Article

Role of Mathematics in Recent Trends of Cosmological Model With Variable Deceleration Parameter

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Abstract

Bianchi type-VI_o cosmological model have been investigated with deceleration parameter $q=-1+H$, which yields scale factor $a=e^{\sqrt{2t+k}}$. The model has non-singular origin and expands exponentially with cosmic time t. We observe that the equation of state parameter (ω) is time dependent. The jerk parameter (j) provides a perfect diagnosis of how dark energy model is closed to ΛCDM dynamics which is in agreement with the recent observations. We also discussed statefinder parameter {r, s} which predicts that the universe in 1the model originates from Einstein era {r, s}={∞, -∞} to ΛCDM model {r, s}={1, 0}. The physical and geometrical properties of cosmological model are also discussed.

Keywords: Bianchi type-VIo, EOS parameter, variable deceleration parameter, jerk parameter, statefinder parameter.

1. Introduction

Many observations of supernova experiments concluded that the expansion of the universe is accelerating, Riess *et. al.* 1998, Perlmutter et. Al. 1997, 1998, Pradhan *et. al.* 2010, Schmidt et. al. 1998, Granavich et. al. 1998. These observations indicates the presence of unknown fluid-dark energy that opposes the self attraction of the matter. This dark energy is the unknown form of energy could not have been detected directly and it does not cluster like ordinary matter. To study cosmological models , one of the important observational quantity is deceleration parameter q. This deceleration parameter q and Hubble's constant H_o play a significant role in describing the nature of evolution of the universe. Berman 1983 proposed a special law of variation for Hubble constant by assuming deceleration parameter $q=-\frac{a\ddot{a}}{\dot{a}^2}$ to find the cosmological solution.

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The equation of state parameter (ω=p/ρ, where p is the pressure and ρ is the energy density) is not necessarily constant, which is considered as an important quantity in describing the dynamics of the universe. The current cosmological data from large scale-structure (Komastu *et al* 2009) does not support the possibility of ω<<-1. If ω=-1 (quitessence dark energy) (Steinhardt et al 1999) less than - 1 (phantom dark energy) (Caldwell 2002). Tiwari and Sonia 2011 investigated the Bianchi Type-I string cosmological model with bulk viscosity and time dependent Λ term. Mukesh and Sonia 2016 investigated the new study of cosmological model with cosmological constant. Kujat et. al. 2002, Bartelmann et. al. 2005 considered the equation of state parameter (ω) as a constant with phase wise value -1, 0, -1/3 and 1 for vacuum fluid, dust fluid, radiation and stiff dominated universe respectively. But, ω is a function of red shift or time (Bartelmann et al 2005, Jimenez 2003, Dass *et al* 2005). In this paper, we study Bianchi Type-VI_o spacetime with EOS parameters (γ and δ). To obtain the explicit solution, we assume deceleration parameter q=-1+H (H=Hubble parameter)

2. The Metric and Field Equations

Consider Bianchi Type-VI_o space time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{-2x} dz^2 \tag{1}$$

Where A, B and C are functions of cosmic time t. This implies that the model is totally anisotropic and spatially homogeneous.

To determine the diagonal form of energy-momentum tensor, the generalization form of EOS parameter of perfect fluid is

$$T_{ij} = diag[T_{oo}, T_{11}, T_{22}, T_{33}] \tag{2}$$

Allowing for anisotropy in the presence of fluid, and thus in its EOS parameter, gives new possibilities for the evolution of the energy source. To see this, we parametrize the energy momentum tensor given in equation (2) as follows:

$$\begin{aligned} T_{ij} &= diag[\rho, -p_x, -p_y, -p_z] \\ &= diag[1, -\omega_x, -\omega_y, -\omega_z]\rho \end{aligned}$$

$$T_{ij} = diag[1, -\omega, -(\omega+\rho), -(\omega+\gamma)]\rho \tag{3}$$

where p_x, p_y, p_z are directional pressure, ρ is energy density, ω_x, ω_y and ω_z are the directional EOS parameters along x, y, z axis respectively, ω is the deviation free EOS parameter, δ and γ are skewness parameters. ω, δ, γ are not necessarily constant, they might be function of cosmic time t.

