_1+\frac{t}{2})\right)\right] \tag{17}$$

$$\rho = \frac{5}{8(Z_1-1)^2}\left[Z_2 + 8Z_4 \tan\left(Z_4(K_1+\frac{t}{2})\right)\right]^2 - \frac{1}{4} \tag{18}$$

$$\Lambda = \Lambda_0 + \frac{\Lambda_1}{3(Z_1-1)}\left[Z_2 + 2Z_4 \tan(K_1+\frac{t}{2})\right] \tag{19}$$

$$p = \Lambda_0 + \frac{5\omega}{8(Z_1-1)^2}\left[Z_2 + 8Z_4 \tan\left(Z_4(K_1+\frac{t}{2})\right)\right]^2 + \frac{2\Lambda_1}{3(Z_1-1)}\left[Z_2 + Z_4 \tan(K_1+\frac{t}{2})\right] \tag{20}$$

$$\xi = \xi_0 + \frac{\xi_1}{3(Z_1-1)}\left[Z_2 + 2Z_4 \tan(K_1+\frac{t}{2})\right] \tag{21}$$

$$\Delta = constant \tag{22}$$

with the suitable values of the constant , the decelerating parameter is as follows

$$q < 0 \tag{23}$$

Case II: The expressions for Hubble parameter is given by

$$H = \frac{2N_0}{3}\cot(l_2 t + K_3) \tag{24}$$

The expressions for expansion scalar is given by

$$\theta = \frac{4N_0}{3}\cot(l_2 t + K_3), \tag{25}$$

The expressions for density is given by

$$\rho = \frac{5N_0^2}{4}\cot^2(l_2 t + K_3) - \frac{1}{4} \tag{26}$$

The expressions for pressure is given by

$$p = \frac{5\omega N_0^2}{4}\cot^2(l_2 t + K_3) + \Lambda_0 - \frac{1}{4}, \tag{27}$$

The expressions for bulk viscosity is given by

$$\xi = \xi_0 + \frac{2\xi_1 N_0}{3}\cot(l_2 t + K_3) \tag{28}$$

The mean anisotropy parameter and deceleration parameter (with suitable values of constant) are given by

$$\Delta = \text{constant} \quad \text{and} \quad q < 0 \tag{29}$$

Case III: The expressions for the density (ρ) and pressure (p) are as follows

$$\rho = \frac{5}{4}e^{2\left(D_0 - \frac{2\Lambda_1}{3}t\right)} - \frac{1}{4} \tag{30}$$

$$p = \frac{5}{4}e^{2\left(D_0 - \frac{2\Lambda_1}{3}t\right)} + \frac{2\Lambda_1}{3}e^{\left(D_0 - \frac{2\Lambda_1}{3}t\right)} - \frac{1}{4} \tag{31}$$

The expressions for Hubble parameter (H), the expansion scalar (θ) and the cosmological term are given by

$$H = \frac{2}{3}e^{\left(D_0 - \frac{2\Lambda_1}{3}t\right)}, \quad \theta = \frac{4}{3}e^{\left(D_0 - \frac{2\Lambda_1}{3}t\right)}, \quad \Lambda = \frac{2\Lambda_1}{3}e^{\left(D_0 - \frac{2\Lambda_1}{3}t\right)} \tag{32}$$

In this case the mean anisotropy parameter is constant and also deceleration parameter is as

$$q < 0$$

Interpretation of our result:

<u>In Case 1</u>:

From eq (16) it is clear that as *t* increases, Hubble parameter (*H*) also increases i.e. for $t \to \infty$ we get $H \to \infty$ and with the present time, H becomes constant. From the eq (18), we have observed that the energy condition is satisfied i.e. $\rho > 0$. Also, it is seen that, $\frac{\sigma}{\theta} \neq 0$ hence the model is anisotropic for the large values of t i.e. the universe is in a state of accelerated expansion. Our model exhibits anisotropic behaviour throughout the evolution of the universe. From eq (18), it is seen that the energy condition is satisfied.

Now it is well-known that the sign of q indicates whether the model inflates or not. It means the "q" in the model is decelerating for $q > 0$ and accelerating for $q < 0$. In this case, we obtain the negative value of q by choosing suitable values of the constants. Hence our resultant model is in accelerating phase.

Case 2:

As t increases θ decreases. It is observed that the energy density ρ is a decreasing function of time t. As time t increases the physical parameters density (ρ) decreases, pressure (p), expansion scalar (θ) are decreases. It is observed that the energy condition is satisfied by eq (26). For the case anisotropy parameter is constant and with the suitable values, it is observed that the deceleration parameter is negative.

Case 3:

For t tends to zero, all the physical parameters become constants. As t increases exponentially then H, θ, ρ and p decreases. Also, it is found that $\frac{\sigma}{\theta} \neq 0$ which implies that the model does not approach to isotropy. When t is zero then density reaches to constant and as t becomes infinity then density is zero which states that the model starts with a big bang singularity at
$t = 0$ which is a point type singularity. In this case also energy condition is satisfied. We have observed that the model is accelerating and anisotropic parameter is constant.

