Exploration

The TGD View of The Possible Role of Spin Glass Phase & p-adic Thermodynamics in Topological Quantum Computation

Matti Pitkänen [1]

Abstract

Topological quantum computation (TQC) or more generally, a TQC-like process (to be referred as TQC), is one possible application of TGD. The latest article summarizes the recent number theoretic view about TQC in TGD inspired biology. There are several new physics elements involved. Mention only the notion of many-sheeted space-time involving the notions of electric and magnetic body; the new view about quantum theory relying on the $M^8 - H$ duality relating number theoretic and geometric views about physics and predicting the hierarchy of effective Planck constants assignable to a hierarchy of extensions of rationals; cognitive representations as unique discretization of space-time surface realizing generalized quantum computationalism; and zero energy ontology (ZEO) suggesting a new vision about quantum error correction. Quantum gravitation plays a key role in the proposal.

The engineering aspects of TQC were not discussed. The question that inspired this article was whether classical computation which relies strongly on non-equilibrium thermodynamics, could provide guidelines to end up with a more detailed view.

This led to a proposal in which p-adic thermodynamics assigned with the TGD based description of spin glasses would play a key role. TQC would involve quantum annealing in the spin glass energy landscape for the fermion states associated with flux tube structures. Anyons would be replaced with representations of the Galois group.

Physical states are however Galois singlets and many fermion states would involve entanglement between irreps of (relative) Galois group associated with spin *resp.* momentum degrees of freedom and give rise to a superposition of Galois singlets. The state function reduction ending TQC would project a tensor product of a given irrep from this superposition.

The entanglement between representations should be engineered in such a manner that the desired outcome of TQC would have the largest entanglement probability. p-Adic thermodynamics could give the entanglement probabilities. A connection with the travelling salesman problem emerges besides the connection with the factorization of the Galois group to prime factors appearing as relative Galois groups, which are simple (prime).

1 Introduction

Topological quantum computation (TQC) or more generally, a TQC-like process, is one possible application of TGD (for simplicity, I will talk in the sequel of TQC rather than TQC). The interested reader can consult the earlier TGD inspired work in TQC [9, 8, 12]. The recent rather concrete model for TQC in living matter utilizing quantum gravitation in the TGD sense see [31].

1.1 Basic ideas of TQC according to TGD

There are several new ideas involved [31].

1. Braidings are represented by monopole flux tubes, which are structures distinguishing between TGD and Maxwellian electrodynamics and are one of the basic implications of the many-sheeted space-time concept. Time-like braidings as TQC programs can be engineered as a flow for the nodes of

[1] Correspondence: Matti Pitkänen http://tgdtheory.fi/. Address: Rinnekatu 2-4 8A, 03620, Karkkila, Finland. Email: matpitka6@gmail.com.

the flux tube network and they induce spatial braidings as memory representations of the time-like braidings - kind of topological Akashic records.

The engineering of the flow involves what might be called quantum hydrodynamics [24]. DNA based TQC would utilize the flow of 2-D liquid crystal defined by a lipid layer of cell membrane to generate braiding [8].

2. The hierarchy of effective Planck constants, assumed to label dark matter as phases of ordinary matter, predicts quantum coherence in arbitrarily long scales and Negentropy Maximization Principle (NMP) [4] favors the generation of negentropic entanglement (NE). NE makes sense only in adelic physics [14, 15] and allows to understand second law as a side effect of the NMP.

   The point is that one can assign to the same entanglement both the ordinary real entanglement entropy and the sum of p-adic variants of entanglement entropies. The sum of two can be negative and the interpretation in this case is as negentropy. NMP tends to make the negentropy positive. The decrease of the negative p-adic entropy would force the increase of real entropy. This view [13] conforms with the vision of Jeremy England about living systems [1].

3. $M^8 - H$ duality as a generalization of momentum-position duality of wave mechanics is a central notion on the number theoretic view of TGD providing a view dual to the geometric view. The complexified M^8 has an interpretation as complexified octonions.

   The roots r_n of rational polynomials P of real variable algebraically continued to complecified octonionic polynomials define 3-D mass shells (hyperbolic spaces H^3) $m^2 = r_n$ of $M^4_c \subset M^8_c$. The mass shells define holographic data for the continuation of these 3-surfaces to 4-D surface X^4 of $M^4_c \subset M^8_c$.

   Dynamics is dictated by the associativity of the normal space of X^4. Associativity in turn makes it possible to map X^4 to a 4-D space-time surface of $M^4 \times CP_2$ by $M^8 - H$ duality.

4. Cognitive representation is a second central concept: one might call it intersection of reality and p-adicities regarded as correlates for ideas and imagination. Originally, the non-determinism of p-adic differential equations motivated this notion.

   Cognitive representation defines a unique discretization of the space-time surface involving a hierarchy of extensions of rationals associated with rational polynomials defining space-time regions via $M^8 - H$ duality. For mass shells cognitive explosion takes place and the representations contain almost all algebraic in rationals. Physical motivations force restriction to algebraic integers and the condition that active points of the cognitive representation contain quark.

   This leads to a generalization of computationalism replacing rationals with the hierarchy of extensions of rationals.

5. Galois confinement is a further key notion. It states that for the physical states the total 4-momentum as a sum of momenta of quarks with components, which are algebraic integers, are real integers.

   One can interpret quark momenta as discretized virtual momenta [26, 27]. The mass squared values as roots of P can be tachyonic in the sense that the real part of mass squared is negative.

   Conformal invariance requires that the scaling generator L_0 annihilates the physical states so that the mass squared for the physical states vanishes. Therefore *all* physical states are analogous to massless particles [34]!