The Einstein's field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{4}$$

where 8πg=1, R_{ij}, R, g_{ij} and T_{ij} are Ricci tensor, Ricci scalar, metric tensor and energy momentum tensor respectively.

The field equation (4), with equation (3) for the line element (1) give rise to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\omega\rho \tag{5}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = -(\omega+\delta)\rho \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\omega+\gamma)\rho \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \rho \tag{8}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{9}$$

We define average scale factor 'a' for Bianchi type VI_o space time as

$$a = (ABC)^{1/3} \tag{10}$$

Also shear scalar (σ), expansion scalar (θ), Hubble's parameter (H), deceleration parameter (q) and Anisotropic parameter $(\bar{A})$ as

$$\sigma^2 = \frac{1}{2}(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2) \tag{11}$$

$$\theta = 3H \tag{12}$$

$$H = \frac{\dot{a}}{a} \tag{13}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{14}$$

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2\sigma^2}{3H^2} \tag{15}$$

where $\Delta H_i = H_i - H$ (i=x, y, z) represents the directional Hubble's parameters.

3. Solution of the Field Equations

Integrating equation (9), we get

$$C = kB \tag{16}$$

where k is constant.

From equations (6), (7) and (16) we obtain that the skewness parameters along y and z axis are equal i.e. γ=δ

Equations (5)-(9) are reduced to

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = -\omega\rho \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=-(\omega+\gamma)\rho \tag{18}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}=\rho \tag{19}$$

Equations (17), (18), (19) are three equations with five unknowns A, B, ω, ϒ, δ. To get the determinate solution's, we require two more conditions. First, we assume that the expansion scalar (θ) is proportional to shear scalar (σ) and using equation (16) we get

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\alpha\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{20}$$

where α is constant, which yields to

$$\frac{\dot{A}}{A}=k_1\frac{\dot{B}}{B} \tag{21}$$

where $k_1=\dfrac{2\alpha\sqrt{3}+1}{1-\alpha\sqrt{3}}$

From equation (21), we get

$$A=k_2B^{k_1} \tag{22}$$

where k_2 is constant. Without any loss of generality we assume $k_1=k_2=1$(Tiwari and Sonia 2011)

$$\Rightarrow A=B \tag{23}$$

The observations of the velocity red-shift relation for extragalactic source says that Hubble expansion of the universe is isotropic at the time within ≈30 (Kristian and Sachs 1966, Kantowski and Sachs 1966, Tiwari, Agrarwal and Shukla 2016). To put more precisely, red shift studies the limit $\frac{\sigma}{H}\leq 0.3$ on the ratio of the shear tensor (σ) and Hubble parameter (H) in the nearest of our galaxy today.

Second, we assume that the deceleration parameter q is a linear function of Hubble parameter (H) i.e.,

$$q = \alpha' + \beta H \tag{24}$$

where α',β are constants and $H = \frac{\dot{a}}{a}$

Recent observations have suggested that the present universe is accelerating and value of deceleration parameter (q) lies between 0 and -1. For the sake of simplicity taking α=-1, β=1, the equation (24) becomes

$$a = e^{\sqrt{2t+k_3}} \tag{25}$$

where k_3 is constant. This shows that at t=0 the scale factor 'a' tends to constant, hence the model is non-singular origin.

Using equation (16), (23) and (25), we have

$$A = k_6 e^{\sqrt{2t+k_3}}$$
$$B = k_4 e^{\sqrt{2t+k_3}}$$
$$C = k_5 e^{\sqrt{2t+k_3}}$$

Where $k_4=k_5=k_6=-1$

$$\therefore A = B = C = -e^{\sqrt{2t+k_3}} \tag{26}$$

$$\therefore ds^2 = -dt^2 + (dx^2 + e^{2x}dy^2 + e^{-2x}dz^2)e^{(2t+k_3)} \tag{27}$$