4. Conclusion

- In the present work, Bianchi type II model is considered with inhomogeneous EoS in the presence of linear form bulk viscous fluid in general relativity. Bianchi type II metric deserves attention and present better pictures of the universe.

- To obtain the more general model, here we assume the physical condition that the shear scalar is proportional to the scalar expansion.

- For the detail study of the model, here we have discussed the three different cases depending upon the nature of on the cosmological term (Λ), the coefficient of bulk viscosity (ξ) and equation of state parameter (ω).

- It is found that in all the cases the energy condition is satisfied i.e. $\rho > 0$ which states that the models are physically realistic as the present day observational data. Therefore, the present models are consistent with observational data. Also, our derived models are stable under perturbations. Our model exhibits anisotropic behaviour throughout the evolution of the universe.

- In big bang scenario all the parameters like shear scalar, expansion scalar and Hubble parameter are finite. Our model exhibits anisotropic behaviour throughout the evolution of the universe.

- It is observed that the current observations of SNIa and CMBR favour accelerating models ($q<0$). Therefore, we can conclude that all the derived models are in accelerating phase.

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References

[1] Nojiri, S. and Odintsov Phys. Rep. 505 (2011) 59.
[2] Li, M. et.al Theor. Phys. 56 (2011) 525.
[3] Meng, X.H. and Wang, P. Class. Quant. Grav. 20 (2003) 4949.
[4] Meng, X.H. and Wang, P. Class. Quant. Grav. 21 (2004) 951.
[5] Meng, X.H. and Wang, P. Class Quantum Grav. 22 (2005) 23.
[6] Nojiri, S. and Odintsov, S. D. Phys. Lett. B 576 (2003) 5
[7] Nojiri, S., Odintsov, S.D. and Tsujikawa, S. Phys. Rev. D. 72 (2005) 023003.
[8] Nojiri, S., Odintsov, S. D. and Tsujikawa S. Phys. Rev. D 71 (2005) 063004.
[9] Babichev, E., Dokuchaev, V. and Eroshenko, Y. Class. Quant. Grav. 22 (2005) 143.
[10] Barrow, J.D. Phy. Lett.B., 180 (1986) 335.
[11] Zimdahl, W. Phys. Rev. D. 53 (1996) 5483.
[12] Pavon, D., Bafaluy, J., Jou, D. Class. Quant.. Gravit. 8 (1991) 347.
[13] Maartens, R. Class. Quantum Gravity 12 (1995) 1455.
[14] Tripathy, S.K., Behera, D., Rautray, T.R. Astrophys. Space Sci, 325 (2010) 93.
[15] Misner, C. W. *Nature* 214 (1967) 40.
[16] Misner, C.W. Astrophy. Jou. 151 (1968) 431.
[17] Weinberg S. Astrophysics J. 168 (1971) 175.
[18] Weinberg, S. Wiley, New York 1972
[19] Waga, L., Fakan, R.C. and Chanda, R. Phys. Rev. D 33 (1986) 1839.
[20] Pacher, T., Stein-Schabas, J.A. and Turner, M.S. Phys. Rev. D 36 (1987) 1603.
[21] Guth, A. Phys. Rev. D 23 (1981) 347.
[22] Murphy, G.L. Phys. Rev D 8 (1973) 4231.
[23] Haung, W. J Math. Phys. 31 (1990) 1456.
[24] Pradhan, A., Yadav, V. K., Chakraborty, I. Int. J. Mod. Phys. D 10 (2001) 339.
[25] Asseo, E. and Sol, H. Phys. Rep. 148 (1987) 307.
[26] Lorenz, D. Phys. Lett .A 79 (1980) 19.
[27] Sharma, S. Int. J. Theo. App. Sci. 4 (2012) 36.
[28] Boutros, H. J. J. Math. Phy. 27 (1986) 1592**.**
[29] Tyagi, A. and Sharma K. Int. J. Thero. Phy. 49 (2010) 1712.
[30] Tiwari, R.K., Tiwari, D. and Shukla, P. Chin. Phys. Lett. 29 (2012) 010403.
[31] Arora S. , Meng Xin-he, Pacif S. K. J. and Sahoo P.K.. Class. Quant. Grav. 37(2020) 20.
[32] Gusu D.M. and Vijaya Santhi. Advances in High Energy Physics, (2021) 1.
[33] Tiwari, R.K., Aroonkumar Beesham, Soma Mishra and Vipin Dubey Universe 7 (2021) 226.
[34] Vinutha T. and Kavya K.S. Int.J of Geometric Methods in Modern Physics 19(2022) 2250046.
[35] Chaoping Mo., Zhang G., Yan D. and Yang S. *ACS Omega* 7 (2022) 9322.
[36] Kumar R. , Tandon R. and Gupta J. J. Phy. *Conference Series* (2022) 012091.
[37] Sen S. , Khadekar G. S. and Samdurkar S. *J. Dyn. Sys. Geo. Theo.*. 19 (2021) 95.