   This leads to a resolution of longstanding interpretational problems of p-adic thermodynamics [3] due to the necessity of tachyonic states and the fact that, in an apparent conflict with conformal invariance, one allows states with non-vanishing value of L_0. The second surprise is that the result actually conforms with the proportionality of blackhole entropy with mass squared and this relation generalizes so that it applies to all systems. Also an analogy with entropic gravity emerges.

For subsystems entangled with the environment, which at elementary particle level in a good approximation reduces to wormhole contacts with an Euclidean induced metric having fundamental fermions at the throats, a superposition of pairs of states for which mass squared values of the members sum up to zero emerges. One must use thermodynamics to describe the non-tachyonic part of the system. The thermodynamic state involves both massless ground state and massive excitations for the tensor factor with non-negative mass squared values.

6. Zero energy ontology (ZEO) predicts that the arrow of time changes in "big" state function reductions (BSRs). This leads to a model for homeostasis [35] as an ability to say near quantum criticality made possible by dissipation with an opposite arrow of time. This would also make healing possible as a time reversed dissipation.

   Concerning TQC, the good news is that the dissipation with a reversed arrow of time could make possible an automatic quantum error correction as a healing.

1.2 Could p-adic thermodynamics be relevant for TQC?

What distinguishes quantum computation (QC) from the classical computation (CC), is that QC is not a deterministic process. For instance, the algorithm for finding the period of function gives outcomes which are its multiples. To obtain the desired result with a high enough probability, one can repeat the QC or use an ensemble of QCs. Could one imagine a more elegant approach than just repeating the TQC sufficiently many times or having an ensemble of TQCs?

Classical computation (CC) is in a good approximation a deterministic process and it is interesting to analyze what makes it possible to physically represent Boolean function as a sequence of steps as a deterministic time evolution. Here non-equilibrium thermodynamics involving a realization of bits as flow equilibria and dissipation are in an essential role so that quantum statistical determinism in microscopic scales is an essential element as also electric and magnetic fields in long scale scales serving as masters of the microscopic dynamics.

This inspires the question whether thermodynamics, not necessarily ordinary thermodynamics but p-adic thermodynamics, could provide a tool of TQC. p-Adic thermodynamics is equivalent with ordinary thermodynamics but has additional constraints forced by the number theoretic existence conditions for Boltzmann weights. For instance, temperature quantization is implied and the convergence of the partition function in powers of p is extremely fast for large p-adic primes such as $p = M_{127} = 2^{127} - 1$.

1. p-Adic thermodynamics is naturally associated with spin glass phases in the TGD based view of spin glasses [23]. One can wonder whether an annealing process could make it possible to end up at the bottom of the deepest valley, possibly assignable with the desired outcome of TQC. p-Adic thermodynamics could assign an analog of free energy minimum to the desired outcome.

   Annealing is a stepwise process involving repeated p-adic heating and cooling. Heating would generate entanglement between anyonic and fermionic degrees of freedom and cooling would allow an SFR to a new deeper local minimum of the p-adic analog of Hamiltonian.

2. The cognitive measurement cascade [21, 28] is an essential part of TQC and decomposes the representation of Galois groups to a product of representations of the relative Galois groups, which are direct sums over irreps.

   The next step involves measurement of the invariants of irreps of relative Galois groups, which project one irrep for each relative Galois group. The measurement however requires entanglement of the irreps with some states. What could these states be? Does nature provide this entanglement automatically or must one engineer it?

3. There is a grave objection against this proposal. The irreducible representations (irreps) of Galois group and relative Galois groups for an extension defined by a functional composite of polynomials

generalizes anyons as group representations. However, Galois confinement states that physical states are Galois singlets!

The resolution of the objection is that bosionic Galois representations represented in terms of momentum space wave functions entangle with the representations realized in terms of fermionic spin degrees of fundamental fermions so that the entangled state is a superposition of Galois singlets as pairs of irreps. The measurement for the analogs of the Casimir operators for the irreps for the either tensor factor would project out a single irrep. Nature could do this automatically or it could be carried out as a quantum measurement.

This process could involve a reduction of h_{eff} (and decomposition of functional composites to product of polynomials) leading to breaking of relative Galois symmetries and reduction of entanglement between momentum and spin degrees of freedom. For $h_{eff} \rightarrow h$, the entanglement could be completely reduced. It might be that this reduction is necessary in order to represent the outcome of the computation at the level of ordinary matter.

4. A connection with the travelling salesman problem emerges. If the pairing of momentum and spin degrees of freedom involves N different relative Galois irreps, there are N! different pairings between momentum and spin representations, which correspond to the number of different solution candidates in the travelling salesman problem. If the problem can be transformed to a travelling salesman problem, it can be solved by using TQC in the TGD sense.

   This suggests a canonical form for the p-adic analog of Hamiltonian for the p-adic thermodynamics selecting the local minimum in the annealing process. The p-adic analog of Hamiltonian could be engineered as in the thermodynamical solution of the travelling salesman problem.

2 Quantum computation *viz.* classical computation

I talked with my friend Tuomas Sorakivi about the relation between quantum computation (QC) and classical computation (CC). The discussion raised a series of questions. For professionals the following ponderings might seem to be trivial but it could give new insights about the TGD counterpart of the topological QC (TQC). In particular, it could give important insights to the basic conceptual and technical problems of QC.

2.1 Meanings of CC and QC

What does one mean with CC?