4. Physical and Geometrical Properties of the Model

The Spatial volume (V), Shear scalar (σ), Expansion scalar (θ), Hubble parameter (H), anisotropic parameter $(\overline{A})$ and deceleration parameter (q) are given by

$$V = e^{3\sqrt{2t+k_3}} \tag{28}$$

$$\sigma = 0 \tag{29}$$

$$\theta = \frac{3}{\sqrt{2t + k_3}} \tag{30}$$

$$H = \frac{1}{\sqrt{2t + k_3}} \tag{31}$$

$$\overline{A} = 0 \tag{32}$$

$$q = -1 + \frac{1}{\sqrt{2t + k_3}} \tag{33}$$

Here we observe that at t=0, the scale factor is constant, hence the model has no singularity at origin. At $t = -\frac{k_3}{2} = t_1$ the expansion scalar θ and Hubble's parameter H are infinite, which shows that the universe starts evolving with zero volume at $t=t_1$ with an infinite rate of expansion. The anisotropic parameter $\left(\overline{A}\right)$ and shear scalar (σ) does not exists.

From equation (19) and (26)

$$\rho = \frac{3}{2t + k_3} - k_4 e^{-2\sqrt{2t+k_3}} \tag{34}$$

$$\omega = -\frac{\left[\dfrac{-2 + 3\sqrt{2t + k_3}}{(2t + k_3)^{3/2}}\right] + k_4 e^{-2\sqrt{2t+k_3}}}{\dfrac{3}{2t + k_3} - k_4 e^{-2\sqrt{2t+k_3}}} \tag{35}$$

$$\gamma = \frac{-2k_4 (2t + k_3) e^{-2\sqrt{2t+k_3}}}{3 - k_4 (2t + k_3) e^{-2\sqrt{2t+k_3}}} \tag{36}$$

From equation (35), we observe that equation of state parameter (ω) is time dependent.

5. The Jerk Parameter (J) and Statefinder Parameter {r,s}

The jerk parameter (j),third derivative of scale factor w.r.t. cosmic time t (Chiba and Nakamura 1998, Sahni 2002, Visser 2004, Blandford *et. al.* 2004) provides a perfect diagnosis of how dark energy model is closed to ΛCDM dynamics. The jerk parameter (j) is defined as

$$j(t)=\frac{\dddot{a}}{aH^3}=\left(\frac{a^2H^2}{2H^2}\right)'' \tag{37}$$

Over dot and primes denotes derivative w.r.t. cosmic time t and scale factor respectively.

Equation (37) can be written as

$$j(t)=q+2q^2-\frac{\dot{q}}{H} \tag{38}$$

From equation (31), (33) and (38), we have

$$j(t)=1-\frac{3}{\sqrt{2t+k_3}}+\frac{3}{2t+k_3} \tag{39}$$

Sahni et. al. 2003 and Alam et. al. 2003 introduced the Statefinder parameter {r, s} defined as

$$r=\frac{\dddot{a}}{aH^2}=1+3\frac{\dot{H}}{H}+\frac{\ddot{H}}{H^3} \tag{40}$$

$$s=\frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{41}$$

Now equation's (31), (33), (40) and (41) result into

$$r=1, s=0$$

Therefore statefinder parameter $\{r,s\}=\{1,0\}$ which is the value for ΛCDM model.

We observe that when $t \to t_1$, $\{r, s\}=\{\infty, -\infty\}$ and when $t \to \infty$, $\{r, s\}=\{1, 0\}$ which depicts that the universe in the model starts from Einstein static era goes to ΛCDM model. This is in agreement with the recent observations (Priyokumar and Jiten 2021), which makes our model observationally acceptable.

6. Conclusions

In summary, we have studied a new class of anisotropic Bianchi type VI_o cosmological model in the presence of variable equation of state parameter (ω) by using the condition for time dependent deceleration parameter (q). As the time t increases, the rate of expansion θ decreases. Thus rate of expansion slows down with the passage of time. σ=0 implies that shear scalar does not exists. Here $\lim_{t\to\infty}\frac{\sigma}{\theta}=0$, therefore the model describes a non-shearing, continuously expanding universe. It is also observe that anisotropic parameter $(\bar{A})$ does not exists. We also depicted that the universe in the model starts from Einstein static era and goes to ΛCDM model. Finally we notice that the jerk parameter (j) and statefinder parameter (r) also in the agreement with recent observations.

Received March 01, 2023; Accepted June 18, 2023

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