Article

Dimensional Reduction as Source of Cosmological Anomalies

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Abstract

Dimensional Reduction (DR) conjectures that the number of spacetime dimensions monotonically drops with the boost in energy scale. According to DR, the expectation is that ordinary space progressively unfolds from being one or two-dimensional near the Big-Bang singularity. This work argues that DR a) can explain some of the observed anomalies challenging the standard model of cosmology and, b) provides a platform for unifying the particle and gravitational interpretations of Dark Matter.

Keywords: Dimensional reduction, Lambda-CDM model, continuous spacetime dimensions, fractional dynamics, Dark Matter, low-dimensional gravity.

1. Newtonian gravitation in arbitrary dimensions

The gravitational field created by a mass distribution M in $D=n+1$ spacetime dimensions takes the form [1],

$$F_D(r) = -G^{(D)} \frac{M}{r^{D-2}} \tag{1}$$

Here, Newton's constant depends on D according to [2]

$$G^{(D)} = G^{(4)} \Lambda^{4-D} \tag{2}$$

in which Λ stands for a reference energy scale. One obtains, by (1) and (2),

$$F_D(r) = -G^{(4)} \Lambda^{4-D} \frac{M}{r^{D-2}} \tag{3}$$

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It is seen from (3) that the Newtonian field carries identical canonical dimension in $D=4$ and $D\neq 4$ dimensions, namely,

$$\left[F_4\right]=\left[F_D\right]=mass \tag{4}$$

We proceed by setting

$$\Lambda = R^{-1} \tag{5}$$

which turns (3) into

$$F_D(r) = -G^{(4)} R^{D-4} \frac{M}{r^{D-2}} \tag{6}$$

By (3) and (6), the field in $D=3$ spacetime dimensions $(n=2)$ reads

$$F_3 = -G^{(4)} \frac{M}{R\,r} \tag{7}$$

with the planar Newtonian potential

$$V_3(r) = G^{(4)} \frac{M}{R} \ln(r/R) \tag{8}$$

Working in arbitrary $D=3-\varepsilon$ non-integer spacetime dimensions ($n=2-\varepsilon$ space dimensions), requires switching from standard calculus to fractional calculus [3]. Among the many choices available for the definition of the potential we may choose

$$F_{3-\varepsilon}(r) = -\frac{\partial^{1-\varepsilon} V(r)}{\partial r^{1-\varepsilon}} \tag{9}$$

where, by (6) and $D=3-\varepsilon$,

$$F_{3-\varepsilon}(r) = -G^{(4)} R^{-(1+\varepsilon)} \frac{M}{r^{1-\varepsilon}} \tag{10}$$

which is a generalization of (7) for $\varepsilon \neq 0$.

We close this section by noting that, 1) according to [4-5], careful consideration must be given to the analysis of low-dimensional gravity, 2) applying fractional dynamics to gravitational problems is far from being a trivial exercise, demanding caution and independent validation or rebuttal.

2. Newtonian gravitation in 2 + 1 and 3 + 1 dimensions

If fractional dimensions are arbitrarily close to integer dimensions (ε << 1), a reasonable assumption is that Newtonian physics in fractional dimensions stays *conservative*. By (7), using polar coordinates and choosing $R=1$, the motion of a test particle in the field created by the mass distribution in 2+1 dimensions is described by the equations,

$$mr^2\dot{\theta}=L=const. \tag{11}$$

$$m\ddot{r}=\frac{L^2}{mr^3}-G^{(4)}\frac{mM}{r} \tag{12}$$

Here, m is the mass of the particle and L stands for its angular momentum. Conservation of the total energy E requires,

$$\frac{1}{2}m\dot{r}^2=E-\frac{1}{2m}\frac{L^2}{r^2}-G^{(4)}M\ln r \tag{13}$$

Equation (13) shows that the trajectory of the particle remains *bounded* to the center of gravitational field located at $r=r_0$, regardless of how large the total energy E is.

Fig.1 displays the phase space portrait in the plane defined by the radial component of momentum and radial coordinate (p_r, r). The equilibrium point defined by $p_r=0$ acts as a *center*. It is located at a radial distance $r_0=Lm^{-1}(MG^{(4)})^{-1/2}$, it corresponds to an energy $E_0=(1/2m)Lr_0^{-2}+MG^{(4)}\ln r_0$ and a period $\tau=2\pi L(mMG^{(4)})^{-1}$ [1].

As a result, all phase space orbits in a planar Newtonian Universe *stay confined* around the center of the field regardless of E.