1. Mathematically CC can be represented as a Boolean map mapping m input bits to n output bits. This map can be decomposed to primitive Boolean maps realized as gates. CC can be represented as a program in which inputs at given time $t = n$ arrive as multi-bits to gates producing output multi-bits.

2. It is highly non-trivial that one can represent Boolean functions to electrical circuits. In the physical realization of this picture, the values of bits correspond to voltage values. Gates are constructed as electric circuits. For instance, gates involving logical conditions have control bits affecting the output. Transistors allow the realization of control bit as bit as base current. The output bits are communicated to the next gates as propagating voltage values.

What does one mean with QC?

1. There are several realizations of QC. The realization of QC as a unitary time evolution is constructed in terms of gates and is nearest to CC using electronic circuits.

2. QC is realized as a TQC in the TGD framework and involves new elements and differs from what might be called standard TQC. Besides topology, also number theory is involved in an essential way and predicts hierarchies labelled corresponding to extensions of rationals.

   In TQC according to TGD [31], the counterpart of the metabolic energy feed is necessary to preserve quantum phases in long scales even at room temperature. Zero energy ontology (ZEO) brings in time reversal as a new element, which allows us to understand homeostasis in living matter.

2.2 How do QC and CC differ?

QC is usually regarded as more advanced than CC since it is conceptually a much more complex and abstract notion.

1. The difficulty to understand QC might be partially due to the missing understanding of state function reduction (SFR). TGD suggests a view of SFR, which is free of paradoxes and means a dramatic conceptual clarification.

2. The practical realization of QC meets huge technical challenges. In the TGD framework, these challenges could reflect the lack of the understanding of what dark matter is. Dark matter as $h_{eff} = nh_0$ phases of ordinary matter could help to overcome these problems.

3. CC seems to have emerged in evolution later than TQC-like information processing, which in TGD is proposed to characterize living matter. Does this mean that CC is more advanced? Probably not. The emergence of CC could be seen as reflecting our high level of evolution: we are the first species that has invented CC and does not imply that CC is more advanced than QC.

   Could also living matter combine some elements of CC with TQC to achieve determinism? Of course, living matter could achieve this by using us to build classical computers!

What distinguishes QC (and TQC according to TGD) and CC?

1. CC can be modelled as a deterministic process. This is what makes it so simple as compared to QC. CC also uses as a tool ordinary matter for which quantum effects occur in very small scales.

   QC in the prototype situation relies on a deterministic unitary time evolution followed by a non-deterministic SFR. Neither energy feed nor dissipation play an active role. The basic goal is to prevent dissipation by isolating the system from the environment.

2. CC involves statistical determinism of quantum theory implying dissipation used to achieve thermodynamic determinism essential for the computation. In CC, dissipation in the presence of energy feed leads to thermodynamic flow equilibria. For instance, external energy feed allows to preserve the values of bits represented as voltages. For this external energy source (battery) is needed.

   The thermodynamic determinism of CC prevails below given length and time scale resolution. The external energy field implies the presence of macroscopic degrees of freedom, which act as masters. For instance, voltages in circuits made possible by the energy feed use ohmic currents in microscopic degrees of freedom as slaves. In the similar way, in TQC according to TGD, the quantum coherence of MB makes it possible to induce ordinary coherence at the lower levels of the hierarchy.

3. In (T)QC according to standard quantum theory energy feed is not an essential element. In TQC according to TGD, metabolic energy feed is necessary to preserve the distribution of h_{eff} since a state with given value of h_{eff} tends to decay to a state with smaller value of h_{eff} having a lower energy. In TGD, metabolic energy feed also makes possible high Tc superconductivity and quantum phases in long length scales.

   The presence of hierarchy of length scales in TGD based TQC assignable to extensions of rationals and labelled by h_{eff} p-adic length scales is also essential.

In ordinary (T)QC only a single scale is involved and one can wonder whether the presence of several scales could make QC deterministic without losing the nice features of QC. One can of course consider an ensemble of TQCs giving an ensemble of possible answers but could one imagine other options?

2.3 What does one mean with a CC program?

A classical computer program can be regarded as a Boolean map, decomposing to a sequence of steps consisting of primitive Boolean maps represented as logic gates, is a sequence of deterministic steps and nodes at which decisions of what is done next. All gates can be seen as Boolean maps assigning to input bits unique output bits. The circuit decomposes to gates representing logical operations as Boolean maps. Gates can be described in terms of control bits which fix the outputs for given inputs.

1. One could argue that the choice of what is done at the next step is non-deterministic. One could argue that the initial values of the program in principle fix the output uniquely in an ideal deterministic classical physics.

   However, it seems very strange that one could engineer a program realizing any desired computation as a deterministic time evolution. Could SFRs be involved in a hidden way and make this engineering possible.

2. Thermodynamics is indeed involved in an essential manner. The physical realization of gates in electric circuits having the bits as flow equilibrium states associated with the non-equilibrium thermodynamics (NET) with external energy feed. This gives the desired thermodynamic determinism. which reduces to statistical determinism of quantum physics.

   NET involves SFRs in short length and time scales for a larger number of electrons, which gives rise to a dissipation leading to a unique flow equilibrium of NET. The description for the dissipation would be in terms of ohmic currents.

Communication of the outputs of gates to the next gates represent an essential element of CC. The bits representing the outputs can be represented as voltages and can be communicated as voltage pulses to the next gates. How does this compare with the TGD view of a TQC.

1. In the TGD framework this communication has a quantum counterpart inspired by the notion of the dark N-particle as an analog and generalization of Bose Einstein condensate [33, 20, 22]. In biology, dark genetic codons would form the basic building brick representing 6 bits and would be represented as dark proton triplets and dark photon triplets having interpretation as 64 chords defining a bioharmony. Codon is received by a cyclotron 3-resonance: this generalizes to 3N-resonance for genes and DNA sequences.

