Article

Bulk Viscous Bianchi Type-I Cosmological Models With Constant Deceleration Parameter

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Abstract

By assuming that the deceleration parameter is constant, Bianchi type-I cosmological models with a variable cosmological term A in the Einstein field equation are taken into consideration. According to what we discovered, the cosmological constant decreases with time. Additionally mentioned were physical and kinematical behaviour.

Keywords: Bulk viscosity, cosmological term, deceleration parameter.

1. Introduction

Recently, it has been discovered that adding viscosity to the cosmic fluid composition can help explain a number of significant physical characteristics of the dynamics of homogeneous cosmological models. The dissipative mechanisms can successfully explain the high entropy per baryon in the current universe in addition to changing the nature of the singularity, which typically occurs for a perfect fluid. The significance of analysing the dissipative effects in cosmology is revealed by the observed physical facts, such as the high entropy per baryon and the astounding degree of isotropy of cosmic microwave background radiation (CMBR).The physical processes such as decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era are expected to give rise to viscous effects. Moreover, according to the Grand Unified Theory (GUT), the phase transition and string creation are also believed to involve viscous cosmological models.

Dissipative forces, such as bulk and shear viscosity, were very important in the early stages of the universe's evolution. By Eckart [1] and Landan and Lifshitz [2], the first theories of relativistic dissipative fluids were attempted. By deriving universal formulas for bulk and shear viscosity, Weinberg [3] was able to assess the pace at which cosmic entropy is produced. Many researchers have looked into how viscosity has changed throughout the universe's history. By accounting for the dissipative process caused by viscosity, Bilinski and Khalatnikov [4] investigated the nature of cosmological solutions for the homogeneous Bianchi type-I model.

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They showed that viscosity cannot remove cosmological singularity but can cause a qualitatively new behaviour of the solutions near singularity. Bianchi type-I solutions in the case of stiff matter with shear viscosity being the power function of energy density were obtained by Banerjee et al. [5] whereas models with bulk viscosity as a power function of energy density and stiff matter were investigate by Huang [6]. The effect of bulk viscosity with a time varying bulk viscous coefficient on the evolution of isotropic FRW models was studied in the context of open thermodynamic system by Desikan [7]. By Krori and Mukherjee [8] for anisotropic Bianchi models, this study was further developed. Chimento et al. [9] discovered cosmological solutions with nonlinear bulk viscosity. Gavrilovet al.[10] have investigated models with both shear and bulk viscosities. Gron [11] has studied the research on viscous cosmological models. Inflationary cosmological models of Bianchi type-I with shear and nonlinear bulk viscosities have been studied. Models with dynamic cosmological term Λ (t) are becoming popular as they solve the cosmological constant problem in a natural way.

There are significant observational evidence for the detection of Einstein's cosmological constant Λ or a component of material content of the universe that varies slowly with time and space to act like Λ. Recent Cosmological Project [12-15] suggest the existence of a positive cosmological term Λ with magnitude $\Lambda\ (Gh / c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type I_a supernova suggest that our universe may be an accelerating function of the cosmological density in the form of the cosmological term Λ. Zeldovich [16], Weinberg [17], Dolgov [18], Betrolomi [19], Ratra and Peebles [20] are a few authors who have written extensively on this subject. Numerous dynamical theories have been put out to explain the density decay, where the cosmological constant fluctuates with cosmic time t. These theories produce an actual cosmological constant that, as the universe expands, decays from a large value at first to the small value that is currently seen. In the past two decades, researchers have examined cosmological models with various decay laws to account for cosmic term variation [21–22].

2. Metric & Field Equation

The line-element describes the spatially homogenous and anisotropic Bianchi type-I space time

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \tag{1}$$

The Einstein's field equations with time dependent Λ are

$$R_i^j - \frac{1}{2}R_k^k g_i^j = -T_i^j + \Lambda g_i^j \tag{2}$$

where energy momentum tensor T_i^j in the presence of bulk viscosity is taken in the form

$$T_i^j = (\rho + \overline{p})v_i v_j + \overline{p} g_i^j \tag{3}$$

where

$$\bar{p} = p - \xi\theta . \tag{4}$$

We assume that the matter content obeys an equation of state

$$p=\omega\rho, \qquad 0\le\omega\le 1. \tag{5}$$

Here ρ, p, ξ and θ are the energy density, isotropic pressure, bulk viscosity and expansion scalar respectively. The flow vector v^i satisfies the condition

$$v_i v^i = -1. \tag{6}$$

In co-moving system of coordinates

$$T_1^1 = \bar{p},\ \ T_2^2 = \bar{p},\ T_3^3 = \bar{p},\ \ T_4^4 = \rho .$$

Then field equations are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\bar{p} + \Lambda , \tag{7}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\bar{p} + \Lambda , \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\bar{p} + \Lambda , \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho + \Lambda . \tag{10}$$

From equations (7)-(10), we obtained

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 0 . \tag{11}$$

Eliminating $\bar{p}$ and Λ from (7)-(9) and integrating, we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC} , \tag{12}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{ABC} , \tag{13}$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_1 + k_2}{ABC} . \tag{14}$$

where k_1 and k_2 are constant of integrations.

We define the average scale factor S by

$$S^3 = ABC .$$

From equations (12)-(14), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{2k_1 + k_2}{3S^3}, \tag{15}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{k_2 - k_1}{3S^3}, \tag{16}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{k_1 + 2k_2}{3S^3} . \tag{17}$$

Integrating it, we get

$$A = m_1 S \exp\left[\left(\frac{2k_1 + k_2}{3}\right)\int \frac{dt}{S^3}\right], \tag{18}$$

$$B = m_2 S \exp\left[\left(\frac{k_2 - k_1}{3}\right)\int \frac{dt}{S^3}\right], \tag{19}$$

$$C = m_3 S \exp\left[-\left(\frac{k_1 + 2k_2}{3}\right)\int \frac{dt}{S^3}\right]. \tag{20}$$

Where m_1, m_2, m_3 are constant of integration .