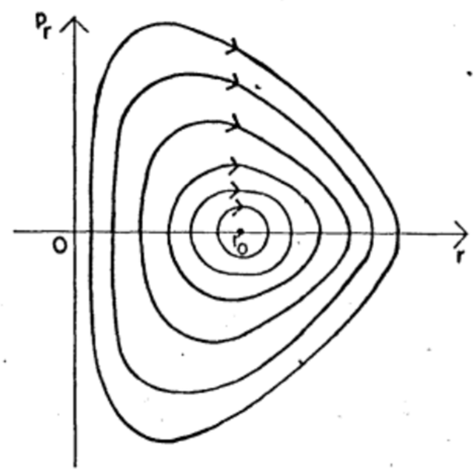

Fig. 1. Phase space portrait of a planar Newtonian Universe [1]

It is instructive to compare these results with the standard Kepler problem in 3+1 dimensions, subject to the initial value conditions [6],

$$\ddot{\vec{r}} + G^{(4)} \frac{M}{r^3}\vec{r} = 0 \tag{14}$$

$$\vec{r}(t_0) = \overrightarrow{r_0} \tag{15}$$

$$\overrightarrow{\dot{r}_0} = \overrightarrow{v_0} \tag{16}$$

Energy and momentum conservation follows from the two prime integrals of motion,

$$\vec{r} \times \vec{v} = \overrightarrow{r_0} \times \overrightarrow{v_0} = K \tag{17}$$

$$h = \frac{1}{2} m\vec{v}^2 - G^{(4)} \frac{mM}{r} = \frac{1}{2} m\vec{v}_0^{\,2} - G^{(4)} \frac{mM}{r_0} \tag{18}$$

Fig. 2 displays the phase portrait of the rectilinear Kepler motion defined by $m = 1,\ K = 0$. The coordinates of phase plane $(\dot{x}, x)$ are given by,

$$\vec{r} = x\frac{\overrightarrow{r_0}}{r_0}, \quad x:[t_0,+\infty) \to \mathbf{R} \tag{19}$$

The orbit $h=0$ separates the region of vanishing velocity ($h<0$) from the region of non-vanishing velocity ($h>0$). It is seen that particles escape at infinity $(x\to\infty)$ if $h\geq 0$, and are trapped inside the potential range if $h<0$.

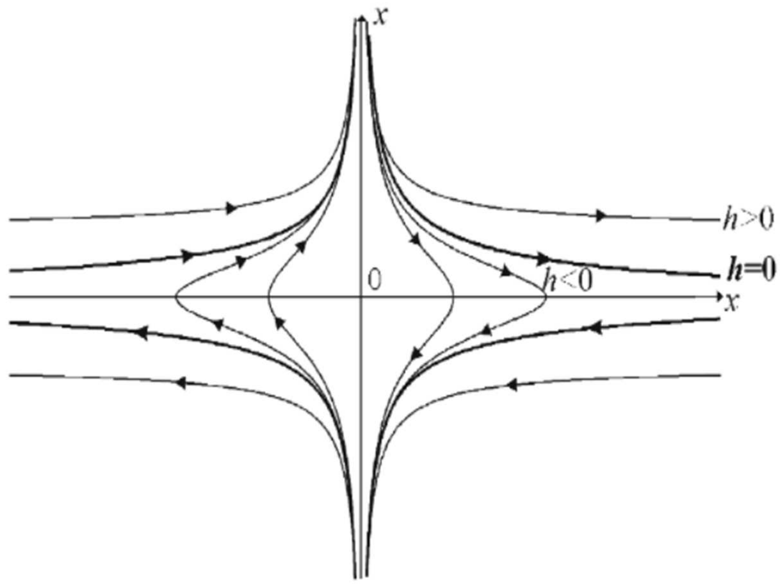

Fig. 2. Phase plane portrait of the classical Kepler problem [6]

A glance at Fig. 1 and Fig. 2 hints that, at least in principle, a spacetime having continuous dimensions ($3<D<4$) enables the emergence of ultra-massive galactic structures whose behavior falls *in between* what is described in Fig.1 and Fig.2 [7].

To some extent, these findings are consistent with the idea that cosmological anomalies are remnants of the *non-trivial topology* of spacetime in the primordial Universe [8-11]. For instance, [11] starts from the hypothesis that the three-dimensional space has an inherent global $R^2\times S^1$ topology. The corresponding correction to the classical Newtonian potential is illustrated in Fig. 3 and can account for the effects induced by Dark Matter on the galaxy rotation curves.

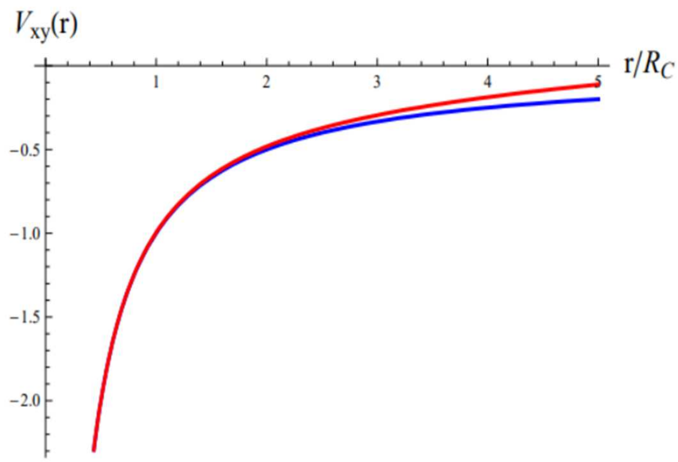

Fig. 3. Newtonian potential in 3 +1 (blue) and 2 +1 dimensions (red) [11]

Since the non-trivial spacetime topology near the Big-Bang singularity is in line with a spacetime endowed with *minimal fractality* above the Fermi scale, the approach discussed here opens the door for unifying the particle and gravitational interpretations of Dark Matter. This possibility was elaborated upon in [12-15].

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