2. The codons serve as addresses determining which receivers get the message: the analogy with computer language LISP is obvious. The information is coded to the modulation of the frequency scale so that the modulation is coded to a sequence of cyclotron N-resonances at the receiving end. If frequencies are modulated independently, a subset of the receiver is selected by the resonance conditions.

   Generalized Josephson junctions produce Josephson radiation with frequency depending on the modulated voltage of the junction [6] [30, 32, 33]. As a special case, one can consider a voltage which is piecewise constant and represents two bit values. Neurotransmitters in synaptic junction actually correspond to miniature potentials, which correspond to a voltage change of few meV as compared to membrane potential of order .05 eV. Miniature potentials relate to the notion of preneural system suggested by the finding that multi-cellulars without nervous system behave as if they had a nervous system: the TGD inspired model is discussed in [29].

One might think that the propagating voltage pulses of rectangular shape as idealized representations of propagating bits could be something totally trivial from the point of view of recent day condensed matter physics. However, it is far from clear what their TGD counterparts could be and I have already years ago considered the possibility that the electric pulses studied by Tesla, which had rather dramatic effects on the environment, might involve new physics. The time reversals of the electric pulses emerged in these considerations much before the precise formulation of ZEO [2, 7].

1. The voltage pulses propagate with a sub-luminal velocity and can be said to have longitudinal electric polarization. Electron-hole pair is the notion of condensed matter physics which comes to mind. These pairs are formed valence electrons transform to conduction electrons. Holes behave as effective positive charges. Could voltage pulse represent a distribution of electron-hole pairs as bound states. Holes and electrons would reside at 2-D surfaces defining analogs of capacitor plates. Since charge carriers are near the surface of the conductor, the one would have annuli instead of plates.

2. What could be the TGD based model for the structure formed by the electron-hole pairs? An interpretation as a moving electric flux quantum is suggestive [25]. Electric flux quantum would be analogous to a moving electric capacitor idealized as a pair of 2-D plates. This suggests a TGD based model for a capacitor plate as a membrane like object, which is a space-time surface with a 2+1-D M^4 projection and therefore defines a planar membrane in E^3. In general relativity these objects are not possible. The CP_2 projection could be a geodesic circle of CP_2.

   The 2-D sheets with the shape of annulus would represent the ends of a hollow cylinder. The hollow cylinder would be minimal surface apart from the singular circles at its ends where the minimal surface property would fail and entire field equations involving sum of terms for the Kähler action and volume term would hold true.

3. The longitudinal electric field could reside at the outer and inner cylindrical walls of the structure. The ends of the annular cylinder could be also connected by thin hollow flux tubes carrying the electric flux parallel to the cylinder. The holes would be assigned to the first annulus and electrons with the second annulus.

4. The electrons could be dark. But can one say that the also hole created in the transition of a dark valence electron to the conduction band is dark. In ZEO, it could be natural to speak about a 4-D time classical evolution leading from a dark valence electron to dark conduction electron so that it makes sense to assign also to the hole the attribute "dark".

5. Debye length λ_D (`https://cutt.ly/QJMNHpq`) as a screening length for the electric field created by electrons gives an estimate for the length of the cylindrical structure. For water at room temperature, one has $\lambda_D = .7$ nm: intuitively, one would expect a considerably larger size scale for the voltage pulse.

   In semiconductors one has $L_D = \sqrt{\frac{\epsilon_r T}{q^2 N_{dop}}}$, where n_{dop} is the doping density. Note that the expression fof L_D has no explicit dependence of Planck constant.

   The generalization for dark particles would replace N_{dop} with the number n_{dark} of dark electrons per volume. The density of dark electrons is expected to be much smaller than atomic density and to scale roughly like $1/L_p^3$.

   One can argue that L_D corresponds to the p-adic length scale defining the length of the electric flux quantum and that there should be roughly 1 dark electron per flux quantum. $n_d 1/\mu m^3$, this gives $L_D \simeq \sqrt{1/n_d}\sqrt{T/300K} \times .45\ \mu$m. For 1 dark electron per volume defined by the p-adic length scale $L(167) = 2.5\ \mu$m assignable to the cell nucleus, one has $L_d = \sqrt{T/300K} \times 1.8\ \mu$m.

3 TQC in TGD

In the TGD counterpart of TQC [9, 8, 12] [31], metabolic energy feed would make quantum phases in long scales possible and the field/magnetic bodies (FBs/MBs) would form a master-slave hierarchy and one could expect that TQC acording to TGD could have a lot of common with CC.

1. The levels of the dark matter hierarchy below a given level could be modells as statistical ensembles much like electrons in an ordinary computer as seen from the level considered. Qubits could be also represented as ferromagnetic multispin systems analogous to ferromagnets.

   Quantum spin glasses are very natural systems in the TGD framework since the action principle reduces to Kähler action in long length scales and it has enormous 4-D vacuum degeneracy analogous to spin glass degeneracy. The TGD based inspired model in terms of flux tube spaghettis and the mathematical description involving p-adic thermodynamics is discussed in [23].

2. QC gives several outcomes in the SFR ending it. The desired outcome, say the minimal period of a function realized as entanglement between qubit registers, is only one of the outcomes, and one should be able to select the desired outcome. One can repeat the QC and find in this manner the shortest period.