We introduce volume expansion θ and σ as usual

$$\theta = v_i^i \qquad \sigma^2 = \frac{1}{2}\left(\sigma_{ij}, \sigma^{ij}\right),$$

σ_{ij} being shear tensor.

In the above the semicolon stands for covariant differentiation. For the Bianchi type-I metric expression for the dynamical scalar come out to be

$$\theta = 3\frac{\dot{S}}{S}, \tag{21}$$

$$\sigma = \frac{k}{\sqrt{3}S^3} . \tag{22}$$

Here $k^2 = k_1^2 + k_2^2 + k_3^2$ In analogy with FRW universe, we define a generalized, Hubble parameter H and the generalized deceleration parameter q as

$$H = \frac{\dot{S}}{S}, \tag{23}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 - \frac{\dot{H}}{H^2}. \tag{24}$$

Equations (7) – (10) can be written in terms of *R*, σ and q as

$$\overline{p} = H^2(2q-1) - \sigma^2 + \Lambda, \tag{25}$$

$$\rho = 3H^2 - \sigma^2 - \Lambda. \tag{26}$$

From (26), we observe that $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Therefore a positive Λ restricts the upper limit of anisotropy whereas a negative more room the anisotropy.

3. Solution & Discussion

The equation (5), (7) and (10) are five equation involving seven unknown term A, B, C, p, ρ, Λ, ζ so in order to close the system, we need two extra condition.

We assume the form of deceleration parameter q given by

$$q = \frac{-S\ddot{S}}{\dot{S}^2} = n \,(\text{Constant}) \tag{27}$$

yields

$$S = [S_0(t+t_0)]^{\frac{1}{1+n}} = (S_0 T)^{\frac{1}{1+n}}. \tag{28}$$

Where S_0 and t_0 are constant at integration and $T = t_0 + t$.

Coefficient of bulk viscosity, ζ in the form

$$\zeta = \zeta_0 \theta^m \tag{29}$$

Using (28) in (18) – (20), we get following expression for the scale factor

$$A = m_1 (S_0 T)^{\frac{1}{1+n}} \exp\left[\frac{2k_1 + k_2}{2}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] \tag{30}$$

$$B = m_2 (S_0 T)^{\frac{1}{1+n}} \exp\left[\frac{k_2 - k_1}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] \tag{31}$$

$$C = m_2 (S_0 T)^{\frac{1}{1+n}} \exp\left[\frac{-k_1 - 2k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] \tag{32}$$

By the transformation, $T = t_0 + t,\ m_1 x = X,\ m_2 y = Y,\ m_3 = z$ the matric (1) reduces

$$ds^2 = dT^2 + (S_0 T)^{\frac{2}{1+n}} \exp 2\left[\frac{2k_1 + k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] dX^2 + (S_0 T)^{\frac{2}{n+1}} \exp 2\left[\frac{k_1 - k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] dY^2$$
$$+ (S_0 T)^{\frac{2}{1+n}} \exp 2\left[\frac{-k_1 - k_2}{3}\left(\frac{n+1}{n-2}\right)(S_0 T)^{\frac{n-2}{n+1}}\right] dZ^2 \tag{33}$$

For model (33), expansion scalar θ, special volume V, shear scalar σ

$$\theta = \frac{3}{(1+n)\ T} \tag{34}$$

$$V = (S_0 T)^{\frac{3}{1+n}} \tag{35}$$

$$\sigma = \frac{k}{\sqrt{3}(S_0 T)^{\frac{3}{1+n}}} \tag{36}$$

Matter density ρ, cosmological term Λ, for the model take the form

$$(1+\omega)\rho = \frac{2}{(1+n)T^2} - \frac{2k^2}{3(S_0 T)^{\frac{6}{(1+n)}}} + \frac{3^{m+1}\zeta_0}{(1+n)^{m+1} T^{m+1}} \tag{37}$$

$$\Lambda = \frac{3\omega - 2n + 1}{(1+\omega)(1+n)^2 (T)^2} - \left(\frac{\omega - 1}{1+\omega}\right)\frac{k^2}{3(S_0 T)^{\frac{6}{(1+n)}}} - \frac{3^{m+1}\zeta_0}{(1+n)^{m+1} T^{m+1}} \tag{38}$$

The model (33), which indicates that the universe begins evolving with zero volume at T = 0 and an unlimited rate of expansion, states that the spatial volume V is zero at T = 0 while the expansion scalar θ is infinite. At T = 0, the scale factor likewise disappears, leading to a point-type singularity in the model at the initial epoch. When T → 0 then σ→∞, Λ→∞, ζ→∞ . It is interesting to note that the deceleration parameter are constant throughout the evolution of universe. As *T*→∞ the scale factors and volume of the universe become infinitely large whereas anisotropy parameter, cosmological term Λ, matter energy density ρ and shear scalar tend to zero.

4. Conclusions

The spatially homogeneous and anisotropic Bianchi type I space-time with bulk viscous matter and time-varying cosmological term has been examined in this study using general relativity. With the help of a constant deceleration parameter, the field equations have been precisely solved. The model of the universe has been obtained, and its physical behaviour is explored. We find that the value of matter density rises when bulk viscosity is present. For the models discovered, $\sigma / \theta \to 0$ as $T \to \infty$. As a result, the models eventually become close to isotropy.

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