   Could one imagine more elegant ways to find the desired outcome? For instance, could the valleys of the quantum spin glass energy landscape correspond to the possible answers of TQC? If this were the case, an annealing involving repeated heating and cooling of the system could lead to the desired answer. The answer of QC (say a minimal period of function) should correspond to the deepest valley. D-wave quantum computers (`https://cutt.ly/QJMNLP7`) rely on an approach, which involves annealing and spin glass ("D-wave" refers to the D-wave superconductors used in the original approach).

3.1 What could be the physical realization of the TQC program?

What could the physical realization of the TQC program in the TGD framework look like?

1. Flux tube network, which reduces as a special case to a braid system, is the key notion. The flux tubes connect nodes and the motion of the nodes give rise to a time-like braiding, which induces space-like braiding, which provides a topological memory representation of the TQC program.

   Anyon replaces qubit as spin state with group representation, which is in the simplest situation (bilocal states of condensed matter Majorana fermions with definite parity introduced by Kitaev) characterized by a parity-like quantum number representing qubit. In the TGD framework the bi-localization of the proton of dark hydrogen bond could give rise to this kind of anyons [31].

   In TGD, the counterparts of anyons would correspond to representations of Galois groups labelled by invariants of the Galois group, and there would be both cognitive measurement cascades using Galois representations for the relative Galois groups as counterparts of qubit and measurements of spins and possibly also momenta of electrons (many-quark states at fundamental level) for these representations

   The TQC program could physically correspond to a dynamical flow for the "liquid" formed by the nodes. There would be large number of realizations of the flow and the physical realizations should correspond to PEs and among them would be PE with minimal action. The braids realized as flux tubes correspond at the fundamental level 2-D string world sheets inside the flux tube orbits and there are many topologically equivalent realization associated with different preferred extremals (PEs).

2. The spin glass energy landscape is realized in terms of manyfermion staes for magnetic flux tube network depends on the detailed realization of a given braiding. Monopole flux tubes induce local

magnetization of the fermions parallel to the flux tube. In the optimal situation there would be 1-1 correspondence between the answers of TQC and the PEs. This would be essentially quantum-classical correspondence (QCC).

The topology preserving modifications of the flux tube network responsible for the TQC would modify the spin glass energy landscape. Could this allow to enhance the probability of the desired outcome of the TQC? It should correspond to the deepest valley: why? Should one engineer this correspondence?

3. p-Adic thermodynamics is a natural mathematical framework for describing the spin glass energy landscape [23]. Could the p-adic thermodynamics be engineered using topology preserving modifications of the braiding. Note that the braiding is determined by 2-D string world sheets so that the modification of the space-time sheets can be considered.

3.2 Quantum analog of annealing

In thermodynamics, the minimization of free energy combined with annealing could lead to the bottom of the deepest valley of the spin glass energy landscape.

In quantum TGD, one can consider two options for the annealing based on the analog of thermodynamics. Free energy could be replaced by the fundamental action or alternative energy could be replaced by a scaling generator L_0 of conformal symmetries. p-Adic thermodynamics is indeed highly suggestive as a description of spin glasses [23].

Consider first the approach based on fundamental action.

1. The outputs from the TQC program should correspond to the PEs with given initial values. One should have a variational principle assigning the desired answer of QC with a minimum the action for a preferred extremal (PEs) representing the deepest valley of the spin glass landscape.

   The exponent of the fundamental action defines the vacuum functional as the sum of Kähler action and volume term. PE is simultaneous extermal of both volume term and Kähler action apart from singularities with various dimensions $d < 4$ defining analogs of frames for a soap film. There is a finite non-determinism associated with the frames present already for the ordinary soap films.

   PE property for the space-time surface realizes almost complete holography and almost complete determinism. The possible answers as PEs could correspond to topologically equivalent braidings realized as space-time surfaces. The action is in general different for these PEs. The PE with a minimal action would have the highest probability proportional the exponent of the action.

2. The braiding corresponds at a fundamental level to string world sheets. Since the string worlds sheets defining the braiding can be associated with a large class of PEs, one can think that the PE could be engineered in such a manner that the desired configuration is strongly favored.

   Only its coupling parameters of the fundamental action that depend on the extension of rationals considered can be varied. This is achieved by varying the polynomial P determining the space-time region considered. The topology of the braiding and also the Galois group should remain unaffected.

   There is a large number of rational polynomials with the same Galois group and extension of rationals so that the engineering could correspond to the variation of the coefficients of P affecting ramified primes. Even the degree of the polynomial could be changed.

   It is however not all clear how one could modify the polynomial in a controlled manner.

It is far from obvious whether it is possible to add to the exponent of the vacuum functional defined by the fundamental action an additional engineerable exponential factor. A more promising approach is based on the engineering of entanglement between fermion states and anyons as Galois representations.

3.2.1 p-Adic thermodynamics as a tool of quantum annealing

Since cognitive measurement cascades for the representations of Galois group are in question, the entanglement engineering would naturally rely on p-adic thermodynamics using the analog of Hamiltonian, whose eigenvalues are analogs of p-adic conformal weights h distinguishing between different outcomes of SFR ending the TQC.

1. The p-adic prime p associated with the engineered p-adic thermodynamics would naturally correspond to the maximal ramified prime of the extension appearing as a factor the discriminant D given as the square $\prod_{i<j}(r_i - r_j)^2$, where $r_i - r_j$ difference of the roots. p-Adic temperature T_p, whose values come as inverse integers, when $log(p)$ is used as a unit, is natural in the modelling of the spin glass energy landscape [23] and $T_p = 1/n$ could serve as the counterpart of temperature varied in the annealing procedure.

2. The thermodynamics would be for the scaling generator L_0 associated with conformal invariance. Although the physical states are are annihilated by L_0, entangled states can have non-vanishing thermal expectation for the entangled factor because tachyonic states are predicted as analogs of virtual particles having roots of polynomials as mass squared values: the real parts of the roots can indeed be negative.

3. Physical states, which satisfy Galois confinement and have vanishing mass squared, consist of virtual quarks (in the simplest scenario in which leptons are 3-quark composites).

   Massless Galois confined states are in general entangled states such that the total momentum is light-like momentum with integer valued components. This is possible because the values of mass squared (conformal weights) for quarks as roots of polynomial P are in general algebraic numbers and can have negative real parts (tachyonicity). In particular, Galois singlets can also have a negative mass squared.

   The total mass squared would be for each pair appearing in the entangled state equal to zero and p-adic thermodynamics would apply to quarks with positive mass squared with Virasoro generator representing the mass squared [34].

4. At the level of H, tachyonic momenta can be assigned with the wormhole contacts having Euclidean signature of the induced metric and associated with elementary particles.

   The twistor lift of TGD requires M^4 to possess the analog of Kähler structure [16, 26, 27]. The massless solution for Dirac equation in H for the second chirality of H-spinor allows a covariantly constant right handed neutrino as a massless solution, which becomes a tachyon as one adds a coupling to the Kähler gauge potential of M^4 required by the twistor lift. Right-handed neutrino could be elementary or could correspond to 3-quark composite with quarks at the same wormhole throat or possibly in the interior of the wormhole contact.

5. In the simplest model in which momenta define Galois representation, the total momentum of Galois singlet would have integer valued components as a sum of quark momenta with algebraic integer valued momentum components. More general representations involve wave functions in momentum and spin degrees of freedom. Even more, by conformal invariance, the states have a vanishing mass squared [34] so that they have vanishing conformal weights as eigenstates of L_0.

6. The basic step of quantum annealing would be p-adic heating increasing the quantized p-adic temperature $T_p = 1/n$ followed by cooling. p-Adic heating would induce entanglement between fermionic states and irreps of the relative Galois group describable in terms of p-adic thermodynamics with an increased temperature T_p. Thermodynamics would therefore be an essential part of TQC in the TGD framework.

3.2.2 An objection against the identification of Galois representations as analogs of anyons

Anyons should correspond to Galois non-singlet representations. The problem is that the representations of the relative Galois groups should be Galois singlets by Galois confinement.

The solution of the problem is analogous to the solution of the problem posed by the basic objection against p-adic thermodynamics. Galois singlets are constructed by entangled pairs of Galois representations in the fermionic momentum and spin degrees of freedom.

Cognitive measurement cascade can take place either in momentum or spin degrees of freedom and select from a superposition of paired representations one particular representation a pair fusing to a Galois singlet. The reduction probabilities are analogous to thermodynamic Boltzmann weights and one can have analog of p-adic thermodynamics with reduction probabilities proportional to non-positive powers of p.

3.3 How the TQC program could be engineered?

In QC there are several alternative outputs (say the multiples of a minimal period of function). QC could be repeated several times to obtain the desired answer. Could one end up with the desired answer by some other method. Could some kind of engineering make this possible?

One can imagine two approaches to the engineering.

3.3.1 Could TQC program involve engineering of preferred extremal?

One can consider two options for what happens as the TQC program runs.

1. The value of the action exponential differs for the different outcomes of the SFR. In this case, the superposition of different outcomes of SFR corresponds to a superposition over different space-time surfaces defining topologically equivalent braidings and SFR selects one of them.

2. An alternative option is that there is only a single space-time surface involved and the fermionic entanglement probabilities between the spin and fermionic degrees of freedom depend on the fermionic state only. Now SFR takes place in the fermionic degrees of freedom and the TQC programmer must engineer the fermionic entanglement.

For the first option, the fundamental action exponential, vacuum functional, as a counterpart of Boltzmann weight, should be maximal for the desired outcome of SFR.

The engineering should select the polynomial determining the space-time surface such that the desired outcome would be achieved.

1. PE as a minimal surface with singularities is analogous to a soap film having frames as singularities and would be the TGD counterpart for a state at the bottom of the valley. The action would depend on the valley and quantum annealing, whatever it could mean, would take the system to the deepest valley.

2. One should identify the quantum counterpart of temperature and Kähler coupling strength is the first guess. If Kähler coupling strength is determined by the extension of rationals, annealing would involve modifications of the polynomial determining the space-time surface. The topology of the braiding must remain unaffected and string world sheets defining the braiding would define part of the holographic data.

3. This option does not look promising in the recent case since it is difficult to imagine how the engineering of the action by modifying the coefficients of P could be possible and whether this engineering could select the desired outcome of SFR as the most probable outcome.

   The physical modification of action would be based on the modification of braiding, which would preserve its topology by modifying the "hydrodynamic" flow defined by the nodes.

3.3.2 Could the entanglement between Galois representations and fermion spins be engineered?

An attractive option is that Nature performs cognitive state function reduction cascade and entanglement engineering guarantees the most probable outcome of TQC for a given relative Galois group. This requires engineering of the entanglement between Galois irreps and some other degrees of freedom. Is there any way to achieve this?

Topological qubits correspond to irreps of a given relative Galois group. One should assign to each irrep a fermionic state. If the irrep is somehow realized, the structure of this state does not matter: what is only required is that it distinguishes between different irreps and the entanglement probability is largest for the desired outcome of SFR. For instance, magnetized many-fermion states with a fixed or slowly varying spin direction could be considered.

Galois confinement however poses a strong additional condition. The irrep must be entangled with a representation of the Galois group such that the outcome is Galois singlet! This suggests that one has two irreps of Galois: the first one in the fermionic spin degrees of freedom and the second one in fermionic momentum degrees of freedom identifiable asd discretized geometric degrees of freedom. These irreps entangle to a Galois singlet and one has a superposition over pairs of irreps of this kind.

1. The entanglement between qubit registers defines a map between integers defined by the qubits sequences of the registers. Should one introduce besides the topological qubit register an additional qubit register entangled with it in such a manner that the desired outcome of TQC corresponds to the most probable outcome of the cognitive SFR cascade?

   One should engineer the entanglement between the registers. The simplest entanglement would be maximal entanglement determined by phase factors in the diagonal representation. This entanglement is determined apart from permutation. The qubits would naturally correspond to quark spins at the fundamental level. At higher level electronic spins would be in question.

   This entanglement would not distinguish between the representation of the Galois group unless the value of the fundamental action correlates with the representations. A more plausible option is that it does not and that the entangled is engineered in such a manner that the reduction probability is largest for the desired outcome of TQC.

2. Galois confinement forces entanglement of the measured system with another system since topological qubits as anyons are generalized to representations of Galois group. Could the quark spin degrees of freedom entangle in 1-1 manner with the bosonic number theoretic degrees of freedom assignable to the orbits of the Galois group?

   In M^8, the orbits of the Galois group correspond to quark momenta with algebraic integers as components and Galois acts also in spin degrees of freedom of quarks. Could the entangled degrees of freedom correspond to fermionic spin and momentum degrees of freedom? Could one think of enhancing the probability of the desired outcome by adding an interaction exponential as a p-adic analog of the exponent of free energy?

3.4 How to engineer the entanglement between many-fermion states and irreps of relative Galois groups?

The many-fermion states and the representations of the relative Galois group as analogs of anyons, would entangle with fermionic spin states. This entanglement would associate to a given geometric irrep of the relative Galois group, realized in the momentum space of fermions, a many-fermion state characterized by fermion spins and entanglement coefficients between fermions.

1. The number theoretical SFR cascade would begin from a state which a the decomposition of irrep of Galois group to a quantum superposition of products of irreps of Galois group to products of irreps of relative Galois groups.

SFR cascade would decompose representation to a product of representations of the relative Galois groups and for the representations as superpositions of irreps would select a particular irrep for each relative Galois group and assign to it a many-fermion state. This many-fermion state would correspond to a valley of spin glass energy landscape. It will be assumed that the space-time surface does not depend on the fermionic states involved.

The projection to an irrep of the relative Galois group induces the selection of a particular outcome. Basically, the number theoretical invariants associated with a particular Galois representation must be measured. Whether Nature does this measurement or whether this measurement must be engineered, is not quite clear.

3.4.1 Fermionic Galois representations

The momentum and spin degrees of freedom of quarks provide the fundamental Galois degrees of freedom.

1. At the level of M^8, the fermionic representation could be constructed as wave functions in the momentum and spin degrees of freedom (also iso-spin degrees of freedom at the quark level). These wave functions are more general than the "classical" many quark states for which the sum of the momentum components as algebraic integers is equal to an integer valued total momentum. For instance, a single quark can be in an analog of s-wave defined as superposition of states at the orbit of the Galois group.

   Homogeneous polynomials of the quark momenta analogous to spherical harmonics would be involved besides spin wave function in which the Galois group should act. Since finite simple groups are in question, the number of irreps is finite. Given irrep can however appear several times.

2. If the number of irreps considered for a given relative Galois group is N, the number of entanglements with a given set of fermion states is the number of permutations $N!$ for N objects. In the travelling salesman problem (`https://cutt.ly/AJMNCbC`) with N cities, the number of ways to visit once in every city and return to the starting city is $N!$. This problem is NP hard.

3. The travelling salesman problem has as special solutions Hamiltonian cycles for which each node has at least one nearest neighbor with a given minimum distance. Each edge of the cycle connects the point of the graph to one of its nearest points so that the path has minimum length and solves the travelling salesman problem. The TGD based model for bioharmony relies on fusion of three icosahedral Hamiltonian cycles with 12 vertices (notes of 12-note scale) and tetrahedral cycle [10] [17, 20, 22]. In this case the notes are scaled by factor 3/2 at each step (quint cycle) for Pythagorean scale. For equal tempered scale the scaling is $2^{7/2)}$.

   For Hamiltonian cycles associated with Platonic solids, one can also consider an ultrametric distance for the points of a given path, not necessarily Hamiltonian cycle. This distance would defined to be the largest scaling as power of x ($x = 3/2$ for icosahedral Hamiltonian cycles) along the path connecting two nodes. This ultmetric distance would be x between all points of the Hamiltonian cycle. p-Adic primes 3 or 2 would be naturally associated with the bioharmony.

The cognitive measurement cascade could therefore have a connection with two problems of computer science.

1. The factorization of the Galois group to prime (simple) factors defined by the relative Galois groups is analogous to the prime factorization of integers.

2. The entanglement between Galois representations and many-fermion states could relate to a solution travelling salesman problem as the path of minimum length connecting all cities.

3.4.2 Travelling salesman problem, entanglement engineering and quantum annealing

The travelling salesman problem for which D-wave quantum computers are proposed as a solution (`https://cutt.ly/QJMNLP7`) suggests a formulation of p-adic thermodynamics allowing to find the desired outcome of SFR by quantum annealing.

1. The path length should appear as an argument in the function to be minimized in the travelling salesman problem. The input data are defined by the distances $d(i,j)$ between the cities. Suppose that the fundamental action is the same for all space-time surfaces considered in QC. In particular, the fundamental action would not depend on the fermionic state paired with the representation of the relative Galois group. The simplest situation is that one has just a single PE.

2. Entanglement engineering would mean that one assigns to a given permutation representing a possible route as the sum of the dimensionless positive integer valued distances $d_{P(i),P(i+1)}$ between subsequent cities in their permutation. The simplest entanglement coefficient defining the reduction probability in the real context is Boltzmann weight $exp(-\sum_i d_{P(i),P(i+1)}/T)$, where T is a real parameter.

   The counterpart of this Boltzmann weight in the p-adic thermodynamics is $p^{\sum_i d_{P(i),P(i+1)}/T_p}$. Here $T_p = 1/n$ is the quantized p-adic temperature using $log(p)$ as a unit. Travelling salesman program is hard since the minima form an analog of spin glass energy landscape. The annealing by varying the value of $T_p = 1/n$ regenerating the entanglement could lead to the deepest valley.

3. The exponent of Boltzman weight could be also seen as a number theoretical analog of a Casimir operator, whose measurement would select the relative Galois representation in the second tensor factor. This kind of operator should have an integer valued spectrum and could define p-adic thermodynamics. The maximal Abelian subgroup of the Galois group could define the observable analogous to free energy.

   The time evolution of spin glass is such that the magnetic relaxation obeys power law rather than exponential law. This suggests that the time evolution for spin glass could correspond to scaling rather than time translation. Therefore the interpretation of the analog of free energy could be in terms of scaling interaction Hamiltonian.

4. This approach could apply to all problems, which can be transformed to the travelling salesman problem. Note that the very large number of problems with varying distances between cities allows the same path as a solution. This would make it possible to transform the problem to a problem in which the distances are integer valued.

3.5 Some delicacies related to the Galois group

The considerations above represent only a general vision and reflect my rather amateurish understanding of number theory.

3.5.1 The isotropy group of the Galois group leaving root fixed

One important point, which is not mentioned above, is that since the action of the Galois group permutes the roots of the polynomial P identified as mass squared values, it does not commute with Lorentz and Poincare transformations. This is one excellent motivation for Galois singlet property of the physical states. The entire Galois group would act along time-like braids and in ZEO, where space-time surfaces are fundamental objects and a small failure of the classical determinism takes place, this would make sense.

For a given root, one can identify its isotropy group as the subgroup of the Galois group leaving the root invariant. The isotropy subgroup respects the value of mass squared and therefore can appear as a physical symmetry group.

For a polynomial of degree n with maximal Galois group S_n, this group is S_{n-1}. For $n = 5$ with A_5 with order 60 as Galois group acting at icosahedron, this group is A_4 with 12 elements acting at tetrahedron. Intriguingly, both groups appear in the model of bioharmony and genetic code [20].

3.5.2 Relationship with Higgs mechanism

Polynomials P have two kinds of solutions depending on whether their roots determine either mass or energy shells. For the energy option a space-time region corresponds by $M^8 - H$ duality to a solution spectrum in which the roots correspond to energies rather than mass squared values and light-cone proper time is replaced with linear Minkoski time [18, 19]. The physical interpretation of the energy shell option has remained unclear.

The energy shell option gives rise to a p-adic variant of the ordinary thermodynamics and requires integer quantization of energy. This option is natural for massless states since scalings leave the mass shell invariant in this case. Scaling invariance and conformal invariance are not violated.

One can wonder what the role of these massless virtual quark states in TQC could be. A good guess is that the two options correspond to phases with broken *resp.* unbroken conformal symmetry. In gauge theories to phases with broken and unbroken gauge symmetries. The breaking of gauge symmetry indeed induces breaking of conformal symmetry and its breaking is more fundamental.

1. Particle massivation corresponds in gauge theories to symmetry breaking caused by the generation of the Higgs vacuum expectation value. Gauge symmetry breaking induces a breaking of conformal symmetry and particle massivation. In the TGD framework, the generation of entanglement between members of state pairs such that members having opposite values of mass squared determined as roots of polynomial P in the most general case, leads to a breaking of conformal symmetry for each tensor factor and the description in terms of p-adic thermodynamics gives thermal mass squared.

2. What about the situation when energy, instead of mass squared, comes as a root of P. Also now one can construct physical states from massless virtual quarks with energies coming as algebraic integers. Total energies would be ordinary integers. This gives massless entangled states, if the rational integer parts of 4-momenta are parallel. This brings in mind a standard twistor approach with parallel light-like momenta for on-mass shell states. Now however the virtual states can have transversal momentum components which are algebraic numbers (possibly complex) but sum up to zero.

   Quantum entangled states would be superpositions over state pairs with parallel massless momenta. Massless extremals (topological light rays) are natural classical space-time correlates for them [11, 5]. This phase would correspond to the phase with unbroken conformal symmetry.

3. One can also assign a symmetry breaking to the thermodynamic massivation. For the energy option, the entire Galois group appears as symmetry of the mass shell whereas for the mass squared option only the isotropy group does so. Therefore there is a symmetry breaking of the full Galois symmetry to the symmetry defined by the isotropy group. In a loose sense, the real valued argument of P serves as a counterpart of Higgs field.

In the maximally symmetric scenario, conformal symmetry breaking would be only apparent, and due to the necessity to restrict to non-tachyonic subsystems using p-adic thermodynamics.

Received June 28, 2022; Accepted January 14, 2023

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